AOE 4134 Problem Sheet Six

Due October 9, 2002

You should have read the following by now:
  Chapter 1, Chapter 3, Chapter 8
You can also read Chapter 6
We now will clean up some loose ends:

30. Show:

\[ e \sin v = \frac{r V^2}{\mu} \sin \phi \cos \phi \quad \text{and} \quad e \cos v = \frac{r V^2}{\mu} \cos^2 \phi - 1 \]

and hence

\[ e^2 = \left( \frac{r V^2}{\mu} - 1 \right)^2 \cos^2 \phi + \sin^2 \phi \]

and

\[ \tan v = \frac{r V^2}{\mu} \frac{\sin \phi \cos \phi}{\cos^2 \phi - 1} \]

These are convenient to use for lots of situations!

31. A space vehicle destined for Mars was first launched into a 200 km circular parking orbit about the Earth. After coasting in the parking orbit for several orbits, the engines were restarted and an increase of velocity of 12 km/s was obtained which placed the vehicle on an interplanetary orbit toward Mars. Determine:

a) The speed in the parking orbit (km/s)

b) The energy in the orbit after the interplanetary insertion burn.

c) Angular momentum of the Earth escape orbit (km^2/s)

d) Eccentricity of the Earth escape orbit

e) Hyperbolic excess speed (km/s)

32. It is desired to send a probe to Venus. Venus’s orbit about the Sun is inclined 3.394 degrees with respect to the Earth’s orbit about the Sun and is assumed circular @ 0.7233 AU. Determine the following:

A) co-planar orbits

a) Assuming the orbits are in the same plane, determine the \( \Delta V \) to leave the Earth’s orbit about the Sun and enter a Hohmann transfer orbit to Venus (\( \Delta V_1 \)). (in AU/TU)

b) With the same assumption as in (a) determine the circularizing burn at Venus to put the probe in Venus’ orbit about the Sun.

c) For parts (a) and (b) determine the total \( \Delta V \) for the mission.

B) non-coplanar orbits

d) Assume now that we will do a plane change maneuver first, and then do the Hohmann burn. Calculate the total of the plane change plus Hohmann insertion burn

e) Calculate the total \( \Delta V \) required for the mission in this (starting like (d)) mode

f) Consider combining the plane change and the Hohmann insertion burns into one burn. Determine the \( \Delta V \) required for the single burn, and compare it with the two-burn method of (d)

g) Consider doing the transfer using an in-Earth-plane transfer and then a circularizing burn and then a plane change burn. Determine the \( \Delta V \) for each and their total. Also determine the total \( \Delta V \) for the mission and compare with the results of parts (c) and (e).

h) Finally, calculate the total \( \Delta V \) for the mission for the case where the Hohmann insertion burn is done at Earth’s orbit, and a single burn is done at Venus to circularize and plane change. Compare with total \( \Delta V \) of (c), (e), (g). Which is best? Note that the total transfer time is the same!