Q1

a. Speed of sound (assume R=287 J/KgK and gamma=1.4)

use 1976 Standard atmosphere

\[ a = \sqrt{\gamma RT} \]  \hspace{1cm} (1)

Station 1.
temperature: \( T = 196.650 \) K
speed of sound: \( a = 281.120 \) m/s

Station 2. NA

Station 3.
temperature: \( T = 237.050 \) K
speed of sound: \( a = 308.649 \) m/s

b. Flight Mach number

\( a = 281.120 \) m/s
\( v = 4000 \) km/h
\( M = \frac{v}{a} = 3.95 \)

c. Vehicle velocity

\( M = 7.6 \)
\( a = 308.649 \) m/s
\( v = Ma = 2345.7324 \) m/s therefore it takes about 15s to reach ground level/crash landing

d. Flow regimes

- Supersonic
- No regime (outer space, no continuum flow)
- Hypersonic

e. Stagnation properties

\[ \frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}} \]  \hspace{1cm} (2)

\[ \frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \]  \hspace{1cm} (3)

Station 1:
pressure: \( p/p_0= 0.00704166, \ p= 0.886280 \) Pa, \( p_0=125.862367 \) Pa

temperature: \( T/T_0= 0.24268899, \ T=196.650 \) K, \( T_0= 810.296338 \) K

Station 3:
pressure: \( p/p_0=0.00014272, \ p=558.924 \) Pa, \( p_0= 3.91622e+006 \) Pa

temperature: \( T/T_0= 0.07966857, \ T= 237.050 \) K, \( T_0= 2975.45192 \) K
Q2.

(a) When the flow is subsonic, just choked, or at the design condition.
(b) It accelerates
(c) M=1 at the throat
(d) Just before the shock
3. \( M_0 = 0.3 \quad p = 900 \text{mBar} = 90 \text{kPa} \)
   \[ T = 5^\circ C = 278.2 \text{K} \]

(a) **Plane sees a free stream of** \( M = 0.3 \) \( \frac{p_0}{p} = 0.9895, \frac{T_0}{T_0} = 0.9823 \)

so plane experiences a free stream stagnation pressure of
\[ p_0 = \frac{90 \text{kPa}}{0.9895} = 91.8 \text{kPa} \text{ or } 91,800 \text{Pa} \]

(b) The stationary observer sees a free stream Mach number of zero, and thus a free stream stagnation pressure of
\[ p_0 = 90,000 \text{Pa} \]

(c) **The answer to (a) is more useful since only in this frame of reference does the flow appear steady and is the energy equation, \( p_0 = \text{const in isentropic flow} \), valid.**

(d) At \( M = 1.1 \)
\[ \frac{p_1}{p_0} = 0.4684, \quad \frac{T_1}{T_0} = 0.8052 \]

so \( p = 0.4684 \times 91.8 \text{kPa} = 44.9 \text{kPa} \)

and \( T_1 = 0.8052 \times 278.2 / 0.9823 = 228.0 \text{K} \)

(e) **Assuming the flow remains adiabatic then**
\[ C_p (T_2 + \frac{u_2^2}{2}) = C_p (T_1 + \frac{u_1^2}{2}) \]
\[ T_2 = T_1 + \frac{1}{2C_p}(v_1^2 - v_2^2) \]

Note: \( v_1 = \alpha v_1 \times M_i = \sqrt{\frac{8R T_1}{M_i}} \times M_i \)
\[ = \sqrt{1.4 \times 287 \times 228} \times 1.1 \]
\[ = 332.9 \text{ m/s} \]

\[ T_2 = 228 + \frac{1}{2 \times 1005} \left( 332.9^2 - 150^2 \right) \]
\[ \Rightarrow T_2 = 271.9 \text{ K} \]
Q4.

(a) The ideal flows of AOE 3014 are adiabatic (since they involved no heat addition) and reversible (since they involved no dissipative effects like viscosity), therefore they would also be isentropic.

(b) A sound wave is also adiabatic (again no heat added) and reversible (since, as discussed in class, it is a very small disturbance that is not significantly influenced by viscosity or heat conduction), and therefore is isentropic.

(c) A shock wave involves no heat addition but it is very thin and intense so dissipative effects including viscosity and heat conduction are important. Thus it is neither isentropic nor reversible.

(d) A sonic line is just the line in a flow where the Mach number passes through 1. No thermodynamic processes occur within the line and it is not a flow feature (just a line we draw on to show the division between sub- and supersonic regions). One could therefore argue either that this question doesn’t apply to a sonic line, or that since no thermodynamic process occurs there the entropy won’t change so it is isentropic and thus reversible and adiabatic.

(e) The boundary layer is not reversible (because it is a dissipative viscous flow) and is losing heat to the space shuttle hull, and thus is not adiabatic either.

(f) Fuel will be added and burnt in the combustor, and thus the flow won’t be adiabatic. One can sometimes model this process as a reversible one, though in practice burning is obviously irreversible (and will usually be accompanied by other irreversible effects like mixing). Both reversible and irreversible are accepted as correct answers on this part.