AOE 3114 Homework 2

Due Date: 02/09/2017

Note: One problem from this homework (to be chosen at random) will be graded for 70% of the credit on this homework. The three remaining questions will be graded for completeness only, with each worth 10%. Students must prepare their homework solutions individually, except as noted below for the ‘recitation problem’. The honor code applies and is strictly enforced. If you are having difficulty, we are glad to give advice on how to approach the homework problems during office hours and recitations.

Recitation Problem. Students will be encouraged to collaborate in small groups during recitation to solve the ‘recitation problem’ (Q1). For this activity to be productive for will need to study this problem before coming to recitation and be prepared to share and discuss your approach with those of your classmates. After recitation you must prepare your final solution to this problem individually but are free to use everything you learned from others in recitation. The recitation problem is as likely to be chosen to be graded as any other.

If not stated explicitly assume $R=287$ J/kgK and $y=1.4$ for perfect gas.

1. (Recitation problem) A large adiabatic reservoir contains Hydrogen at a certain pressure and a temperature of 200K. The gas exhausts the reservoir into a larger tank with a back pressure of 50 kPa through a convergent nozzle with a throat area $A_e = 0.2m^2$. The pressure in the exhaust tank is kept constant. An engineer wishes to study the dependence of mass flow rate and Mach number at a particular section on the chamber pressure as the first step in order to estimate the optimum chamber pressure. Using Matlab,

   a. Plot the flow speed at the nozzle exit as a function of the chamber pressure.
   b. Plot the mass flow rate through the nozzle as a function of the chamber pressure.
   c. Plot the Mach number at a station $X$, whose area is 1.2 times the throat area as a function of the chamber pressure.

Please list all the assumptions about the nature of the gas, type of flow etc, that are essential to tackle this challenge. Also, include the axes labels and the title on the plots.

In each case consider chamber pressures in the range 0 to 200kPa in steps of 1kPa. Your homework submission must include a listing of your Matlab code, which also should be submitted on Canvas for the homework 2 assignment. The submitted code must run to produce your plots without modification. It is an honor code violation to adapt or otherwise include components from anyone else’s code, whether a student in the
course or not. **Hint:** A code that determines the inverse of the Area-Mach number relation will be distributed for use in this problem.

2. The figure below shows a test setup used to perform experiments on an engine nozzle. Air (gas constant, specific heats can be found in Anderson) is exhausted through the nozzle from a large chamber. The chamber is big enough so that the flow velocity there is basically zero, even when the nozzle is operating. The flow passes from the chamber through the nozzle entrance (at location 1) and then to its exit (at location 2). The cross-sectional area of the nozzle is 0.05m² at location 1 and 0.07m² at location 2.

(a) During the first test, the velocity at location 1 is measured (using a hot-wire probe) to be 221m/s. A thermocouple senses the temperature here as 7 degrees Celsius. A probe at location 2 records a Mach number of 0.75. A shock wave is known to be present in the flow between locations 1 and 2. Determine the Mach number of the flow at location 1, and the temperature at location 2.

(b) From this test the pressure is measured at location 1 and found to be 37kPa. You are given that the nozzle boundary layer remains thin and, since the flow is entirely subsonic upstream of station 1, there are no shocks here. Estimate the pressure, density and temperature of the air in the chamber feeding the nozzle.

![Figure 3. Converging Diverging Nozzle of Engine](image)

Compare and comment on the ratio of $T_2/T_1$ determined from the two cases.

3. A rocket exhaust nozzle consists of a converging-diverging nozzle with an exit-to-throat area ratio of 11, a throat area of 0.05m². The nozzle is tested by exhausting air through it from a chamber in which the pressure is 500kPa while the back pressure is
varied. Determine: (a) the design pressure ratio of the nozzle, (b) the design back pressure, (c) the design Mach number of the nozzle, (d) the minimum and maximum possible Mach numbers at the nozzle throat, (e) the maximum possible mass flow rate through the nozzle (in terms of the chamber temperature), and what range of back pressures will produce this, (f) the range(s) of back pressures will produce a wave free flow. Be sure to show the working behind each of your answers.

4. The purpose of this question is to introduce you to the converging-diverging nozzle - one of the most important pieces of compressible flow hardware. Read carefully the material at the end of this homework paper entitled “Some background on Converging-Diverging nozzles”, then answer the following questions:

a. At what condition(s) is the flow through and out of a converging diverging nozzle free of any kind of waves?
b. At what condition(s) additional to those identified in (a) is the flow inside the nozzle free of any waves?
c. What happens to a supersonic flow when it passes through a duct of increasing cross-sectional area?
d. Under what flow conditions are the expansion waves formed? Can they be useful? What happens to the pressure after the expansion waves?
e. Is the pressure in the jet always the same as ambient pressure (back pressure)? Explain in brief.
Some background on Converging-Diverging nozzles

The usual configuration for a converging diverging (CD) nozzle is shown in figure 1. Gas flows through the nozzle from a region of high pressure (usually referred to as the chamber) to one of low pressure (referred to as the ambient or tank). The chamber is usually big enough so that any flow velocities here are negligible. The pressure here is denoted by the symbol $p_c$. Gas flows from the chamber into the converging portion of the nozzle, past the throat, through the diverging portion and then exhausts into the ambient as a jet. The pressure of the ambient is referred to as the 'back pressure' and given the symbol $p_b$.

A simple example

To get a basic feel for the behavior of the nozzle, imagine performing the simple experiment shown in figure 2. Here we use a converging diverging nozzle to connect two air cylinders. Cylinder A contains air at high pressure, and takes the place of the chamber. The CD nozzle exhausts this air into cylinder B, which takes the place of the tank.

Imagine you are controlling the pressure in cylinder B, and measuring the resulting mass flow rate through the nozzle. You may expect that the lower you make the pressure in B the more mass flow you'll get through the nozzle. This is true, but only up to a point. If you lower the back pressure enough you come to a place where the flow rate suddenly stops increasing all together and it doesn't matter how much lower you make the back pressure (even if you make it a vacuum) you can't get any more mass flow out of the nozzle. We say that the nozzle has become 'choked'. You could delay this behavior by making the nozzle throat bigger (e.g. grey line) but eventually the same thing would
happen. The nozzle will become choked even if you eliminated the throat altogether and just had a converging nozzle.

The reason for this behavior has to do with the way the flows behave at Mach 1, i.e. when the flow speed reaches the speed of sound. In a steady internal flow (like a nozzle) the Mach number can only reach 1 at a minimum in the cross-sectional area. When the nozzle isn't choked, the flow through it is entirely subsonic and, if you lower the back pressure a little, the flow goes faster and the flow rate increases. As you lower the back pressure further the flow speed at the throat eventually reaches the speed of sound (Mach 1). Any further lowering of the back pressure can't accelerate the flow through the nozzle anymore, because that would entail moving the point where M=1 away from the throat where the area is a minimum, and so the flow gets stuck. The flow pattern downstream of the nozzle (in the diverging section and jet) can still change if you lower the back pressure further, but the mass flow rate is now fixed because the flow in the throat (and for that matter in the entire converging section) is now fixed too.

The changes in the flow pattern after the nozzle has become choked are not very important in our thought experiment because they don't change the mass flow rate. They are, however, very important however if you were using this nozzle to accelerate the flow out of a jet engine or rocket and create propulsion, or if you just want to understand how high-speed flows work.

The flow pattern
Figure 3a shows the flow through the nozzle when it is completely subsonic (i.e. the nozzle isn't choked). The flow accelerates out of the chamber through the converging section, reaching its maximum (subsonic) speed at the throat. The flow then decelerates through the diverging section and exhausts into the ambient as a subsonic jet. Lowering the back pressure in this state increases the flow speed everywhere in the nozzle.

Lower it far enough and we eventually get to the situation shown in figure 3b. The flow pattern is exactly the same as in subsonic flow, except that the flow speed at the throat has just reached Mach 1. Flow through the nozzle is now choked since further
reductions in the back pressure can't move the point of \( M=1 \) away from the throat. However, the flow pattern in the diverging section does change as you lower the back pressure further.

As \( p_b \) is lowered below that needed to just choke the flow a region of supersonic flow forms just downstream of the throat. Unlike a subsonic flow, the supersonic flow accelerates as the area gets bigger. This region of supersonic acceleration is terminated by a normal shock wave. The shock wave produces a near-instantaneous deceleration of the flow to subsonic speed. This subsonic flow then decelerates through the remainder of the diverging section and exhausts as a subsonic jet. In this regime if you lower or raise the back pressure you increase or decrease the length of supersonic flow in the diverging section before the shock wave.

If you lower \( p_b \) enough you can extend the supersonic region all the way down the nozzle until the shock is sitting at the nozzle exit (figure 3d). Because you have a very long region of acceleration (the entire nozzle length) in this case the flow speed just before the shock will be very large in this case. However, after the shock the flow in the jet will still be subsonic.

Lowering the back pressure further causes the shock to bend out into the jet (figure 3e), and a complex pattern of shocks and reflections is set up in the jet which will now involve a mixture of subsonic and supersonic flow, or (if the back

Figure 3. Flow Patterns
pressure is low enough) just supersonic flow. Because the shock is no longer perpendicular to the flow near the nozzle walls, it deflects it inward as it leaves the exit producing an initially contracting jet. We refer to this as overexpanded flow because in this case the pressure at the nozzle exit is lower than that in the ambient (the back pressure)- i.e. the flow has been expanded by the nozzle too much.

A further lowering of the back pressure changes and weakens the wave pattern in the jet. Eventually we will have lowered the back pressure enough so that it is now equal to the pressure at the nozzle exit. In this case, the waves in the jet disappear altogether (figure 3f), and the jet will be uniformly supersonic. This situation, since it is often desirable, is referred to as the 'design condition'.

Finally, if we lower the back pressure even further we will create a new imbalance between the exit and back pressures (exit pressure greater than back pressure), figure 3g. In this situation (called 'underexpanded') what we call expansion waves (that produce gradual turning and acceleration in the jet) form at the nozzle exit, initially turning the flow at the jet edges outward in a plume and setting up a different type of complex wave pattern.

The pressure distribution in the nozzle
A plot of the pressure distribution along the nozzle (figure 4) provides a good way of summarizing its behavior. To understand how the pressure behaves you have to remember only a few basic rules
• When the flow accelerates (sub or supersonically) the pressure drops
• The pressure rises instantaneously across a shock
• The pressure throughout the jet is always the same as the ambient (i.e. the back pressure) unless the jet is supersonic and there are shocks or expansion waves in the jet to produce pressure differences.
• The pressure falls across an expansion wave.
The labels on figure 4 indicate the back pressure and pressure

Figure 4  Pressure distribution along the nozzle.
Labels refer to flow regimes in figure 3.
distribution for each of the flow regimes illustrated in figure 3. Notice how, once the flow is choked, the pressure distribution in the converging section doesn't change with the back pressure at all.