Applications of the Normal Shock Relations
Pitot Static Probes

http://ttjstk.diy.myrice.com/wqxh/su27/small/Su-27_pitot.JPG

http://www.clubhyper.com/reference/images/f14dkittybg_1.JPG
Pitot Probe in Supersonic Flow

$M_\infty = 4$

Schlieren picture of Mach number measurement inside VT SST for $M=4$
(Dr. Eric Marineau)
Example

If $M_\infty = 1.5$ and $p_{atm} = 50\text{kPa}$, find $p_s$ and $p_p$
Variations

\[
\frac{M_\infty}{p_\infty = p_{atm} = 50 \text{kPa}}
\]

What if...

- asked for pressure on vehicle nose rather than Pitot static?
  - \( M_\infty < 1 \), say 0.8?
  - \( M_\infty > 1 \), and asked for \( T_p \)?
Variations \[ M_\infty \frac{p_\infty}{p_\infty = p_{atm} = 50\text{kPa}} \]

What if...

- given \( p_p \) and \( p_{atm} \) and asked for \( M_\infty \)?

- given \( p_p \) and \( p_s \) and asked for \( M_\infty \)?
Critical Area Ratio Across a Shock

\[ \frac{M=1 A_1^*}{M=1 A_2^*} \]
Critical Area Ratio

For Air

\[ \frac{A_2^*}{A_1} \]

\[ M_1 \]
Example

Find $p_b/p_c$ and $u_j$ if $A_e/A_t = 5$, $A/A_t = 2.5$
Variations

$P_c$  
$T_c=500K$

What if asked for…

• highest $p_b/p_c$ for shock in nozzle?

• lowest $p_b/p_c$ for shock in nozzle?

• $p_b/p_c$ range for entirely subsonic flow? choked flow? design?
Variations

\( \frac{P_c}{T_c=500K} \)

What if asked for…

• \( \frac{p_b}{p_c} \) range for over-expanded flow?

• \( \frac{p_b}{p_c} \) range for under-expanded flow?

• flow (including shock position if relevant) given any \( \frac{p_b}{p_c} \)?
CD nozzle analysis given $A_e/A_t$ and any $p_b/p_c$

Flow Regime

Borderline case

Subsonic

Flow just choked

Shock in Nozzle

Shock at exit

Over-expanded

Design

Under-expanded

Subsonic Flow just choked

$M < 1 \Rightarrow p_e = p_b \Rightarrow M < 1$

Shock in Nozzle

$M < 1 \Rightarrow M > 1 \Rightarrow p_2 = p_b \Rightarrow M < 1$

Shock at exit

$M < 1 \Rightarrow M > 1 \Rightarrow p_e = p_b \Rightarrow M > 1$

Over-expanded

$M < 1 \Rightarrow M > 1 \Rightarrow p_e = p_b \Rightarrow M > 1$

Design

$M < 1 \Rightarrow M > 1 \Rightarrow p_e = p_b \Rightarrow M > 1$

Under-expanded
Shock in Nozzle

Given $A_e/A_t$ and $p_b/p_c$ need to find area $A$ or Mach number $M_1$ at the shock $M_e \Rightarrow$ tables $\Rightarrow p_e/p_{oe}=p_b/p_{o2}$ then $p_b/p_{o2} * p_c/p_b = p_{o1}/p_{o2}$

- $p_{o1}/p_{o2}$ $\Rightarrow$ tables $M_1 \Rightarrow A/A_1^* \Rightarrow A$

Not simple, but problem is solved if we know $M_e$

From the Area-Mach number relation and the definition of stagnation pressure one can show...

\[
M_e^2 = -\frac{1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma - 1)^2} + \frac{2}{\gamma - 1} \left( \frac{2}{\gamma + 1} \right)^{\gamma + 1/\gamma - 1} \left( \frac{p_c}{p_b} \frac{A_t}{A_e} \right)^2}
\]
Supersonic Wind Tunnel

Tunnel has two CD nozzles with the test section in between

- Ideal: Both nozzles have same throat area, this way there are no shock waves and thus no losses due to shocks.
- Unfortunately such a tunnel cannot be started because the first nozzle will choke before any supersonic flow is established in the test section.
- In practice the tunnel must have:
  - Adaptable walls
  - A second throat with an area larger than the critical area downstream of the shock when it is in the largest part of the test section. Tunnel will operate with a shock just downstream of the second throat.