Trade-Off Between Fuel and Time Optimization

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Interplanetary Mission Design

- **Use natural dynamics for fuel efficiency**
  - Dynamic channels connecting planets and moons
  - Trajectory generation using invariant manifolds in the 3-body problem suggests new numerical algorithms for interplanetary missions

- **How to balance fuel efficiency with reasonable flight times?**
  - Gravity assists, ballistic captures can take a long time
  - Short flight times important for challenging missions, e.g. *Multi-Moon Orbiter* to multiple Jovian moons
Multi-Moon Orbiter

- *Orbit each moon in a single mission*
  - Other Jovian moons are also worthy of study
    - All may have oceans, evidence from *Galileo* suggests
Multi-Moon Orbiter

\[ \Delta V \text{ is low (} \sim 20 \text{ m/s)}, \text{ but flight time } \sim 4 \text{ years} \]

Low Energy Tour of Jupiter’s Moons
Seen in Jovicentric Inertial Frame
Fuel vs. Time Trade-Off

Motivating Example

Earth to Moon Trajectories

- Consider a transfer from Earth orbit to lunar orbit
- Previously addressed by Belbruno and Miller, where nonlinear $n$-body effects were used to lower $\Delta V$ at the expense of a longer time of flight compared to Hohmann
- A good example problem for seeking efficient numerical algorithms to find fuel vs. time trade-off, applicable for many other situations
Use planar circular restricted 3-body model
  • Consider the effect of only the Earth and Moon

Compare with earlier methods; Hohmann, Bollt and Meiss, Schroer and Ott
  • Hohmann: simple and fast, but fuel-expensive
  • Remove recurrent loops in a chaotic trajectory
  • Target passes between mean motion resonances

We will give some background on these methods
Some Background

■ Starting/final orbital parameters
  □ Starting orbit: 59559 km circular Earth orbit
  □ Final orbit: lunar orbit with perilune 13970 km

■ Transfer trajectory
  □ Classical method: Hohmann transfer
    • Two maneuvers: $\Delta V_1$ and $\Delta V_2$, both large
  □ Total $\Delta V = 1220$ m/s, TOF = 6.6 days
  □ First Goal: use the same orbital parameters, but lower the total $\Delta V$
  □ Second Goal: keep time of flight reasonable
Bollt & Meiss [1995]: target through recurrence
- Find chaotic solution for fixed energy, remove recurrent loops with very small $\Delta V$’s

Poincare section: chaotic and regular motion intermixed.
Some Background

- Trajectory found: 750 m/s, 748 days

Spacecraft trajectory in the rotating frame
Schroer & Ott [1997]: target passes between resonances

- Leap-frog between a series of resonances using very small $\Delta V$'s until trajectory reaches the moon

Resonances: target the passes between them with small controls.
Some Background

- Trajectory found: 749 m/s, 378 days

![Spacecraft trajectory in the rotating frame](image)
Our Approach

Our approach

- Take full advantage of all known phase space structures: seek intersections between resonances and tubes leading to ballistic capture by the Moon

Building blocks

- **Appropriate energy:** transfer at three-body energy where resonances and tube dynamics are important
- **Poincare section:** reduces problem to 2D
- **Resonant gravity assists:** maximize change in orbit during every lunar encounter
- **Tube dynamics:** get captured by the Moon
Appropriate Energy

- Transfer to occur on a single 3D energy surface
- Poincaré surface-of-section: motion on 2D map

Case 2: $E_1 < E < E_2$

Forbidden Realm (at a particular energy level)

Poincare Section

Earth

Moon
Study Poincaré surface of section at fixed energy $E$, reducing system to a 2-dimensional area preserving map.
Resonant Gravity Assists

- Unstable resonances: Periodic orbits forming a dynamical “back-bone,” via their stable/unstable manifolds.
- Physically, these manifolds correspond to orbits undergoing repeated close encounters with the Moon.

Unstable resonances and their manifolds.
Jump to the Moon’s vicinity via invariant manifold tubes associated to a periodic orbit about $L_1$.

Track orbits via exits/entrances on Poincaré sections.

Tube dynamics: going from one Poincaré section to another.
Resonances and Tubes

Resonances and tubes are linked

- It has been observed that the tubes of capture orbits are coming from certain resonances.
  - Koon, Lo, Marsden, Ross [2001]

Designing an efficient transfer

- First, from the starting Earth orbit, perform a $\Delta V$ placing the spacecraft on a trajectory near one of the resonances which is linked to capture tubes.
- Then perform small maneuvers to steer into a capture tube – none may be needed!
Resonances and Tubes

- Poincaré section: tube cross-sections are closed curves and resonance manifolds are windy curves.
Invisible structure: both of them reveal the structure in the “chaotic” part of phase space.

Poincaré section showing tubes and resonances and background points.
Tubes and resonance manifolds intersect, i.e., there is a free transfer from the resonance to the Moon’s vicinity.

Poincaré section showing tubes and resonances.
Resonances and Tubes

- The free transfer corresponds to a flight time of over 250 days, but we can do better.
- The tubes and resonance manifold come “close” to intersecting in several places, i.e., a small $\Delta V$ can drastically cut out unnecessary flight time.
- By searching for $\Delta V$’s of a particular size, one can systematically get a curve of $\Delta V$ vs. time of flight.
**Results**: much shorter transfer times than previous authors for only slightly more $\Delta V$.
Resonances and Tubes

Compare with Boltt and Meiss [1995]

- A tenth of the time for only 100 m/s more

Current Result
65 days, $\Delta V = 860$ m/s

Boltt and Meiss [1995]
748 days, $\Delta V = 750$ m/s

TOF = 65 days
$\Delta V = 860$ m/s
1 Day Tick Marks

Earth
Moon
e.g., GEO to Lunar Orbit

GEO to Moon Orbit Transfer

Seen in Geocentric Inertial Frame

TOF = 63 days
ΔV = 1211 m/s
1 Day Tick Marks

Moon’s Orbit
Earth
e.g., GEO to Lunar Orbit

GEO to Moon Orbit Transfer

Seen in Lunar Rotating Frame
e.g., GEO to Lunar Orbit

GEO to Moon Orbit Transfer
Seen in Geocentric Inertial Frame

Moon's Orbit

Earth
References


For papers, movies, etc., visit the websites:

- [http://www.cds.caltech.edu/~marsden](http://www.cds.caltech.edu/~marsden)
- [http://www.cds.caltech.edu/~shane](http://www.cds.caltech.edu/~shane)