BALANCE RECOVERY STRATEGY: ACROBOT VS. WOBBLE CHAIR

Frontiers in Dynamical Systems

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Introduction

• Multiple segment inverted pendulums
  • Used in biomechanics to model balance recovery and postural control in humans.

• Present Study
  • model two configurations of double segment inverted pendulums to analyze the strategy used to recover from external perturbation
Acrobot Model: Strategy 1

- **Model:**
  - Under actuated double segment inverted pendulum modeling “hip strategy”

- **Strategy**
  - Forward or backward sway displacement elicited rotation of the hip that moved the trunk in the direction of the initial body displacement.

Figures: Experimental work showing use of hip strategy when subjects stand on a narrow beam.
Wobble Chair Model: Strategy 2

• Model:
  • Under actuated double segment inverted pendulum

• Strategy
  • Tanaka et al. (2010) recovery of COM location was achieved by causing flexion of trunk when overall center of mass was posterior to pivot point.
Strategy: Acrobot Vs. Wobble Chair

- Purpose
  - Attempt discovery of the opposing strategies used between the acrobot and the wobble chair to recover balance after an external perturbation
Models’ Description

\[ \theta_{COM} = \tan^{-1}\left( \frac{m_1 \cdot X_{COM_1} + m_2 \cdot X_{COM_2}}{m_1 \cdot Y_{COM_1} + m_2 \cdot Y_{COM_2}} \right) \]
**Wobble Chair**

**Segment 1**

\[ X_{COM1} = \frac{m_p \cdot r_p \cdot \sin \theta_1 + m_t \cdot r_t \cdot \sin(\theta_1 + \alpha) + m_s \cdot (L_t \cdot \sin(\theta_1 + \alpha) + r_s \cdot \cos(\theta_1 + \alpha))}{m_p + m_t + m_s} \]

\[ Y_{COM1} = \frac{-m_p \cdot r_p \cdot \cos \theta_1 - m_t \cdot r_t \cdot \cos(\theta_1 + \alpha) + m_s \cdot (-L_t \cdot \cos(\theta_1 + \alpha) + r_s \cdot \sin(\theta_1 + \alpha))}{m_p + m_t + m_s} \]

\[ r_1 = \sqrt{X_{COM1}^2 + Y_{COM1}^2} \]

\[ \varphi + \theta_1 = \tan^{-1}(X_{COM1}/Y_{COM1}) \]

\[ m_1 = m_p + m_t + m_s \]
**Models’ Parameters**

- **Simplified models’ Parameters**

<table>
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<tr>
<th></th>
<th>$m_p$ (kg)</th>
<th>$m_t$ (kg)</th>
<th>$m_s$ (kg)</th>
<th>$m_1$ (kg)</th>
<th>$m_2$ (kg)</th>
<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
<th>$L_1$ (m)</th>
<th>$L_2$ (m)</th>
<th>$I_1$ (kg.m$^2$)</th>
<th>$I_2$ (kg.m$^2$)</th>
<th>$\alpha$ (deg)</th>
<th>$\phi$ (deg)</th>
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<td>0.0833</td>
<td>0.0833</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Wobble chair</td>
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<td>0.4</td>
<td>0.4</td>
<td>1</td>
<td>1</td>
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<td>0.0500</td>
<td>0.0833</td>
<td>150</td>
<td>176</td>
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Equations of motion (EQM)

• Lagrangian Method

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \left( \frac{\partial L}{\partial q_j} \right) = Q_j \quad q_1 = \theta_1 \quad q_2 = \theta_2 \quad L = K - V \]

\[ K = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2 + \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} I_2 \dot{\theta}_2^2 \]

\[ V = -m_1 g \mid \vec{r}_1 \mid \cos(\theta_1 + \varphi) - m_2 g \mid \vec{L}_1 \mid \cos \theta_1 - m_2 g \mid \vec{r}_2 \mid \cos \theta_2 \]

• EQM

\[ M \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) = Q \]

\[ M = \begin{bmatrix} l_1 + m_1 \mid \vec{r}_1 \mid^2 + m_2 \mid \vec{L}_1 \mid^2 & m_2 \mid \vec{L}_1 \mid \mid \vec{r}_2 \mid \cos(\theta_1 - \theta_2) \\ m_2 \mid \vec{L}_1 \mid \mid \vec{r}_2 \mid \cos(\theta_1 - \theta_2) & l_2 + m_2 \mid \vec{r}_2 \mid^2 \end{bmatrix} \]

\[ C = \begin{bmatrix} 0 & m_2 \mid \vec{L}_1 \mid \mid \vec{r}_2 \mid \sin(\theta_1 - \theta_2) \dot{\theta}_2 \\ -m_2 \mid \vec{L}_1 \mid \mid \vec{r}_2 \mid \sin(\theta_1 - \theta_2) \dot{\theta}_1 & 0 \end{bmatrix} \]

\[ G = \begin{bmatrix} m_1 g \mid \vec{r}_1 \mid \sin(\theta_1 + \varphi) + m_2 g \mid \vec{L}_1 \mid \sin \theta_1 \\ m_2 g \mid \vec{r}_2 \mid \sin \theta_2 \end{bmatrix} \quad Q = \begin{bmatrix} -T + T_{sp} \\ T \end{bmatrix} \]

\[ T_{sp} = k_s \cdot d^2 \cdot \sin(\theta_1) \]
Controller: PID

\[ C_{pd} = K_d \cdot \dot{\theta}_{COM} + \begin{cases} 
K_p \cdot \theta_{COM} + K_i \cdot \int_0^t \theta_{COM} \, dt & \text{if } |\theta_{COM}| < \theta_{critical} \\
T_{pmax} & \text{Otherwise}
\end{cases} \]
Stability Criteria: Hilbert Envelope

\[ \theta_{\text{COM}} \] converging to zero indicating a stable system
Stability regions
Different Strategies

Acrobot

Wobble Chair
Transformation:

\[ L_1 = L_{1A}(x) + L_{1S}(1 - x) \]
\[ m_1 = m_{1A}(x) + m_{1S}(1 - x) \]
\[ L_t = L_s \]
\[ 1 = L_p + L_s + L_t \]
\[ M_t = M_s \]
\[ 1 = M_p + M_s + M_t \]
Transformation Results
10% and 15% acrobot
Animation

Acrobot

Strategy 1

Wobble Chair

Strategy 2

perturbation

perturbation
Summary

• Two segment inverted pendulum models were developed representing the acrobot and wobble chair configurations.

• Two opposite recovery strategies were observed for the models subjected to an external perturbation.

• Using a search method with variations in controller gains revealed the presence of these opposing strategies in our models.
Future Work

- Transforming the model in x and y direction
  - The location of the COM of segment 1

- Find out why the models choose different strategies
Thank you