Aeroelasticity and Flight Dynamics of Oblique Flying Wings

Brijesh Raghavan* and Mayuresh Patil†

Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0203.

First proposed in the 1950’s, oblique flying wings offer superior aerodynamic performance in transonic and supersonic flight. Recent advances in technology have revived interest in such configurations for supersonic transports. These configurations will exhibit aeroelastic and flight dynamic characteristics which are different from those encountered in symmetric wing-body configurations. This paper seeks to understand aeroelasticity and its coupling with flight dynamics in oblique flying wing configurations. A sample configuration is analysed using geometrically-exact beam theory to model the structure and a combination of vortex-lattice and doublet-lattice models for the aerodynamics.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(B)</td>
<td>integration matrix</td>
</tr>
<tr>
<td>(b)</td>
<td>reference semi-chord</td>
</tr>
<tr>
<td>(C)</td>
<td>rotation matrix</td>
</tr>
<tr>
<td>(C_a)</td>
<td>rotation matrix from aerodynamic frame to local beam frame</td>
</tr>
<tr>
<td>(C_D)</td>
<td>Coefficient of drag</td>
</tr>
<tr>
<td>(c)</td>
<td>local chord</td>
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<tr>
<td>(D)</td>
<td>aerodynamic influence coefficient matrix</td>
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<tr>
<td>(dl)</td>
<td>structural element length</td>
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<tr>
<td>(E)</td>
<td>matrix for extracting aerodynamic moments from (F_{aero})</td>
</tr>
<tr>
<td>(e_1, e_2, e_3)</td>
<td>unit vector along x, y and z axis respectively</td>
</tr>
<tr>
<td>(F)</td>
<td>internal force</td>
</tr>
<tr>
<td>(F_{aero})</td>
<td>generalized aerodynamic force in the aerodynamic frame</td>
</tr>
<tr>
<td>(f)</td>
<td>external force</td>
</tr>
<tr>
<td>(G)</td>
<td>matrix for extracting aerodynamic forces from (F_{aero})</td>
</tr>
<tr>
<td>(g)</td>
<td>gravity vector</td>
</tr>
<tr>
<td>(g_0)</td>
<td>magnitude of gravity vector</td>
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<tr>
<td>(H)</td>
<td>angular momentum</td>
</tr>
<tr>
<td>(I)</td>
<td>moment of inertia per unit span</td>
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<tr>
<td>(k)</td>
<td>initial curvature of wing, also used for reduced frequency</td>
</tr>
<tr>
<td>(M)</td>
<td>internal moment, also used for Mach number</td>
</tr>
<tr>
<td>(m)</td>
<td>external moment</td>
</tr>
<tr>
<td>(P)</td>
<td>linear momentum</td>
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<tr>
<td>(p)</td>
<td>pressure across panel</td>
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<tr>
<td>(q)</td>
<td>dynamic pressure</td>
</tr>
<tr>
<td>(R, S, T)</td>
<td>cross-sectional flexibility coefficient</td>
</tr>
<tr>
<td>(r_i)</td>
<td>turn rate in inertial frame</td>
</tr>
<tr>
<td>(T)</td>
<td>thrust value</td>
</tr>
<tr>
<td>(r_R)</td>
<td>position vector of the beam axis from the origin of the reference frame (R)</td>
</tr>
<tr>
<td>(u_R)</td>
<td>deformation in the (R) frame</td>
</tr>
</tbody>
</table>

*Graduate Student, Department of Aerospace and Ocean Engineering; brijeshr@vt.edu, Student Member AIAA.
†Assistant Professor, Department of Aerospace and Ocean Engineering; mpatil@vt.edu, Senior Member AIAA.
I. Introduction

The oblique wing configuration was first suggested by R.T. Jones in 1958 as it offered significant drag reduction in transonic and supersonic flight.\(^1\) Though the concept was studied extensively, it did not result in a prototype capable of supersonic flight. However, with improvements in technology over the last few decades, the oblique flying wing concept is once again being considered for supersonic transport with research being carried out in aerodynamics and controls.\(^2\)–\(^4\) Over the years, the aeroelasticity of oblique wings was subject to considerable research\(^5\)–\(^12\). Both oblique-winged aircrafts with fuselage and oblique flying wings were studied. These studies suggested significantly improved divergence characteristics of oblique flying wings due to freedom in roll.\(^5\)–\(^7\) Flutter analysis of asymmetrically swept wings revealed two basic modes of flutter instability.\(^8\),\(^9\) Studies on the influence of aeroelasticity on lateral trim and improvements using aeroelastic tailoring were carried out.\(^10\)–\(^12\)

In this paper, the interaction of aeroelasticity with flight dynamics for a simplified oblique flying wing configuration will be studied. Previous research on aeroelasticity and flight dynamics of flying wing configurations will be used as a starting point.\(^14\),\(^15\)
II. Details of Modeling

The flying-wing configuration being studied is shown in Fig. (1) and is modeled as having symmetric taper about its mid-point. The engine is located at the mid-point, and can be pivoted about an axis normal to the wing to be aligned in the plane of the freestream. The trailing edge is divided into four sections. The inner two sections are deflected symmetrically for pitch trim, while the outer sections are deflected differentially for roll control. The wing is trimmed in yaw by changing the thrust of the auxiliary engine located at the spanwise tip of the right wing. The orientation of the auxiliary engine relative to the direction of the freestream is preset. This can be seen as being analogous to vertical tail sizing in conventional configurations. The wing is made to fly obliquely with the right wing-tip positioned into the free-stream.

The structural dynamics of the flying wing is modeled using geometrically-exact, intrinsic equations for a general, non-uniform, twisted, curved, anisotropic beam, undergoing large deformation. These equations are augmented by kinematic relations defined in Ref. 17 derived from the generalized strain-displacement and generalized velocity-displacement equations. The equations can be solved for velocity and angular velocity without computing nodal displacement and rotation.

The aerodynamics of the oblique flying wing is modeled in two parts. Quasi-steady aerodynamics is modeled using a non-planar Vortex-Lattice Theory and is used for trim computation. This can augmented by Doublet-Lattice Theory to account for unsteady aerodynamic effects during stability analysis.
A. Structural Model

The structural equations are formulated in the deformed beam axis system. The local ‘X’ axis at any point on the beam is aligned along the span of the wing, and the ‘Z’ axis is defined to be perpendicular to the surface and points vertically upwards. The local ‘Y’ axis is defined so as to complete the triad. The sweep angle Λ is defined to be the angle between the beam ‘X’ axis and the normal to the freestream as shown in Fig. (1). The dynamics of the structure is modeled by the following sets of equations:

\[ F' + (\tilde{k} + \bar{k})F + f = \tilde{P} + \bar{\Omega}P \]  
\[ M' + (\tilde{k} + \bar{k})M + (\tilde{\epsilon}_1 + \bar{\epsilon})F + m = \tilde{H} + \bar{\Omega}H + \bar{V}P \]  
\[ V' + (\tilde{k} + \bar{k})V + (\tilde{\epsilon}_1 + \bar{\epsilon})\Omega = \dot{\epsilon} \]  
\[ \Omega' + (\tilde{k} + \bar{k})\Omega = \dot{\kappa} \]  

where, ( )' denotes a derivative with respect to the beam x co-ordinate, ( . ) denotes a derivative with respect to time, and f and m are the external forces including gravity \((f_g, m_g)\), aerodynamic loads \((f_{aero}, m_{aero})\), and thrust \((f_T, m_T)\). The first two equations in the above set are the equations of motion\(^{16}\) while the latter two are the intrinsic kinematical equations.\(^{17}\)

1. Cross-sectional constitutive laws

The secondary beam variables are modeled as linear functions of the primary variables using the cross-sectional constitutive laws (flexibility and inertia matrices) given below:

\[ \begin{bmatrix} \epsilon \\ \kappa \end{bmatrix} = \begin{bmatrix} R & S \\ S^T & T \end{bmatrix} \begin{bmatrix} F \\ M \end{bmatrix} \]  

\[ \begin{bmatrix} P \\ H \end{bmatrix} = \begin{bmatrix} \mu \Delta & -\mu \xi \\ \mu \xi & I \end{bmatrix} \begin{bmatrix} V \\ \Omega \end{bmatrix} \]  

\(R, S,\) and \(T\) are 3×3 matrices of cross-sectional flexibility coefficients; and \(\mu, \xi, I\) are the mass per unit length, mass center offset, and mass moment of inertia per unit length, respectively. The above model is based on assumptions of small strain and slenderness.

2. Finite-element discretization

Eqs. (1)-(4) are solved using a finite element discretization that conserves energy.\(^{17}\) As an example of discretization, consider a variable \(X\). The nodal values of the variable after discretization be represented by \(\hat{X}_i^n\) and \(\hat{X}_r^n\), where the superscript denotes the node number, the subscript denotes the left or right side of the node, and the hat denotes that it is nodal value. For the element \(n\)

\[ X' = \frac{\hat{X}_i^{n+1} - \hat{X}_r^n}{dl} \]  
\[ X = \overline{X}^n = \frac{\hat{X}_i^{n+1} + \hat{X}_r^n}{2} \]

Eqs.(1)-(4) are re-written in discretized form as,

\[ \frac{\hat{F}_i^{n+1} - \hat{F}_r^n}{dl} + (\tilde{\kappa}^n + \tilde{k}^n)\hat{F}^n + \hat{F}^n - \tilde{\Omega}^n\hat{P}^n = 0 \]  
\[ \frac{\hat{M}_i^{n+1} - \hat{M}_r^n}{dl} + (\tilde{\kappa}^n + \tilde{k}^n)\hat{M}^n + (\tilde{\epsilon}_1^n + \tilde{\epsilon}^n)\hat{F}^n + \hat{m}^n - \tilde{\Omega}^n\hat{H}^n - \hat{V}^n\hat{P}^n = 0 \]  
\[ \frac{\hat{V}_i^{n+1} - \hat{V}_r^n}{dl} + (\tilde{\kappa}^n + \tilde{k}^n)\hat{V}^n + (\tilde{\epsilon}_1^n + \tilde{\epsilon}^n)\hat{\Omega}^n - \hat{\epsilon}^n = 0 \]  
\[ \frac{\hat{\Omega}_i^{n+1} - \hat{\Omega}_r^n}{dl} + (\tilde{\kappa}^n + \tilde{k}^n)\hat{\Omega}^n - \hat{\kappa}^n = 0 \]
where, the barred quantities correspond to the mean values of the variables at the boundary of the element while the hatted quantities are nodal values at the boundary. The barred and hatted quantities of the primary variables are related as

\[
\begin{align*}
\mathbf{T}^n &= \frac{\bar{F}_r^{n+1} + \bar{F}_r^n}{2} \\
\mathbf{M}^n &= \frac{\bar{M}_r^{n+1} + \bar{M}_r^n}{2} \\
\mathbf{F}^n &= \frac{\bar{V}_r^{n+1} + \bar{V}_r^n}{2} \\
\mathbf{O}^n &= \frac{\hat{\Omega}^{n+1} + \hat{\Omega}^n}{2}
\end{align*}
\]

The barred secondary variables and the barred primary variables are related as shown in Eq. (5)-(6).

3. Gravity loads

The force and moment due to gravitational effects are modeled as

\[
\begin{align*}
\mathbf{f}_g &= \mu g \\
\mathbf{m}_g &= \mu \hat{g}
\end{align*}
\]

where \( g \) is the gravity vector.

The components of \( g \) are known in the inertial frame. The components of the gravity vector \( g \) in the \( B \) frame at all the nodes can be calculated using the following equations:

\[
\begin{align*}
g' + (\kappa + \bar{k}) g &= 0 \\
\dot{g} + \hat{\Omega} g &= 0
\end{align*}
\]

which in the discretized form can be written as

\[
\begin{align*}
\frac{\hat{g}_r^{n+1} - \hat{g}_r^n}{dl} + (\bar{\kappa} + \bar{k}) \hat{\mathbf{g}}^n &= 0 \\
\hat{\dot{g}} + \hat{\Omega} \hat{\mathbf{g}} &= 0
\end{align*}
\]

In the above set of equations, the time-differentiated one is to be satisfied at one reference node; following which the spatially-differentiated equation can be used to obtain components of the \( g \) vector at other nodes. Both equations are matrix equations, i.e. a set of three scalar equations. Since the magnitude of the \( g \) vector is a constant (represented by \( g_0 \)), one of the three dynamic equations is replaced by this magnitude constraint. The dynamic equation at the reference node \( n_k \) is re-written as

\[
\begin{align*}
(e_1 \varepsilon_1^T + e_2 \varepsilon_2^T) \hat{g}^{n_k} + (e_1 \varepsilon_1^T + e_2 \varepsilon_2^T) \bar{\hat{\mathbf{g}}}^{n_k} + e_3 |\bar{\hat{\mathbf{g}}}^{n_k}| &= e_3 g_0
\end{align*}
\]

4. Engines, nodal masses, and slope discontinuities

To account for the presence of nodal mass, nodal force (thrust) and slope discontinuities the force on one side of the node is modeled as being different from the force on the other side of the node. Thus Eqs. (1)-(2) at the nodes are given by,

\[
\begin{align*}
\bar{F}_r^n - \bar{\bar{C}}_{ir}^{n} \bar{F}_i^n + \bar{\bar{\bar{F}}}_r^n + \bar{\mu}^n \hat{g}_r^n - \bar{\bar{\hat{F}}}_r^n - \bar{\bar{\hat{\mathbf{p}}}}_r^n &= 0 \\
\bar{\bar{M}}_r^n - \bar{\bar{\bar{M}}}_r^n + \bar{\bar{\bar{\mu}}}^n \hat{g}_r^n - \bar{\bar{\bar{\hat{\mathbf{p}}}}} - \bar{\bar{\bar{\hat{\mathbf{O}}}}}_r^n - \bar{\bar{\bar{\hat{\mathbf{V}}}}}_r^n &= 0
\end{align*}
\]

where \( \hat{F}_r^n \) is the discrete nodal thrust force defined in the \( (\tau) \) reference frame, \( \bar{\bar{\bar{\mu}}}^n \) is the corresponding nodal moment, \( \bar{\bar{\bar{\mu}}}^n \) is the concentrated nodal mass, and \( \hat{\xi}_n \) is the corresponding mass offset. \( \bar{\bar{\bar{\hat{\mathbf{p}}}}}_r^n \) and \( \bar{\bar{\bar{\hat{\mathbf{O}}}}}_r^n \) are the linear and angular momenta of the concentrated nodal mass, given by

\[
\begin{align*}
\left\{ \bar{\bar{\bar{\hat{\mathbf{p}}}}}_r^n \right\} &= \left[ \begin{array}{c} \bar{\bar{\bar{\mu}}}^n \Delta \\
\bar{\bar{\bar{\mu}}}^n \hat{\xi}_n \\
\end{array} \right] \left\{ \hat{\mathbf{v}}_r^n \right\} + \left\{ \begin{array}{c} 0 \\
\bar{\bar{\bar{\Omega}}}_r^n \\
\end{array} \right\}
\end{align*}
\]
where $\mathbf{I}^n$ is the mass moment of inertia matrix of the concentrated mass and $\mathbf{H}_{\text{engine}}^n$ is the angular momentum of the engine.

A slope discontinuity in the beam will also change all the other variables, such that

$$\hat{V}_t^n = \hat{C}_{l_t}^n \hat{V}_r^n$$

(26)

$$\hat{\Omega}_i^n = \hat{C}_{l_i}^n \hat{\Omega}_r^n$$

(27)

$$\hat{g}_i^n = \hat{C}_{l_i}^n \hat{g}_r^n$$

(28)

The $(\cdot)_r$ variables can be used to replace $(\cdot)_t$ variables for $V, \Omega$ and $g$, thus reducing the number of variables.

5. Final structural equations

The complete set of primary structural equations for a free-flying aircraft is given by

$$\mathbf{F}^n - \hat{C}_{l_t}^n \mathbf{F}_t^n + \hat{F}_t^n + \hat{\mu}^n \hat{g}_r^n - \hat{F}_r^n - \hat{\Omega}_t^n \hat{P}_r^n = 0$$

(29)

$$\hat{M}_r^n - \hat{C}_{l_t}^n \hat{M}_t^n + \hat{m}_r^n + \hat{\mu}^n \hat{\xi}_r^n \hat{g}_r^n - \hat{H}_r^n - \hat{\Omega}_r^n \hat{H}_r^n - \hat{V}_r^n \hat{P}_r^n = 0$$

(30)

$$\frac{\hat{F}_t^n - \hat{F}_r^n}{dl} + (\hat{\pi}^n + \hat{k}^n) \mathbf{F}^n + \mathbf{f}_w^n + \mu^n \mathbf{g}_w^n - \hat{T}_r^n \mathbf{P}_r^n = 0$$

(31)

$$\frac{\hat{M}_t^n + \hat{M}_r^n}{dl} + (\hat{\pi}^n + \hat{k}^n) \hat{M}^n + (\hat{\xi}^n + \hat{\tau}^n) \mathbf{F}^n + \mathbf{f}_w^n + \mu^n \mathbf{g}_w^n - \hat{T}_r^n \hat{P}_w^n = 0$$

(30)

and, where the secondary variables are linked to the primary variables by:

$$\begin{bmatrix} \xi^n \\ \eta^n \end{bmatrix} = \begin{bmatrix} R^n & S^n \\ S^n & T^n \end{bmatrix} \begin{bmatrix} \mathbf{F}^n \\ \mathbf{M}^n \end{bmatrix}$$

(36)

$$\begin{bmatrix} \mathbf{F}^n \\ \mathbf{H}^n \end{bmatrix} = \begin{bmatrix} \mu^n \Delta & -\mu^n \xi^n \\ \mu^n \xi^n & I^n \end{bmatrix} \begin{bmatrix} \mathbf{V}^n \\ \mathbf{\Omega}^n \end{bmatrix}$$

(37)

$$\mathbf{F}^n = \frac{\hat{F}_t^n + \hat{F}_r^n}{2}$$

(38)

$$\mathbf{M}^n = \frac{\hat{M}_t^n + \hat{M}_r^n}{2}$$

(39)

$$\mathbf{V}^n = \frac{\hat{C}_{l_t}^n \hat{V}_t^n + \hat{V}_r^n}{2}$$

(40)

$$\mathbf{\Omega}^n = \frac{\hat{C}_{l_t}^n \hat{\Omega}_t^n + \hat{\Omega}_r^n}{2}$$

(41)

$$\mathbf{g}^n = \frac{\hat{C}_{l_t}^n \hat{g}_t^n + \hat{g}_r^n}{2}$$

(42)

$$\begin{bmatrix} \hat{P}_r^n \\ \hat{H}_r^n \end{bmatrix} = \begin{bmatrix} \hat{\mu}^n \Delta & -\hat{\mu}^n \xi^n \\ \hat{\mu}^n \xi^n & \hat{\tau}_n \end{bmatrix} \begin{bmatrix} \hat{V}_r^n \\ \hat{\Omega}_r^n \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{H}_{\text{engine}}^n \end{bmatrix}$$

(43)
6. Boundary conditions

The following boundary conditions complete the set of equations required to analyze the free-flying aircraft problem as formulated above:

\[
\begin{align*}
\hat{F}_l^1 &= 0 \\
\hat{M}_l^1 &= 0 \\
\hat{F}_r^{N+1} &= 0 \\
\hat{M}_r^{N+1} &= 0 \\
(\epsilon_1 e_1^T + \epsilon_2 e_2^T) \hat{g}^{n_g} + (\epsilon_1 e_1^T + \epsilon_2 e_2^T) \hat{\Omega}^{n_g} \hat{g}^{n_g} + e_3(|\hat{g}^{n_g}| - g_0) &= 0
\end{align*}
\]

\(N\) denotes the total number of elements and \(n_g\) denotes the reference node for gravity.

Having solved for the state variables, the geometry before and after deformation can be generated by using the following post-processing relations which relate the strains and curvatures to displacements and rotations:

\[
\begin{align*}
(r_R + u_R)' &= C^{RB}(\epsilon + e_1) \\
C^{BR'} &= -(\vec{\kappa} + \vec{k})C^{BR}
\end{align*}
\]

where \(r_R\) is the position vector of the beam axis from the origin of the reference frame \(R\), and \(u_R\) is the deformation in the \(R\) frame. The deformed beam axis could be plotted if \(r_R + u_R\) is known.

To generate the wing surface one needs the vector defining the position vector of the points on the wing surface from the beam axis. Assuming the cross section to be rigid, the deformed surface can be generated by \(r_R + u_R + C^{RB} \zeta\), where \(\zeta\) is the cross-sectional position vector.

The above equations are expressed in discretised form as

\[
\hat{C}^{BR_{n+1}} = \left( \Delta \frac{\Delta^2}{2} \right)^{-1} \left( \Delta \frac{\Delta^2}{2} \right) \hat{C}^{BR_n}
\]

\[
r_{R_{n+1}} + u_{R_{n+1}} = r_{R_n} + u_{R_n} + \hat{C}^{RB_{n}}(\tau_{n} + e_1)dl
\]

The geometry of the wing before and after deformation is required for generating the aerodynamic model using Vortex Lattice and Doublet Lattice theory.

B. Aerodynamic Model

1. Vortex-Lattice and Doublet-Lattice Model

The reference frame used for aerodynamic computations can be obtained from the the structural frame, by a rotation of \(\Lambda\) about the negative z-axis so that the free-stream has no component normal to the aerodynamic \(Y-Z\) plane.

In order to compute the aerodynamic forces and moments, the wing is divided into discrete panels such that the chordwise panel boundaries are parallel to the freestream while the spanwise panel boundaries are uniformly distributed between the leading and trailing edges (or control surface hinge line). A non-planar vortex-lattice model\(^{18}\) is applied by placing a horse-shoe vortex of unknown strength in each panel, such that the inbound and outbound trailing vortices are along the streamwise panel boundaries and the bound vortex is placed along the panel quarter chord line. The doublet lattice method\(^{19,20}\) accounts for unsteady effects by placing a string of acceleration-potential doublets at the quarter chord line of each panel. The Kutta condition is applied by ensuring that the down-wash generated at the three-quarter chord of the span-wise center of each panel is equal to the corresponding velocity of the structure at that point.

Based on the positions of the vortices and the doublets, an aerodynamic influence coefficient can be generated which relates the downwash at the three quarter-chord of each box with the pressure across the box. This relation is represented by,

\[
\{w\} = [D(X, k, M)] \{p\}
\]
The aerodynamic influence matrices are a function of the reduced frequency \((k = \frac{\omega b}{V})\), free-stream Mach number \((M_\infty)\), and the geometry of the lifting surface (related to the state-vector \(X\)). The aerodynamic influence coefficient is the combination of the effect due to vortices and doublets and can be represented by

\[
[D(X, k, M)] = [D^\text{vort}_s(X, M_\infty)] + [\Delta D^\text{doub}(X, k, M_\infty)]
\]

where \(D^\text{vort}_s(X, M_\infty)\) represents the aerodynamic influence coefficient generated by the vortex lattice method and \(\Delta D^\text{doub}(X, k, M_\infty)\) represents the aerodynamic influence coefficients generated by the doublet lattice method.

The forces and moments acting on the wing can be computed at each section by integrating the pressure over the chordwise boxes. This is represented as

\[
\{F_{\text{Aero}}\} = q \{B\} \{p\}
\]

where \(F_{\text{Aero}}\) represents the generalised aerodynamic force. Eqs. (53)-(55) can be combined to give the expression for generalised aerodynamic force as

\[
\{F_{\text{Aero}}\} = q \{B\} ([D^\text{vort}_s(X, M_\infty)] + [\Delta D^\text{doub}(X, k, M_\infty)])^{-1}\{w\}
\]

The aerodynamic forces and moments are then separated and transferred from the aerodynamic reference frame to the beam reference frame.

\[
\begin{align*}
\bar{f}^n_{\text{aero}} &= C^n_{\text{a}} [G] \{F_{\text{Aero}}\} \\
\bar{m}^n_{\text{aero}} &= C^n_{\text{a}} [E] \{F_{\text{Aero}}\}
\end{align*}
\]

2. Augmentation to account for \(C_D\).

The expression for \(\bar{f}^n_{\text{aero}}\) obtained in the previous section is augmented to include the effect of skin-friction drag. The modified expression for aerodynamic force per unit span in the beam reference frame is given by,

\[
\bar{f}^n_{\text{aero}} = C^n_{\text{a}} [G] \{F_{\text{Aero}}\} - \frac{q e^n C_D}{V_\infty} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}
\]

C. Trim Computation for Aeroelastic System

Eqs. (29)-(48) together with the aerodynamic model given by Eqs. (58)-(59) represents the coupled aeroelastic system. Representing the structural part of the equations by \(f_s\) and the aerodynamic part by \(f_a\) the equations can be represented as,

\[
f_s(X, \dot{X}) + f_a(X, k, M_\infty) = 0
\]

The solutions of interest for the above equation can be expressed in the form

\[
\{X\} = \{\bar{X}\} + \{\dot{X}(t)\}
\]

where \(\bar{X}\) denotes the steady-state values of the state variables at trim and \(\dot{X}(t)\) denote the small perturbations about the steady-state. The trim solution \(\{\bar{X}\}\) can be obtained by solving a set of nonlinear equations given by

\[
f_s(\bar{X}, 0) + f_a(\bar{X}, M_\infty) = 0
\]

The steady-state solution is computed using the Newton-Raphson method. The static Jacobian required for this purpose is given by

\[
[J_{\text{static}}] = \frac{\partial f_s}{\partial \bar{X}}|\bar{X} + \frac{\partial f_a}{\partial \bar{X}}|\bar{X}
\]

The aerodynamic part of the static Jacobian for trim calculations is given by

\[
\frac{\partial f_a}{\partial \bar{X}}|\bar{X} = q [B] ([D^\text{vort}_s(X, M_\infty)])^{-1} [W]
\]
where the downwash is linearised as,

\[ \{w\} = [W] \{X\} \tag{65} \]

The aerodynamic influence coefficient matrix from the doublet lattice method is not included while generating the Jacobian for trim computations.

Eqs. (60) can be solved to obtain the trim state for a given set of control parameters or, having augmented the equations with as many equations as free control parameters, they can be solved to obtain the control parameters for a pre-specified flight condition.

D. Linear Stability Analysis

The linear stability of the system about the computed trim point can be studied by computing the eigenvalues of the linearised system given by

\[ [J_{dyn}] \{\dot{X}\} + [J_{static}] \{\dot{X}\} = 0 \tag{66} \]

where

\[ [J_{dyn}] = \frac{\partial f_s}{\partial X}|_{X} \tag{67} \]
\[ [J_{static}] = \frac{\partial f_s}{\partial X}|_{X} + \frac{\partial f_a}{\partial X}|_{X} \tag{68} \]

and

\[ \frac{\partial f_a}{\partial X}|_{X} = q \left[ B \left( [D_v^{\text{pers}}(X, M_\infty)] + [\Delta D_d^{\text{doub}}(X, k, M_\infty)] \right)^{-1} [W] \right] \tag{69} \]

Stability analysis is carried out by the Non-Iterative \( p-k \) method, wherein eigenvalues are computed for multiple values of reduced frequency \( k \). These complex eigenvalues give multiple frequency and damping values for each \( k \) value. However, for a given trim speed, the actual frequency can be computed as a function of the reduced frequency. Only these frequencies and the corresponding damping values correspond to the actual physical system being studied.

E. Augmented Equations for computing control parameters

For a flight condition specified by a trim speed \( V_\infty \), flight path angle \( \gamma \), sweep angle \( \Lambda \) and turn rate in the inertial frame \( r_i \), values of four control parameters have to be computed. These are the values of main engine thrust \( T_{\text{main}} \), auxiliary engine thrust \( T_{\text{aux}} \), flap deflection \( \delta_f \) and aileron deflection \( \delta_a \). Multiple sets of control parameter values can be obtained by varying the orientation of the auxiliary engine with respect to the freestream \( \delta_{aux} \). The four equations that augment the set of equations for trim computation are given by,

\[ V_1^n s_2 + V_2^n s_2 + V_3^n s_2 - V^n_\infty = 0 \tag{70} \]
\[ \Omega^n_{gs}. (g^n_{gs}/g_0) - r_i = 0 \tag{71} \]
\[ V^n_1 s - V_2^n s \tan \Lambda = 0 \tag{72} \]
\[ V^n_1 g_1^n + V^n_2 g_2^n + V^n_3 g_3^n + V^n_\infty g_0 \sin \gamma = 0 \tag{73} \]

III. Example configuration

The sample configuration being studied is partly based on values given in Ref. 1. The flying wing has an unswept wing span of 80ft. The chord at the center of the unswept wing is 8ft long, and each half of the wing has a taper ratio of 0.5. Aerodynamic control surfaces are 20 percent of the local chord length. The trailing edge of each half-wing section is divided into two equal parts; with the outer half functioning as the aileron and the inner half functioning as the elevator. Other parameters that characterise structural properties are given in Table (1).
<table>
<thead>
<tr>
<th>Elastic (reference) axis</th>
<th>20% chord</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsional rigidity</td>
<td>$1.2 \times 10^9$ lb ft$^2$</td>
</tr>
<tr>
<td>Bending rigidity</td>
<td>$7.5 \times 10^9$ lb ft$^2$</td>
</tr>
<tr>
<td>Bending rigidity (chordwise)</td>
<td>$90 \times 10^9$ lb ft$^2$</td>
</tr>
<tr>
<td>Mass per unit length</td>
<td>6000 lbs/ft</td>
</tr>
<tr>
<td>Additional mass at wing center</td>
<td>12000 lbs</td>
</tr>
<tr>
<td>Center of gravity</td>
<td>20% chord</td>
</tr>
<tr>
<td>Centroidal Mass Mom. Inertia:</td>
<td></td>
</tr>
<tr>
<td>about $x$-axis (torsional)</td>
<td>$18 \times 10^4$ lb ft</td>
</tr>
<tr>
<td>about $y$-axis</td>
<td>$3 \times 10^4$ lb ft</td>
</tr>
<tr>
<td>about $z$-axis</td>
<td>$15 \times 10^4$ lb ft</td>
</tr>
<tr>
<td>Aerodynamic Coefficient $C_{D_0}$</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Table 1. Wing cross-sectional properties

A. Trim computation

1. Rigid structure

The airplane is trimmed at multiple subsonic flight speeds for the same value of auxiliary engine angle and wing sweep. Static aeroelastic deformations of the wing are also accounted for in trim computations. A representative chart of the variation of thrust, aileron deflection angle, flap deflection angle, angle of attack and angle of sideslip with trim speed is given in Table (2). The wing is discretized into 30 spanwise panels and 5 chordwise panels. The airplane is trimmed in straight and level flight at sea level where the air density has a value of $0.076 \text{ lb/ft}^3$. The wing sweep is set at 35 degrees while the auxiliary engine deflection is set at 15 degrees for all three cases. Flexibility values are set to zero when the structure is assumed to be rigid.

<table>
<thead>
<tr>
<th>Trim speed</th>
<th>260 ft/s</th>
<th>500 ft/s</th>
<th>775 ft/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach number</td>
<td>0.23</td>
<td>0.45</td>
<td>0.69</td>
</tr>
<tr>
<td>Main engine thrust</td>
<td>55312.33 lb</td>
<td>55360.33 lb</td>
<td>110198.73 lb</td>
</tr>
<tr>
<td>Auxiliary engine thrust</td>
<td>138.82 lb</td>
<td>515.28 lb</td>
<td>1238.24 lb</td>
</tr>
<tr>
<td>Flap deflection</td>
<td>-2.73 degrees</td>
<td>-0.70 degrees</td>
<td>-0.26 degrees</td>
</tr>
<tr>
<td>Aileron deflection</td>
<td>0.99 degrees</td>
<td>0.28 degrees</td>
<td>0.13 degrees</td>
</tr>
<tr>
<td>Angle of attack</td>
<td>6.17 degrees</td>
<td>1.60 degrees</td>
<td>0.60 degrees</td>
</tr>
</tbody>
</table>

Table 2. Trim results for Rigid Structure

2. Flexible structure

For computing the trim for a flexible structural model, flexibility values are set to be equal to those shown in Table (1). A representative chart of the variation of thrust, aileron deflection angle, flap deflection angle, angle of attack and angle of sideslip with trim speed is given in Table (3).

B. Stability analysis

1. Rigid Structure and Quasi-steady Aerodynamics

Stability analysis for a model using quasi-steady aerodynamics can be carried out by studying the eigenvalues generated by the linearized system of equations represented by Eqs. (66), without resorting to use of the ‘p-k’ method. The eigenvalues corresponding to aeroelastic modes and rigid body flight dynamic modes are identified by studying the corresponding eigenvectors.
<table>
<thead>
<tr>
<th>Trim speed</th>
<th>260 ft/s</th>
<th>500 ft/s</th>
<th>775 ft/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach number</td>
<td>0.23</td>
<td>0.45</td>
<td>0.69</td>
</tr>
<tr>
<td>Main engine thrust</td>
<td>55446.72 lb</td>
<td>55544.01 lb</td>
<td>110616.85 lb</td>
</tr>
<tr>
<td>Auxilllary engine thrust</td>
<td>12.05 lb</td>
<td>328.50 lb</td>
<td>812.74 lb</td>
</tr>
<tr>
<td>Flap deflection</td>
<td>-2.74 degrees</td>
<td>-0.71 degrees</td>
<td>-0.27 degrees</td>
</tr>
<tr>
<td>Aileron deflection</td>
<td>1.11 degrees</td>
<td>0.43 degrees</td>
<td>0.38 degrees</td>
</tr>
<tr>
<td>angle of attack</td>
<td>6.19 degrees</td>
<td>1.62 degrees</td>
<td>0.61 degrees</td>
</tr>
</tbody>
</table>

Table 3. Trim results for Flexible Structure

<table>
<thead>
<tr>
<th>260 ft/s</th>
<th>500 ft/s</th>
<th>775 ft/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.009075 - 0.007409i</td>
<td>0.001517</td>
<td>0.000355</td>
</tr>
<tr>
<td>0.009075 + 0.007409i</td>
<td>0.027155</td>
<td>0.043286</td>
</tr>
<tr>
<td>-0.023680</td>
<td>-0.039311</td>
<td>-0.060143</td>
</tr>
<tr>
<td>-0.003177 - 0.118823i</td>
<td>-0.003109 - 0.061708i</td>
<td>-0.004536 - 0.039402i</td>
</tr>
<tr>
<td>-0.003177 + 0.118823i</td>
<td>-0.003109 + 0.061708i</td>
<td>-0.004536 + 0.039402i</td>
</tr>
<tr>
<td>-0.864097</td>
<td>-1.679958</td>
<td>-2.650486</td>
</tr>
<tr>
<td>-1.054797 - 1.925599i</td>
<td>-2.140080 + 3.806982i</td>
<td>-3.732429 + 6.299807i</td>
</tr>
<tr>
<td>-1.054797 - 1.925599i</td>
<td>-2.140080 + 3.806982i</td>
<td>-3.732429 + 6.299807i</td>
</tr>
</tbody>
</table>

Table 4. Eigenvalues for rigid structure and quasi-steady aerodynamics

2. Rigid Structure and Unsteady Aerodynamics

The eigenvalues for each value of reduced frequency are plotted in 3D, with the reduced frequency along the x-axis, the imaginary part of the eigenvalue along the y-axis and the real part of the eigenvalue along the z-axis. Following this, the eigenvalues corresponding the actual system are isolated by plotting a plane whose projection on the x-y plane at all z-locations is given by the line $\omega = kV_{\infty}/b$. The points at which the eigenvalue-k plots intersect this plane correspond to the eigenvalues of the actual system.

Alternately, the eigenvalues corresponding to the actual system at each value of $k$, can be identified by subtracting $i\omega = ikV_{\infty}/b$ from each eigenvalue. Among the complex numbers thus generated, those that have an imaginary part equal to zero give the damping of the real system.

<table>
<thead>
<tr>
<th>260 ft/s</th>
<th>500 ft/s</th>
<th>775 ft/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.009075 - 0.007409i</td>
<td>0.001517</td>
<td>0.000355</td>
</tr>
<tr>
<td>0.009075 + 0.007409i</td>
<td>0.027155</td>
<td>0.043286</td>
</tr>
<tr>
<td>-0.023680</td>
<td>-0.039311</td>
<td>-0.060143</td>
</tr>
<tr>
<td>-0.003177 - 0.118823i</td>
<td>-0.003109 - 0.061708i</td>
<td>-0.004536 - 0.039402i</td>
</tr>
<tr>
<td>-0.003177 + 0.118823i</td>
<td>-0.003109 + 0.061708i</td>
<td>-0.004536 + 0.039402i</td>
</tr>
<tr>
<td>-0.003177 + 0.118823i</td>
<td>-0.003109 + 0.061708i</td>
<td>-0.004536 + 0.039402i</td>
</tr>
<tr>
<td>-0.864097</td>
<td>-1.679958</td>
<td>-2.650486</td>
</tr>
<tr>
<td>-2.276 + 1.579i</td>
<td>-4.628 + 3.089i</td>
<td>-8.36 + 4.84i</td>
</tr>
<tr>
<td>-2.276 - 1.579i</td>
<td>-4.628 - 3.089i</td>
<td>-8.36 - 4.84i</td>
</tr>
</tbody>
</table>

Table 5. Eigenvalues for rigid structure and unsteady aerodynamics
Table 6. Eigenvalues for flexible structure and unsteady aerodynamics

<table>
<thead>
<tr>
<th></th>
<th>260 ft/s</th>
<th>500 ft/s</th>
<th>775 ft/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001724</td>
<td>-0.0006</td>
<td>-6.8318e-005</td>
<td></td>
</tr>
<tr>
<td>0.027</td>
<td>0.0362</td>
<td>0.045006</td>
<td></td>
</tr>
<tr>
<td>-0.034</td>
<td>-0.046</td>
<td>-0.061538</td>
<td></td>
</tr>
<tr>
<td>-0.0031 + 0.118i</td>
<td>-0.003 - 0.06169i</td>
<td>-0.004 - 0.039380i</td>
<td></td>
</tr>
<tr>
<td>-0.0031 + 0.118i</td>
<td>-0.003 + 0.06169i</td>
<td>-0.004 + 0.039380i</td>
<td></td>
</tr>
<tr>
<td>-0.8431</td>
<td>-1.531</td>
<td>-2.13</td>
<td></td>
</tr>
<tr>
<td>-2.53 - 1.44i</td>
<td>-4.628 - 3.089 i</td>
<td>-8.36 - 4.84i</td>
<td></td>
</tr>
<tr>
<td>-2.53 + 1.44i</td>
<td>-4.628 + 3.089i</td>
<td>-8.36 + 4.84i</td>
<td></td>
</tr>
<tr>
<td>0 - 18.2i</td>
<td>-0.5 - 17.5i</td>
<td>-1.74 - 16.69i</td>
<td></td>
</tr>
<tr>
<td>0 + 18.2i</td>
<td>-0.5 + 17.5i</td>
<td>-1.74 + 16.69i</td>
<td></td>
</tr>
<tr>
<td>0.059 - 21.35i</td>
<td>0 - 21.25i</td>
<td>-0.57 - 20.13i</td>
<td></td>
</tr>
<tr>
<td>0.059 + 21.35i</td>
<td>0 + 21.25i</td>
<td>-0.57 + 20.13i</td>
<td></td>
</tr>
<tr>
<td>0.09926 - 36.45i</td>
<td>0.22 - 35.96i</td>
<td>0.2 - 34.88i</td>
<td></td>
</tr>
<tr>
<td>0.09926 + 36.45i</td>
<td>0.22 + 35.96i</td>
<td>0.2 + 34.88i</td>
<td></td>
</tr>
</tbody>
</table>

3. Flexible Structure and Unsteady Aerodynamics

IV. Conclusions

This paper presents a model for studying the aeroelasticity and flight dynamics for oblique flying wings. Sample results are presented for control inputs to obtain a pre-specified trim condition. Unlike conventional configurations which are trimmed in the plane of symmetry, the longitudinal and lateral flight dynamics for an oblique flying wing are expected to be coupled. Work is currently in progress to study the stability of these configurations using both quasi-steady and unsteady aerodynamic models.

References


