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Limit Cycle Oscillations of Aircraft due to Flutter-Induced Drag

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The present work is a continuation of earlier work by the author on the energetics of flutter. It has been shown in earlier work that the energy for flutter comes from propulsion, i.e., flutter leads to increase in drag and thus the thrust has to be increased in order for the wing to maintain instability. In reality, the thrust is not increased to maintain the speed while the wing goes into flutter. The present paper seeks to explain the effect of flutter-induced drag on the response of the aircraft. It is shown that the aircraft can undergo limit cycle oscillations (LCOs) even if the oscillations are in the linear structural and aerodynamics lift ranges. This LCO occurs due to the inability of the engine to support exponentially increasing oscillations. Such a LCO is composed of variations in airspeed and amplitude of vibration. This modality of LCOs has not been presented in the literature before and leads to better understanding of the LCO problem.

Introduction

The paper presents a new type of Limit Cycle Oscillation (LCO) in flexible aircraft. The results from this work lead to better understanding of energy flow which forms the basis of aeroelastic response and instability. As such the topic of the present work fits right into the structural dynamics area of AIAA-SDM conference.

LCO

Limit Cycle Oscillations of wings and aircraft is an important problem that has received a lot of attention over the decades. Aeroelastic LCO is normally associated with flutter. It is assumed that nonlinearities in the structure or the aerodynamic forces leads to limiting of the amplitude of oscillations of an unstable system. Dugundji,1 Dowell,2 Strganac,3 Hodges4 have done experimental (wind tunnel) and/or theoretical investigation of LCOs in wings and airfoil.

Energy Considerations

Earlier work by the author5 investigates the energy transfer pathways in aeroelastic flutter. The paper presents a new paradigm for understanding flutter. It integrates aeroelastic interactions with applied and induced propulsive effects. The framework is based on conservation of energy. Energy produced, energy lost and/or work done due to structural vibration, aerodynamic wake and propulsion are taken into account. It is shown that there exist three types of modes in an aeroelastic system, viz., i) unstable (flutter) mode producing drag, ii) stable mode producing drag, and, iii) stable mode producing thrust (flapping).

Fig. 1 illustrates the energetics of these modes. It shows three energy sinks/sources, 1) Structure: denotes the energy of the structure, the direction of energy flow indicating the work done by or on the structure, 2) Wake: is the energy lost in the fluid, and, 3) Propulsion: is the work done by the engine to maintain a given speed. If the structural energy is increasing it indicates an unstable (flutter) mode while decreasing structural energy indicates a damped mode, i.e., work has to be done on the structure to maintain a level of oscillations. Wake energy is the kinetic energy of the fluid due to the vorticity shed in an otherwise stationary flow. This energy is constantly increasing. Finally, flow of energy from propulsion indicates a requirement of higher thrust to maintain a given speed and thus denotes a drag producing mode whereas flow of energy into propulsion indicates that the mode produces thrust. It should be noted that this propulsive energy is in addition to that required to maintain the trim condition.

Flutter and Drag

It is clear that when the wing flutters, the energy comes from propulsion, i.e., when the wing starts to
oscillate in the flutter mode it leads to increase in the drag. Thus, to maintain the aircraft speed the engine needs to increase the thrust and this is the increase in energy expenditure that results in flutter and increase in structural (and wake) energy. This phenomenon is not obvious if the airspeed is assumed constant while solving aeroelastic problems as is usually done. With the assumption of constant velocity it is implicit that whatever thrust required to maintain that velocity is provided by the engine.

Now, in actual flight the thrust is not increased to maintain the flutter speed but rather a specified thrust is applied (as required for steady trimmed flight). If this is the case, then, as the system starts to flutter, the drag on the aircraft will increase and the aircraft will decelerate. One can only speculate at this point on the response of the system in a dynamic sense, but it can be seen that the problem is now coupled between the airspeed and the oscillation amplitude. The airspeed determines the stability of the wing and thus the oscillation amplitude of the wing (whether it will be damped or it will explode exponentially). On the other hand, the amplitude of oscillation determines the drag on the aircraft and thus the airspeed of the aircraft (whether it will accelerate or decelerate).

One possible solution is if the aircraft goes down to the critical flutter speed and oscillates at an amplitude which leads to loss of energy in aerodynamic wake equal to the excess of propulsive energy. This is the only possible steady state. Other solution is a periodic solution which involves periodic change in the flight velocity and the amplitude of vibration. A chaotic variation in the flight speed and oscillation amplitude cannot be ruled out either. The present aeroelastic problem is similar to non-ideal power source problem that has been analyzed in non-linear dynamic literature.

**Problem Statement**

The objectives of the present work are;

1. Derive a theoretical basis for aeroelastic analysis of aircraft that takes in to account induced drag and flight dynamics.

2. Write a computer program based on the aeroelastic formulation

3. Generate results pertaining to the aeroelastic behavior of a complete aircraft

4. Understand the aeroelastic results leading to further insight into the energetics of flutter and the dynamics of LCO.

**Analysis Methodology**

The analysis methodology is based on simple beam model for structure and unsteady vortex lattice method for aerodynamics. In unsteady vortex lattice method, vortices are distributed on the wing as well as in the wake. Various conditions on the vortex strengths are then imposed to solve the problem. The discrete unsteady vortex lattice relations are summarized below:

\[
K_b \Gamma_b^{n+\frac{1}{2}} + K_w \Gamma_w^{n+\frac{1}{2}} - W^{n+\frac{1}{2}} = 0
\]  

where, \(\Gamma_b\) and \(\Gamma_w\) denote the strengths of bound and wake vortices, \(K_b\) and \(K_w\) are the corresponding induced velocity coefficients, \(W\) denotes the downwash due to structural deflections, and the superscript denotes the time iteration at which the variable are calculated.
b) Shed vorticity - bound vorticity relation relates the change in bound vorticity to the wake vorticity shed in that time interval.

\[ \Sigma \Gamma_b^n - \Sigma \Gamma_b^{n+1} - \Gamma_w^{n+1} = 0 \]  

(2)

where, \[ \Sigma \Gamma_b \] is the shedding vorticity at the vortex location due to the trailing vortices and wake vorticity at the next vortex element.

\[ \Gamma_{n+1}^{w_1} - \Gamma_{n}^{w_1} = 0 \]  

(3)

The equations given above form a complete set of equations representing the dynamics of bound and wake vortices. The equation are in discrete time. The equations can be converted to continuous time form using finite difference equations. The continuous-time wake dynamic equations are given by,

\[ \{ \dot{\Gamma} \} = [A_1]\{\Gamma\} + [A_2]\{W\} \]  

(4)

where, \[ [A_1] \] and \[ [A_2] \] are coefficient matrices which relate the vorticity (\( \Gamma \)) to structural deformations (\( W \)). \( W \) can be written in terms of the structural displacement variables.

The aerodynamics equations can be coupled with the structural equations to form a complete aeroelastic set. The structural equations as derived using the beam modeshapes are given by,

\[ [M]\{\dot{q}\} + [K]\{q\} = \{Q\} \]  

(5)

where, \([M]\) and \([K]\) are the mass and stiffness matrices, \( \{q\} \) are the modal displacement, and, \( \{Q\} \) are the modal forces.

The modal forces are calculated from the aerodynamic lift distribution. The lift distribution is given by,

\[ L = \rho U \Gamma + \rho \int_{-b}^{b} \dot{\Gamma}_b dx \]  

(6)

Eqns. 4 and 5 form a set of linear aeroelastic equations for the dynamics of the complete aircraft.

\[ \Sigma \Gamma_b^n - \Sigma \Gamma_b^{n+1} - \Gamma_w^{n+1} = 0 \]  

(7)

where, \( \Sigma \Gamma_b \) is the shedding vorticity at the vortex location due to trailing and wake vortices and \( \alpha \) is the geometrical angle-of-attack.

Eqns. 4, 5, and 7 together form a complete set of aeroelastic equations for the dynamics of the complete aircraft.

A MATLAB program has been written based on the aeroelastic analysis described above. The program gives the aeroelastic response of an aircraft. The program can be used to:

- simulate the energy transfer during cantilevered wing aerelastic response
- show the increase in thrust requirement at flutter
- investigate the possibility of LCO if the thrust is kept constant.

\[ D_{\text{induced}} = \rho w_z \Gamma + \rho \alpha \int_{-b}^{b} \dot{\Gamma}_b dx \]  

(8)

\[ \Sigma \Gamma_b^n - \Sigma \Gamma_b^{n+1} - \Gamma_w^{n+1} = 0 \]  

(9)

where, \( w_z \) is the downwash at the vortex location due to trailing and wake vortices. The induced drag can be expressed as:

\[ M_{\text{aerodynamic}}\dot{U} = T - \frac{1}{2} \rho U^2 SC_{d_\alpha} - D_{\text{induced}} \]  

(10)

where, \( M_{\text{aerodynamic}} \) is the mass of the aircraft, \( U \) is the airspeed, \( T \) is the thrust, \( C_{d_\alpha} \) is the viscous drag coefficient, and \( D_{\text{induced}} \) is the induced drag.

Induced drag is a second-order nonlinear term, but, in the absence of a linear drag term, it is quite important even for small motions. The induced drag has a steady component due to the trailing vortices and a unsteady component due to the shed wake vortices. The induced drag can be expressed as:

\[ D_{\text{induced}} = \rho w_z \Gamma + \rho \alpha \int_{-b}^{b} \dot{\Gamma}_b dx \]  

(8)

Results

Results are presented for a high-aspect-ratio wing with ten equally spaced flaps. The geometric and structural data for the wing is given in Table 1. The structure is modeled using five bending and five torsion modes. The airflow is modeled using 50 wing vortices and 125 wake vortices distributed over 5 spanwise and 25 chordwise station (over five chord lengths).

2-D Comparisons

Before studying the aeroelastic characteristics of a wing, it is essential to gauge the accuracy of the unsteady vortex lattice method (VLM) in predicting the aerodynamic forces, especially the drag. Since exact solutions are not available for the 3-D problem, a 2-D VLM time-marching solution is checked against 2-D Theodorsen results.

Let us consider a 2-D airfoil oscillating in pitch and plunge. Using von Karman and Burgers,\(^3\) Theodorsen\(^8\) and Garrick\(^10\) one can write the expression for the lift (\( L \)) and drag (\( D \)) as,

\[ L = 2\pi \rho b b \left( U \hat{\alpha} + \hat{h} \right) + C(k) U \left( U \hat{\alpha} + \hat{h} + \frac{1}{2} b \hat{\alpha} \right) \]  

\[ D = L \alpha - 2\pi \rho b b \left( C(k) \left( U \hat{\alpha} + \hat{h} + \frac{1}{2} b \hat{\alpha} \right) - \frac{1}{2} b \hat{\alpha} \right)^2 \]  

(9)

where, \( \hat{\alpha} \) and \( \alpha \) are the midchord plunge and pitch angle, \( \rho, b, U \) are the air density, semi-chord, airspeed respectively, and \( C(k) \) is the Theodorsen’s function which depends on the reduced frequency \( k = \frac{b \alpha}{2} \). The
above expressions are frequency domain solutions for harmonic structural deformations.

The lift and drag given by Theodorsen/Garrick (TG) can be compared with the results of the 2-D time-domain unsteady vortex lattice method. Figs. 2 - 5 plot the aerodynamic response to harmonic input as calculated by the VLM. The lift and drag from the VLM calculations should approach the TG results after a few oscillation. Fig. 2 shows the normalized lift response to a cosine plunge input. As can be seen in the figure, the lift predicted by the VLM is almost the same as that predicted by TG. Fig. 3 shows the corresponding drag plot. Again the VLM results are right on top of the TG results. Also, as expected the drag plot shows that, i) average drag is not zero, ii) drag varies with twice the frequency as the deformation, iii) for pure plunging motion, drag is negative (i.e. there is thrust). Figs. 4 and 5 show the lift and drag for pitch oscillations. Again the comparison with TG is excellent. This shows that VLM can capture the unsteady flow field quite accurately, and consequently, leads to accurate prediction of the aerodynamic forces.

3-D Lift and Drag

Figs. 6 - 9 show the aerodynamic response plots for a 3-D wing. Here the wing is oscillated in either the first bending or the first torsion mode. The amplitude of oscillations is such that the tip displacement is 1 m for the bending mode while the tip twist is 1 rad for the torsion mode. Fig. 6 shows the total unsteady lift acting on the wing oscillating in the first bending mode. A reduced frequency of 0.25 and a velocity of 35 m/s is used for the calculations. Fig. 7 shows the corresponding induced drag on the wing. Again, as expected, a bending mode produces thrust. Finite-wings produce trailing vortices which produce drag and thus thrust generated by finite-wings is less than that by a corresponding 2-D airfoil. It is seen that unlike the 2-D case the thrust is negative over a small portion of the oscillation.

It should be noted that the magnitude of the drag/thrust is of the same order as the magnitude of the lift. This goes to show the importance of drag even for not-so-large oscillation (1 m tip displacement on a 16 m wing). It should be reiterated that the drag/thrust is a quadratic function of the oscillation amplitude while the lift is linearly related to the oscillation amplitude. Thus the drag would increase at a higher rate with increase in amplitude as compared to lift. This again goes to point out the importance of including the drag/thrust forces in aeroelastic analysis.

Figs. 8 and 9 show the lift and drag for oscillations in the first torsion mode. Again, a reduced frequency of 0.25 and a velocity of 35 m/s is used for the calculations. Here we see that the first torsion mode produces drag and not thrust. Also, the drag amplitude is of the same order as the lift amplitude.

Post-Flutter Response

The aeroelastic model described in the earlier section is now used to calculate the aeroelastic response of the wing and aircraft. The flutter calculations give the flutter speed for the wing to be 33.4 m/s. A wing flying at speeds above the flutter speed is analyzed. Linear stability theory predicts that any disturbances to the wing will grow exponentially. Exponentially increasing response will continue to infinity for a linear system.

Fig. 10 shows the tip displacement, tip twist and the airspeed for a wing at a trim condition of 35 m/s. A disturbance of 0.01 rad tip twist in the first torsion mode is provided to the system. The wing is assumed to maintain the trim airspeed of 35 m/s. It is seen that the response (tip displacement and tip twist) increases exponentially, exceeding practical limits within 10 sec. One can analyze the role of structural and aerodynamic nonlinearities in restricting the amplitude of oscillations. Analysis of such LCOs has been presented earlier and is not the focus of the present work.

The focus of the present work is to expose the role of flutter-induced drag. Fig. 10 also presents the response of the aircraft as a whole to the same initial velocity and disturbance. The response of the aircraft is calculated using Eq. 7. The equation only takes into account the dynamics of the aircraft in terms of speed changes. The pitch and plunge dynamics of the aircraft are neglected as the present focus is on the effects on airspeed. Now, as the wing oscillation amplitude increases exponentially, there is a corresponding increase in the drag on the wing. This leads to decrease in the airspeed. As the airspeed decreases continuously, it will fall below the flutter speed and the wing oscillation amplitude will start to decrease exponentially. Thus as show in the figure, taking into account the aircraft dynamics, it is clear that the oscillation amplitude will not tend to infinity.

LCO

When the aircraft is trimmed at a flight speed above flutter, the wing will oscillate, the drag will increase, and thus the airspeed will change. Essentially, the wing oscillation amplitude as well as the airspeed change with time. Fig. 11 shows the aircraft response over a longer duration (100 sec). The figure shows that the the aircraft speed oscillates about the flutter speed. The amplitude of vibrations also oscillates. This is expected because, i) as the airspeed decreases below flutter speed, the oscillation...
amplitude starts to decrease, and ii) as the oscillation amplitude decreases, so does the drag, and the aircraft then accelerates to a speed above flutter speed leading to an increase in the amplitude of vibration.

Fig. 11 shows that the oscillations in the airspeed decrease and the airspeed approaches a steady airspeed equal to the flutter speed. There is a corresponding decrease in the oscillation of vibration amplitude and it approaches a constant amplitude oscillation. Thus the aircraft trimmed at a speed above the flutter speed approaches the flutter speed if the thrust is held constant. At the flutter speed, the flutter mode is neutrally stable and a constant amplitude vibration is maintained. The amplitude of vibration is such that the effective drag is equal to the difference between the thrust provided for the trim flight and the thrust necessary for the flight at the flutter speed. Thus the drag induced by flutter leads to a LCO such that the structure vibrates with a constant amplitude in the flutter mode and the airspeed is maintained at the flutter speed.

Now, if the thrust is higher, then the trim speed will be higher. The difference between the thrust provided and the thrust required (at the flutter speed) will be higher and thus the LCO amplitude will be higher. Fig. 12 shows the LCO of the aircraft at various trim conditions. It is seen that for speeds below the flutter speed, the disturbance dies out quickly and thus the airspeed is maintained. For trim velocities above the flutter speed the disturbance initially grows exponentially and then the vibration amplitude and airspeed oscillate. For higher speed the oscillations in the vibration amplitude and the airspeed are higher. Also, as expected, the steady state LCO vibration amplitude is higher.

Fig. 13 shows the plot of steady-state LCO amplitude as a function of trim flight speed. It should be noted that at steady state the aircraft will be at the flutter speed for all the different trim speeds. In essence, the trim speed is an indication of the trim thrust. The higher trim speed indicates higher thrust from the engine. As the trim speed/thrust is increased above the flutter speed there is LCO. The amplitude of LCO increases with the trim speed/thrust.

Figs. 14 and 15 show the effect of the drag coefficient and the fuselage mass on the LCO of the aircraft. The plots show the response in terms of the airspeed. From Eq. 7 it is clear that if \( CD_0 \) increases or \( M_{aircraft} \) decreases, the frequency of oscillation will increase. The plots show such a change in the LCO with variation in \( CD_0 \) and \( M_{aircraft} \). The plots also show that as the \( CD_0 \) increases or \( M_{aircraft} \) decreases, there is a decrease in the damping in the oscillations and thus it takes longer to reach the steady-state LCO.

Conclusions

The paper presents a new type of LCO. The LCO occurs due to the induced drag accompanying flutter vibrations. If the complete aircraft is considered, the induced drag couples the wing aeroelastic equations with the flight speed equation. Thus, the vibration amplitude and flight speed are interdependent. When disturbed at a speed above the flutter speed, the flight speed as well as the amplitude of vibrations oscillate initially. Eventually the oscillations converge to a steady, periodic, sinusoidal oscillation of the wing with the airspeed at the flutter speed. LCOs for various trim flight speeds, viscous drag coefficients and aircraft weights are presented.

The present work does not take into account the nonlinear characteristics of the wing. If the vibration amplitude is in the nonlinear range, still the actual response of the aircraft can be completely described only if flutter-induced drag is included in the analysis. Thus, the analysis as described in this paper should be an integral part of post-flutter analysis of aircraft.

References

Fig. 2  Comparison of normalized unsteady lift for $h = \cos k\bar{t}$ ($k = 0.5$)

Fig. 3  Comparison of normalized induced drag for $h = \cos k\bar{t}$ ($k = 0.5$)

Fig. 4  Comparison of normalized unsteady lift for $\alpha = \cos k\bar{t}$ ($k = 0.5$)

Fig. 5  Comparison of normalized induced drag for $\alpha = \cos k\bar{t}$ ($k = 0.5$)
Fig. 6  Unsteady lift for $h_{\text{bend}} = \cos \omega t$ ($k = 0.25$, $U = 35\text{m/s}$)

Fig. 8  Unsteady lift for $\alpha_{\text{torsion}} = \cos \omega t$ ($k = 0.25$, $U = 35\text{m/s}$)

Fig. 7  Induced drag for $h_{\text{bend}} = \cos \omega t$ ($k = 0.25$, $U = 35\text{m/s}$)

Fig. 9  Induced drag for $\alpha_{\text{torsion}} = \cos \omega t$ ($k = 0.25$, $U = 35\text{m/s}$)
Fig. 10  Comparison of post flutter response with and without induced drag effects ($U = 35m/s$)
Fig. 11 Limit cycle oscillations due to flutter induced drag at $U = 35m/s$. 

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Fig. 12 LCOs at various trim velocities
**Fig. 13** LCO amplitude as a function of trim flight speed

**Fig. 14** Effect of drag coefficient on the LCO at $U = 35 m/s$

**Fig. 15** Effect of aircraft weight on the LCO at $U = 35 m/s$

**Table 1** Model data

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<td>Torsional rigidity</td>
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<td>Bending rigidity (edgewise)</td>
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**FLIGHT CONDITION**

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