Parallel Multipoint Variable-Complexity Approximations for Multidisciplinary Optimization

Susan L. Burgee and Layne T. Watson
Departments of Computer Science
and Mathematics
Virginia Polytechnic Institute
and State University
Blacksburg, VA 24061-0106

Anthony A. Giunta, Bernard Grossman,
Raphael T. Haftka, and William H. Mason
Department of Aerospace Engineering
Virginia Polytechnic Institute
and State University
Blacksburg, VA 24061-0203

Abstract

The design of modern aerospace vehicles involves multidisciplinary interactions and presents a formidable challenge to the designer in a competitive marketplace. Multidisciplinary optimization technology is a major tool for addressing this need, but its current use in the design process is limited by its enormous computational burden. Even refined models for recent applications such as the High Speed Civil Transport fall short of the level of complexity required for realistic designs. The goal of our research is to apply the power of emerging parallel computation systems to permit a level of model refinement that would produce realistic designs. We propose a new variable-complexity approximation strategy that will take maximum advantage of parallel computation capabilities.

1: Introduction

The design of modern aerospace vehicles involves multidisciplinary interactions and presents a formidable challenge to the designer in a competitive marketplace. Multidisciplinary optimization (MDO) technology is a major tool for addressing this need, but its current use in the design process is limited by its enormous computational burden. To address this challenge, our group at Virginia Tech recently developed a variable-complexity-modelling approach, involving the use of refined and computationally expensive models together with simple and computationally inexpensive models ([8]). We have applied this approach to the combined aerodynamic-structural optimization of transport wings focusing recently on the High Speed Civil Transport (HSCT) ([5]–[7]). However, in these applications, even the refined models, based on linear aerodynamic and structural models, fell short of the level of complexity required for realistic designs. Our goal is to apply the power of emerging parallel computation systems to permit a level of model refinement that would produce realistic designs.

The application of parallel computation strategies to multidisciplinary analysis has attracted much interest, particularly in aeroelasticity research. Coupled aerodynamic and structural problems are amenable to parallel computation as demonstrated by the work of Byun and Guruswamy ([1]). The problem decomposition and parallelization techniques developed by these and other researchers will be applied to our existing analysis methods and utilized in the HSCT design optimization process.

Rather than simply relying on the promise of increased computational power, we consider a new variable-complexity approximation strategy that will take maximum advantage of parallel computation capabilities. The increased efficiencies inherent in this new approximation strategy together with parallel computational power will permit us to employ aerodynamic models based on Euler equations to supplement the simple algebraic models and panel models for the HSCT that we used before.

The new concept stipulates that approximations of aircraft response should span only those regions in design space that have some potential of being traversed by the search for the optimal design. In contrast to present approximations that define appropriate regions of design space only by move limits, the new approximation concept adds constraint and objective function limits to design variable limits in order to further reduce the approximation region. With this reduction in approximation region and parallel execution of refined analyses at selected points in the region, we hope to achieve an efficient and accurate approximation of both aerodynamic and structural response.

We envision an initial three-levels-of-complexity strategy. First we use the lowest level and least expensive models to arrive at a reasonable initial design. Next we use these models to define approximation domains for the more complex models. We use the two

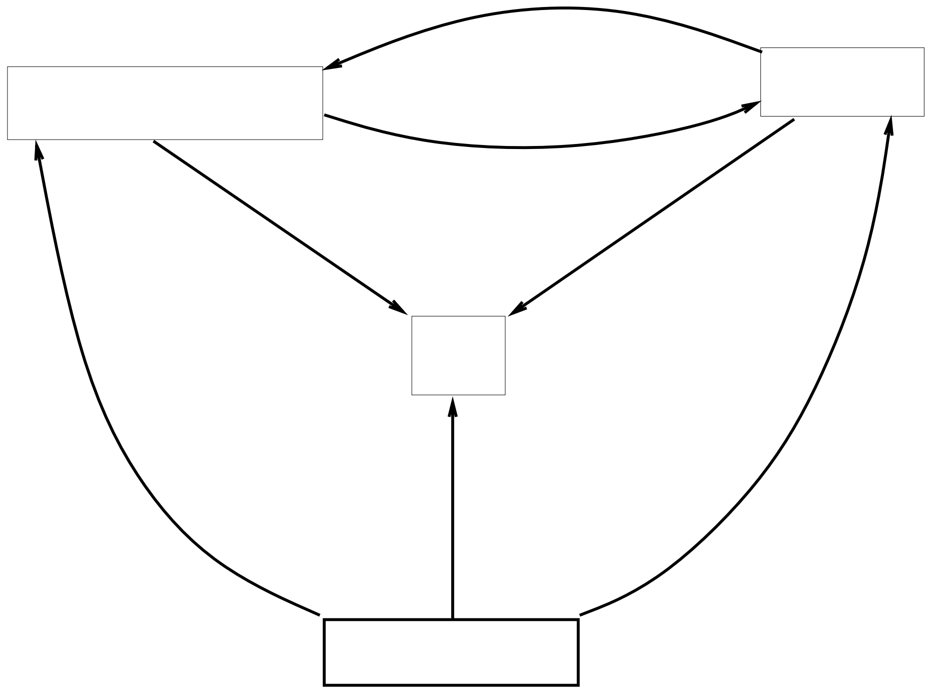

Figure 1. Inter-relationship of the technical tasks to the overall HSCT design. The application of all three research efforts to the HSCT will cement the activities into a single, coordinated group effort.

more complex models for constructing a multipoint variable-complexity approximation.

The overall research plan, in which multipoint approximation plays a crucial role, is geared toward redefining variable-complexity modelling in a parallel computing environment, as illustrated in Figure 1. The focus here is on the development of a parallel multipoint approximation with primary application to the aerodynamic design of the HSCT. Research challenges include the development of a strategy for selecting points in the approximation domain, constructing the multipoint approximation in parallel, and the coordination of algebraic models, panel models and Euler/Navier-Stokes models for predicting aerodynamic drag.

2: Multipoint approximations

Our previous work on scaled approximations ([6], [4], [2]) utilized a *single* refined analysis, possibly with its derivatives with respect to design variables, to construct the variable-complexity approximation. We now seek to develop approximations based on *multiple* refined analyses, executed in parallel. Multipoint approximations have attracted growing interest in the past few years, (e.g., [3]), and we believe that parallel computation advances will give them an edge over single-point approximations.

Past attempts to construct multipoint approximations worked well only for small to moderate numbers of design variables. For large numbers of design variables the *'curse of dimensionality'* dictated a very large number of analyses even for a low order approximation. For example, a quadratic approximation in n variables requires about n^2 analyses. We use the simple analysis to *limit* the region in design space where the approximation is utilized to alleviate the problem of constructing a good approximation when the number of design variables is large. This requires the following steps: (i) Define the approximation domain by using simple analysis. (ii) Select points in the approximation domain where we execute the refined analysis. (iii) Construct an approximation based on the refined and simple analyses.

2.1: Approximation domain definition

The approximation domain is centered around a nominal design point, usually obtained from the previous approximate optimization cycle. In current practice, the approximation domain is bounded only by move limits on the design variables. We bound the domain by additional limits on the values of the constraints and the objective function obtained from an inexpensive approximation. When the simple analysis is several orders of magnitude cheaper than the refined

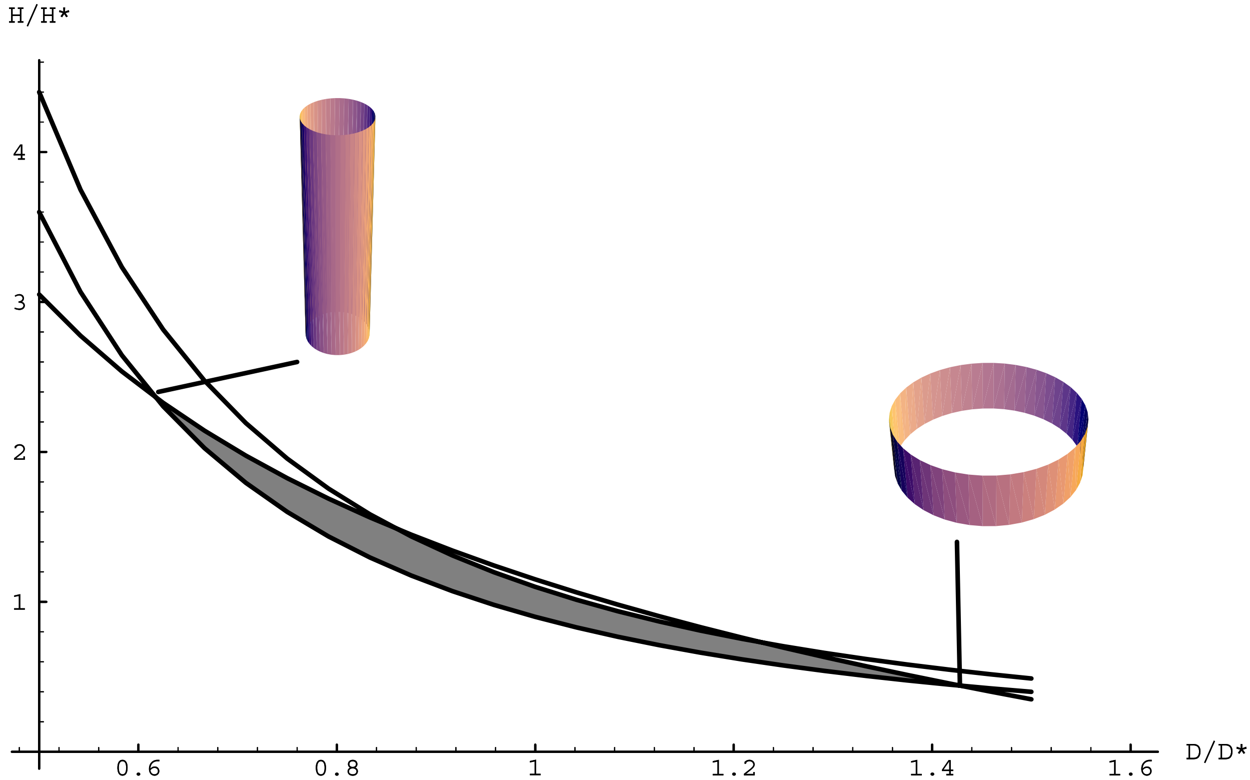

Figure 2. Cylindrical container design example illustrating the utility of the approximation domain concept. An approximation domain is based on a limitation of the objective function and constraints based on a simplified analysis. For the cylindrical container problem, the cost is a complicated function which is approximately proportional to the surface area. The approximation domain, indicated by the shaded area, is calculated for a 10% change in volume and surface area from an optimum based on the simple analysis. A large variation in design variables is indicated by the sketched shapes, but the approximation domain is only a thin sliver of design space.

analysis then the simple analysis defines directly the approximation domain. Otherwise, a linear approximation to the simple analysis is used. Constraints can bound the approximation domain by requiring that *the approximation to the constraint not be violated by more than a certain margin*, say 20 percent. Similarly, the objective function can bound the approximation domain by *limiting the change in the objective function.*

To illustrate the effectiveness of bounding the approximation domain by the constraint and objective function we consider the problem of designing a cylindrical container of given volume for minimum cost. Assume that from a simple analysis the cost is *proportional to the surface area.* Assume that a refined analysis of the cost requires a complex finite-element analysis of the can to determine the required thickness of the top, bottom and sides, as well as an analysis of the manufacturing cost. Based on a simple analysis the optimum container has a ratio of diameter to height of *one*. We use this as a nominal design and define the approximation domain to include all designs where the volume and surface area change by 10 percent from the nominal design. Figure 2 shows the approximation domain with the diameter D normalized by the optimum diameter D^*, and the height H normalized by the optimum height, H^*. We see that the limits on the constraint (volume) and objective function permit large changes in the design variables and the shape of the cylinder, as illustrated by the two cylinders sketched in the figure. However, the approximation domain is a thin sliver (shaded area) of the total domain.

2.2: Point selection for refined analysis

We next consider the problem of optimal placement of N points in the often elongated domain S produced

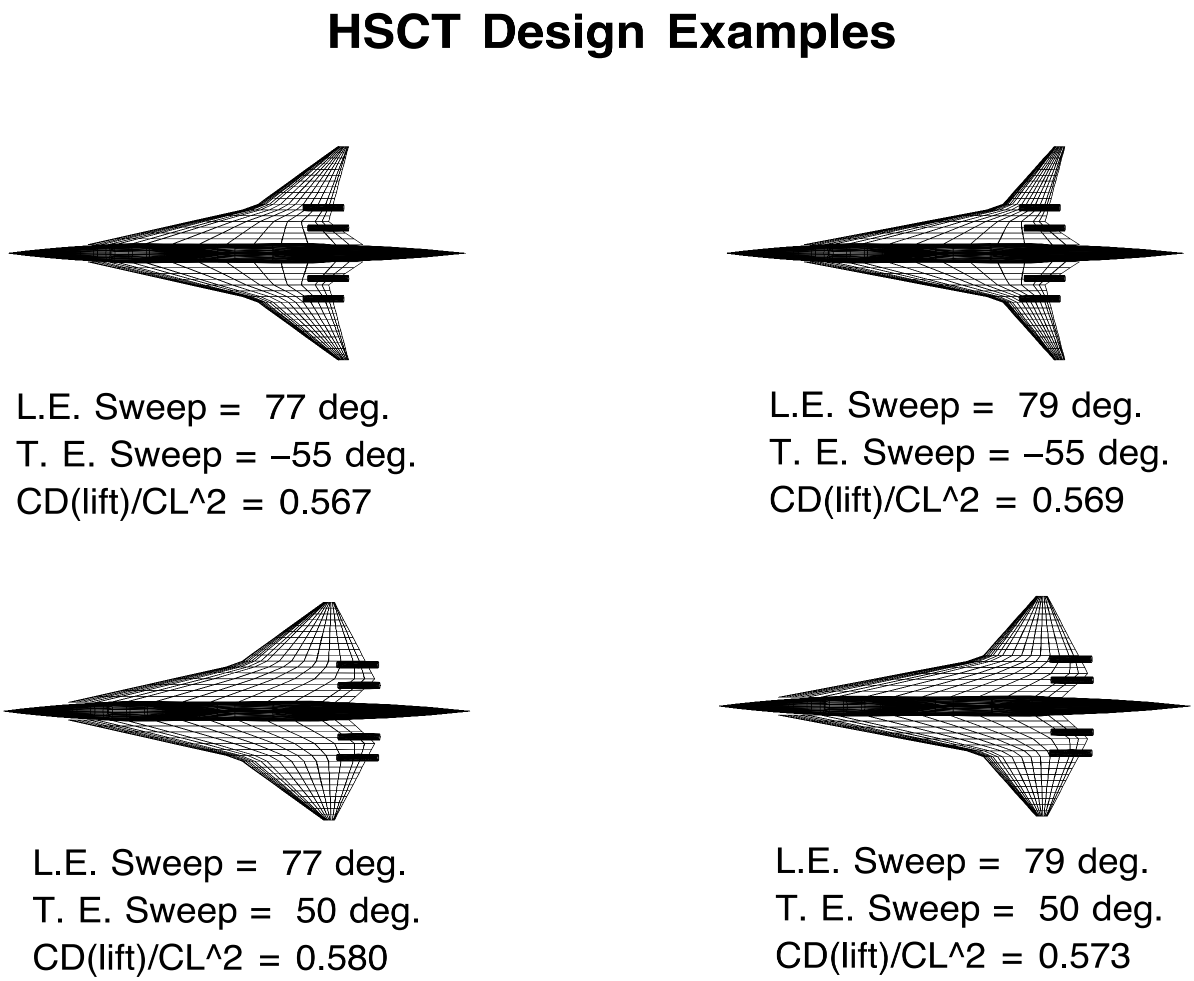

Figure 3. HSCT planforms corresponding to four extreme design points.

by bounds on design variables, constraints, and objective function in n dimensional space. We will evaluate several competing approaches, one based on treating the points as particles that repel one another, say like electrons, and the other based on an orderly progression using simplices.

Placing the points by treating them as electrons requires us to *minimize the potential energy associated with the forces between all pairs*. That is, we select the points x^i, $i = 1, \ldots, n$, so as to minimize

$$\sum_{i=1}^{n}\sum_{j=0}^{i-1}\frac{1}{\|x^i - x^j\|}, \qquad x^i \in S,$$

where x^0 denotes the nominal design. Of course, we must scale or normalize the design variables properly for the distances to be meaningful. This approach has an intuitive appeal, but the problem of minimizing the potential may prove to be computationally expensive.

The other scheme is based on the use of the simplex, which is the generalization of the triangle and the tetrahedron to higher dimensions. We start with an initial simplex with x^0 as one of its vertices, and create additional vertices by reflecting one vertex through the face opposite to it. For a two dimensional domain the result is a triangulation of the domain.

2.3: Construction of approximation

We seek to approximate a function $f(x)$ given a simple analysis $s(x)$ and a set of points x^i where the function was calculated by both a simple analysis $s(x^i)$ and refined analysis $r(x^i)$. These correction functions are chosen so as to minimize the difference between $f(x^i)$ and $r(x^i)$. We propose to begin with the approximation

$$f(x) \approx c_1(x)s(x) + c_2(x),$$

where $c_1(x)$ and $c_2(x)$ are multiplicative and additive corrections to the simple models. We plan to begin

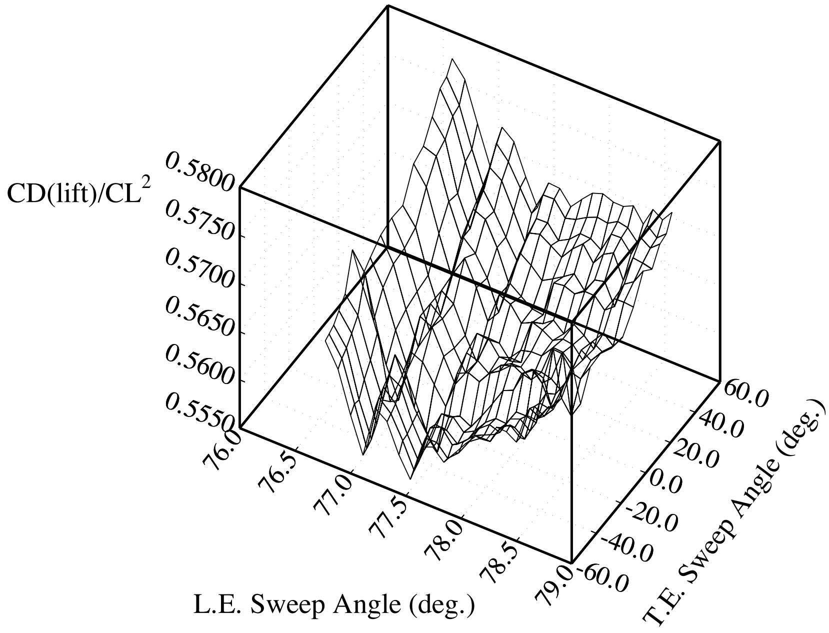

Figure 4. Noisy calculated surface for drag due to lift.

with linear polynomials with coefficients obtained by a least-squares fit to the data. Obviously, we can consider higher order polynomials for the correcting polynomials. Because of the special shape of the approximation domain, polynomial approximations may not do well, even if they only need to correct the simple analysis. Instead we plan also to explore the use of *rational functions* and *neural networks* to determine c_1 and c_2.

2.4: HSCT drag calculation

We have used ([6]) variable-complexity modelling to calculate the wave drag and drag due to lift of the HSCT. However, the refined approximations, the Harris drag code, and a vortex lattice approach were still not very accurate. We expect to test the proposed approach to upgrade the refined analysis to Euler-flow analysis. We plan to use all three analysis levels simultaneously. The least expensive analysis will define the approximation domain, while the other two analyses will be used in the approximation itself.

2.5: HSCT example problem

Our current HSCT design optimization research involves up to twenty-three design variables and fifty-three constraints. Although difficult to visualize in higher dimensions, the design space is easily visualized with only two design variables. When the objective function is plotted as a function of the two design variables, a three-dimensional surface is formed revealing the topography of the design space. Therefore, an example case was selected using two design variables, with the remaining variables held constant.

For the example case the two design variables were the wing leading-edge and trailing-edge break points. The design variables were changed by 5 percent from the current baseline HSCT configuration used in our studies. This corresponded to a variation of the leading-edge sweep angle from 77 to 79 degrees and a variation of the trailing-edge sweep angle from -55 to 50 degrees. The extreme HSCT designs produced by these variations are shown in Figure 3. The design surface was than discretized using a 21 x 41 uniform grid, and drag due to lift was calculated for the HSCT configuration at each point on the mesh. This produced a detailed map of the surface topography as shown in Figure 4. This view of the drag demonstrates the jagged surface features which reduce the effectiveness of derivative based optimization techniques. Similar results were obtained for the volumetric wave drag.

Response surface approximations to the surface in Figure 4 were constructed using both polynomial functions and rational approximations. The most simple polynomial model was the bilinear tensor product

$$f(x, y) = \left(a_1 x + a_2\right)\left(a_3 y + a_4\right).$$

Also employed were the quadratic polynomial

$$f(x, y) = a_1 x^2 + a_2 xy + a_3 x + a_4 y^2 + a_5 y + a_6$$

and the biquadratic tensor product

$$f(x, y) = \left(a_1 x^2 + a_2 x + a_3\right)\left(a_4 y^2 + a_5 y + a_6\right).$$

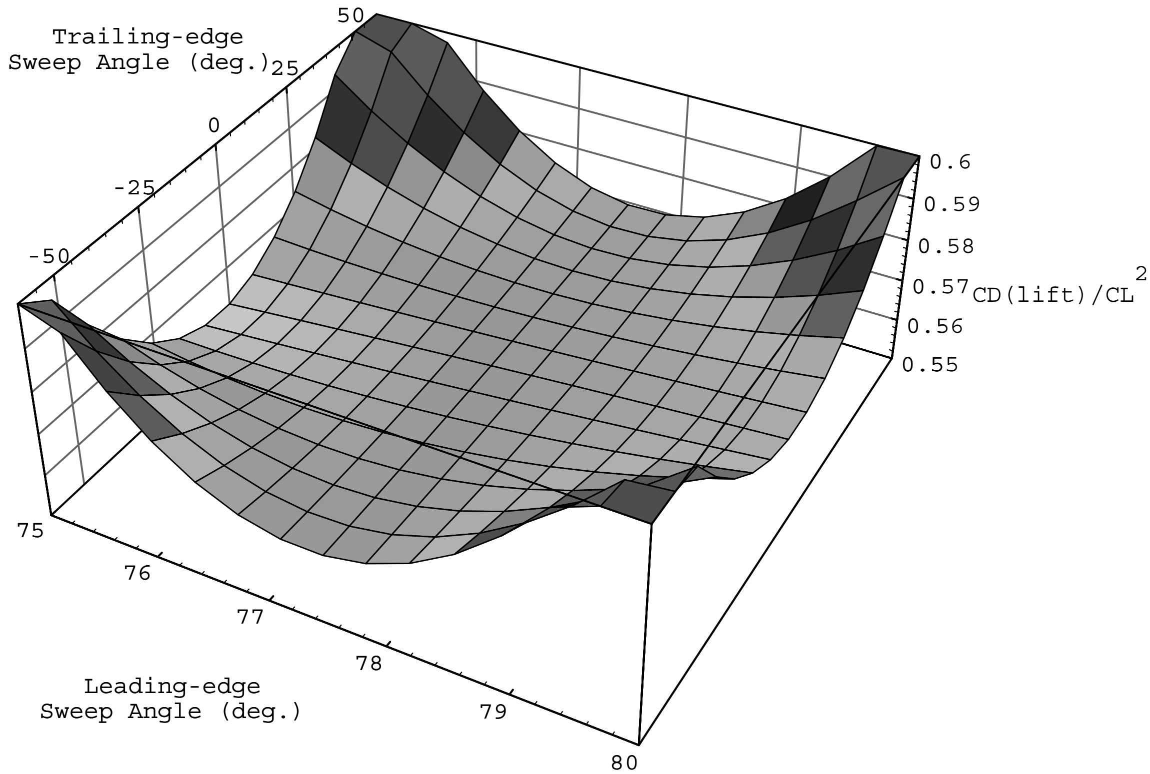

Figure 5. Biquadratic tensor product least squares approximation to noisy drag due to lift surface.

In addition to these models, the rational approximation

$$f(x,y) = \frac{a_1 x + a_2 y + a_3}{a_4 x + a_5 y + a_6}$$

was utilized. Because of the computational costs associated with evaluating the large number of point designs required for multipoint approximation, the present work focuses on methods to determine the minimum number of calculations required to construct various response surfaces. From this work with two design variables, the simple models and the mathematical techniques developed will be extended to design problems involving many variables where computational costs are significant.

The construction and evaluation of a response surface involved several steps. Initially, a number of points were selected at random from the design surface and the drag due to lift was calculated for each point design. A three-dimensional surface was then fit to the data. For the bilinear, quadratic, and biquadratic functions, this surface was found by performing a linear least-squares surface fit to the drag due to lift data points. For the rational function approximations, a Levenberg-Marquardt nonlinear least-squares surface fitting routine was used. To evaluate the quality of the surface fit, the infinity-norm, 1-norm, and 2-norm of the residual were calculated between the discrete points on the exact, known surface and corresponding points on the response surface.

As an example of this method, Figure 5 shows a response surface model produced using the biquadratic tensor product. This response surface modeled the exact surface topography fairly accurately both in the region near the global minimum and along the boundaries of the design surface. However, it required approximately 150 to 200 drag calculations to model the design surface before the surfaces became independent of the number of points in the analysis. In this example problem, the computational costs of calculating the response surface using a biquadratic tensor product were minimal. The computational costs would become prohibitively expensive if more accurate drag calculation procedures were utilized or if this model were extended to the twenty-three dimensions of the entire HSCT design problem.

Additional research is planned to validate the application of multipoint approximation methods to the HSCT optimization problem. Future work will focus on the example problem with respect to the techniques for point selection and on methods for quantifying the accuracy of the response surface fits. In addition, the response surface methodology will be extended to problems involving many variables and higher dimensional design spaces. Because the response surface technique requires numerous independent calculations, it offers an excellent way to exploit coarse grained parallel computation.

References:

[1] C. Byun and G. P. Guruswamy, *A Comparative Study of Serial and Parallel Aeroelastic Computation of Wings*, NASA TM 108805, Jan. 1994.

[2] K. J. Chang, R. T. Haftka, G. L. Giles, and P.-J. Kao, *Sensitivity-Based Scaling for Approximating Structural Response*, J. Aircraft, 30, No. 2, 1993, pp. 283–287.

[3] G. N. Fadel, M. F. Riley, and J.-F. Barthelemy, *Two-Point Exponential Approximation Method Structural Optimization*, Structural Optimization, 2, 1990, pp. 117–124.

[4] R. T. Haftka, *Combining Global and Local Approximations*, AIAA J., 29, No. 9, 1991, pp. 1523–1525.

[5] M. G. Hutchison, X.-M. Huang, W. H. Mason, R. T. Haftka, and B. Grossman, *Variable-Complexity Aerodynamic-Structural Design of a High-Speed Civil Transport*, Paper 92–4695, Fourth AIAA/ USAF/ NASA/ OAI Symposium on Multidisciplinary Analysis and Optimization, Sept. 1992.

[6] M. G. Hutchison, E. R. Unger, W. H. Mason, B. Grossman, and R. T. Haftka, *Aerodynamic Optimization of an HSCT Wing Using Structural Weight Equations*, Paper 92–0212, AIAA 30th Aerospace Sciences Meeting, Jan. 1992; see also *Variable-Complexity Aerodynamic Optimization of an HSCT Wing*, J. Aircraft, 31, No. 1, 1994, pp. 110-116.

[7] M. G. Hutchison, E. R. Unger, W. H. Mason, B. Grossman, and R. T. Haftka, *Aerodynamic Optimization of an HSCT Configuration Using Variable-Complexity Modeling*, Paper 93–0101, AIAA 31st Aerospace Sciences Meeting, Jan. 1993.

[8] E. R. Unger, M. G. Hutchison, R. T. Haftka, and B. Grossman, *Variable-Complexity Design of a Transport Wing*, Internat. J. Systems Automation: Research and Applications (SARA), 2, No. 2, 1992, pp. 87–113.