SUMMARY REPORT FOR AN UNDERGRADUATE RESEARCH PROJECT TO DEVELOP PROGRAMS FOR AIRCRAFT TAKEOFF ANALYSIS IN THE PRELIMINARY DESIGN PHASE

by

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BLACKSBURG, VIRGINIA

11 MAY 1994

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Slight editing of the report for the web, March 1997.

Nomenclature

AEO	= all engines operating
BFL	= balanced field length
C_D	= coefficient of drag
CFL	= critical field length
C_L	= coefficient of lift
C_{Lmax}	= maximum lift coefficient
D	= drag force
F	= applied force
g	= gravitational acceleration
L	= lift force
OEI	= one engine inoperative
S	= aircraft wing area
T	= thrust or time
T_0, T_1, T_2	= quadratic thrust terms
u	= horizontal velocity component
v	= vertical velocity component
V	= total velocity
W	= aircraft weight
x	= horizontal direction
y	= vertical direction
Λ	= thrust deflection angle (pos CCW)
γ	= climb angle
μ	= friction coefficient
ρ	= air density

Subscripts

air	= climb regime
brk	= braking
crit	= critical engine failure
fr	= friction
grd	= ground regime
lo	= liftoff
MC	= minimum control
MU	= minimum unstick
obs	= at obstacle passage
r	= rotation
s	= stall
1	= OEI takeoff/brake decision
2	= at obstacle passage (FAR)

1.0 Introduction

The objective of this two-semester undergraduate research project was to research, design and implement several programs to analyze aircraft takeoff for use during the preliminary design phase. The project took place during the Fall and Spring semesters of the 1993-1994 school year and the project results were given at the 1994 AIAA Student Conference in Morgantown, WV. This report is the summary of those two semesters of work and is designed to perform several functions. First, a definition of takeoff will take place to give a background from which the developed programs can be discussed. Next the two developed programs will be discussed in some detail. The first methodology used was one proposed by S.A. Powers in a 1981 AIAA paper [2], while the second methodology was proposed by Krenkel and Salzman in a 1968 AIAA paper [1]. For each methodology the equations of aircraft motion will be discussed as well as the proposed and implemented solutions to the equations of motion. Then, the algorithmic implementation of each program will be discussed. The third topic of this summary report will be a comparison of the developed programs with flight test data and results of a program of higher order. Finally, a discussion of possible programmatic improvements will occur.

2.0 Takeoff Definitions

Many standards are used to define the stages of an aircraft takeoff run, depending on the country and type of aircraft. Because this project dealt only with commercial jet aircraft, Part 25 of the Federal Aviation Regulations (FAR) was used as the basis for defining takeoff. Under FAR 25, an aircraft taking off performs a ground roll to rotation velocity, rotates to liftoff attitude, lifts off and climbs to a height of 35 ft. This definition can be applied to two types of takeoff: takeoff with all engines operating (AEO) and takeoff with engine failure, usually prescribed as one engine inoperative (OEI). Each of these types of takeoff will be discussed in turn.

Takeoff with all engines operating is the type dealt with in most day-to-day situations and follows the same steps mentioned above. The aircraft accelerates from a stop or taxi speed to the velocity of rotation, V_r, rotates to the liftoff attitude with corresponding velocity V_{lo}, and climbs over an obstacle of 35 feet as shown in Fig. 1. The velocity at the end of the 35 ft climb is usually called the takeoff safety speed and given the designation V_2. FAR 25 prescribes limits to these velocities based on the stall velocity, V_s, the minimum control velocity, V_{MC}, and the minimum unstick velocity, V_{MU}. These three velocities are physical minimum velocities under which the aircraft can operate.

The stall velocity is the aerodynamically limited velocity at which the aircraft can produce enough lift to balance the aircraft weight. This velocity occurs at the maximum aircraft lift coefficient, C_{Lmax}, and is defined as:

$$V_s = \sqrt{\frac{2(W/S)}{\rho\, C_{L_{\max}}}} \tag{1}$$

The *minimum control velocity* is the “lowest airspeed at which it has proved to be possible to recover control of the airplane after engine failure” [3]. This is an aerodynamic limit which is difficult to predict during the preliminary design stage, but may be obtained from wind tunnel data during later phases of design. The *minimum unstick velocity* is the “airspeed at and above which it can be demonstrated by means of flight tests that the aircraft can safely leave the ground and continue takeoff” [3]. This velocity is usually very close to the stall velocity of the aircraft.

With these reference velocities defined, the FAR places the following broad requirements on the velocities of takeoff [3]:

$$\begin{aligned} V_r &\geq 1.05\, V_{MC} \\ V_{lo} &\geq 1.1\, V_{MU} \\ V_2 &\geq 1.2\, V_{MC} \end{aligned} \tag{2}$$

Because the minimum unstick velocity is usually very close to the stall velocity the liftoff velocity is often referenced as greater than 1.1 times the stall velocity, rather than 1.1 times the minimum unstick velocity.

Engine failure brings another level of complexity to the definitions and requirements of takeoff. Usually takeoff of this nature is categorized by the failure of one engine, or one engine inoperative (OEI). OEI takeoff includes an AEO ground run to engine failure, an OEI ground run to liftoff, and a climb to 35 ft, also with OEI as illustrated in Fig. 1. A takeoff with an engine out will take a longer distance than AEO takeoff due to the lower acceleration produced by the remaining engines. The obvious questions to ask are if the OEI takeoff field length required is longer than the field length available and if the distance to brake to a stop after engine failure is longer than the available field length. These questions are often answered by solving for a critical or balanced field length (CFL or BFL); the distance at which OEI takeoff distance equals the distance needed to brake to a full stop after engine failure.

Defining the CFL leads back to the time, or more specifically, the velocity at which engine failure occurs. As it turns out, by imposing the CFL definition, there is an engine failure velocity which uniquely defines the critical field length. This velocity is often called the critical velocity, V_{crit}. It must be noted that during an aborted takeoff some amount of time will be required after the engine fails for the pilot to actually begin braking, both because of the pilot's reaction time and the mechanics of the aircraft. During this passage of time, the aircraft continues to accelerate on the remaining engines and will finally reach the decision velocity, V_1.

Careful inspection of the above definitions will show that engine failure at a velocity lower than the critical velocity will require an aborted takeoff, while engine failure after the critical velocity has been reached will require a continued takeoff. With the above definitions in place, FAR 25 imposes additional requirements for OEI takeoff [3]:

$$\begin{aligned} V_r &\geq V_1 \\ V_{lo} &\geq 1.05\, V_{MU} \end{aligned} \tag{3}$$

Note that although other standards for aircraft takeoff exist, most use the same four velocities in their takeoff analysis: V_1, V_r, V_{lo}, V_2.

3.0 Simplified Powers Methodology

The first of the methods developed during this project was proposed by S.A. Powers in his 1981 AIAA paper [2]. Powers proposed two separate methodologies: full and simplified. The simplified Powers method was selected for two reasons. First, the solution to the equations of motion was easier to implement. Second, and more important was the fact that, for typical aircraft, there was very little variance between the full and simplified solutions. This simplified method has no thrust vectoring capability, nor is able to deal with head or tail winds. More important is the assumption that aircraft thrust is constant throughout the takeoff run.

Powers uses only one equation of motion throughout the aircraft takeoff. It is found through the balance of forces on the aircraft in the horizontal direction. The forces on the aircraft are shown in Fig. 2 where, for the simplified Powers methodology, the thrust deflection angle, Λ, is zero. Summing the forces and introducing nondimensional coefficients for lift and drag yields:

$$\frac{du}{dt} = \frac{g}{W}\left[(T - \mu W) + 0.5\rho S(\mu C_L - C_D)u^2\right] \tag{4}$$

With Powers' assumption of constant thrust, this equation of motion is able to be integrated analytically for the time and distance needed to accelerate between two velocities. V_a is the initial velocity and V_b is the final velocity. The resulting time and distance equations are shown below.

$$t_b - t_a = \frac{G}{\bar{V}}\ln\left[\left|\frac{(\bar{V} + V_b)(\bar{V} - V_a)}{(\bar{V} - V_b)(\bar{V} + V_a)}\right|\right] \tag{5}$$

$$X_b - X_a = -G\ln\left|\frac{\bar{V}^2 - V_b^2}{\bar{V}^2 - V_a^2}\right| \tag{6}$$

where,

$$G = \frac{W/S}{\rho g(C_D - \mu C_L)} \tag{7}$$

and,

$$\bar{V} = \sqrt{\frac{T - \mu W}{\rho S(C_D - \mu C_L)}} \tag{8}$$

It is critical to note that, although the above equations are specifically for ground roll, Powers uses them along with climb aerodynamic input to approximate the times and distances of the climb segment as well. Thus, the two dimensional qualities of climb-out are not represented in the Powers' solution. As will be seen, this assumption works fairly well for "conventional" commercial aircraft, but seriously misrepresents the actual takeoff for aircraft like a High Speed Civil Transport (HSCT). This is one of the largest potential problems with the simplified Powers methodology.

The time and distance equations, Eq. 5 and Eq. 6 of the simplified Powers method were coded into FORTRAN. The following algorithm was used to provided AEO takeoff solutions:

1) Given basic aircraft geometry and aerodynamics calculate V_s from Equation 1.
2) Calculate V_{lo} by applying user input, usually the FAR 25 limitations given in Equation 2.
3) Calculate V_2 by applying user input, usually the FAR 25 limitations given in Equation 2.
4) Calculate the ground distance between V=0 and V=V_{lo} as well as the time taken, using Eqs. 5 & 6.
5) Calculate the climb distance between V=V_{lo} and V=V_2 as well as the time taken, using Eqs 5 & 6.

In a similar manner the solution for BFL is calculated, although iteration is needed to provide a solution for V_{crit}. The algorithm used in the developed FORTRAN code uses the following steps:

1) Given basic aircraft geometry and aerodynamics calculate V_s from Equation 1.

2) Calculate V_{lo} by applying user input, usually the FAR 25 limitations given in Equation 2.

3) Guess two initial critical velocities and for each guess velocity, both within the ground run:

  a) Solve for distance travelled for an OEI takeoff using Eq. 6.

  b) Solve for distance travelled for a takeoff aborted after engine failure using Eq. 6..

4) Compare the distances travelled for OEI takeoff and aborted takeoff for equality.

5) If the distances are not equal, a bisection method routine calculates the new estimated V_{crit}.

6) Iterate through 3, 4, and 5 until convergence is met with new values from the bisection method.

The FORTRAN code for the simplified Powers method is copiously documented and contains around 600 lines of code. The program was designed to stand alone. In other words, it require no users manual. The input file attempts to provide a description of and/or input values for each piece of data. The input not only specifies aircraft takeoff information, but run control information. Through the input file, the user has the ability to control the number of incremental outputs for each run segment as well as whether the output goes to the printer, screen or a file.

The simplified Powers method provides two primary advantages over more complex codes. The number of input parameters is very small. Only 13 basic input parameters are necessary to provide solutions to both the AEO and BFL problems for any aircraft *[Note: Table 1 provides a list of the actual input required. The code for this approach is called TAKEOFF1]*. The other advantage is that solutions, even those with considerable incremental data, are produced in no more than 15 seconds on a 386PC.

Along with the advantages, the simplified Powers method does contain several disadvantages. Foremost among these is the fact that climb-out is not well modeled in this method. As will be shown, the method produces surprisingly good results for conventional aircraft, but aircraft with large high lift devices or with STOL capability will not produce good results with this methodology. Second, while the small amount of input data may work toward simpler program operation, it does not allow STOL or unconventional high lift devices to be properly modeled. Finally, the absence of a rotation phase has proven to under-predict ground runs in many situations. This has proven to be more of a problem in aircraft with high thrust to weight ratios (*T/W*) and in aircraft with especially high wing loadings (*W/S*).

4.0 Modified Krenkel and Salzman Methodology

The second aircraft takeoff methodology implemented was proposed by Krenkel and Salzman in their 1968 AIAA paper [1]. The original method calculated the aircraft equations of motion for ground roll and climb. The equations provided for vectored thrust and introduced the assumption that thrust varied with velocity. The equations were then solved parametrically, creating charts from which aircraft ground roll, climb and total takeoff distance could be calculated. The original method did not solve for balance field length. The equations of motion for Krenkel and Salzman's methodology relied on the balance of force equations during the two segments of takeoff: ground roll and climb. The ground roll equations were obtained by summing the forces on the aircraft in the horizontal direction and substituting them into Newton's second law ($\sum F=ma$). Figure 2 shows the aircraft forces during the ground run. With non-dimensionalized lift and drag coefficients, the governing equation developed for the ground run is:

$$\frac{du}{dt} = \frac{g}{W}\left[T\cos\Lambda - 0.5\rho S C_D u^2 - \mu\left(W - 0.5\rho S C_L u^2 - T\sin\Lambda\right)\right] \tag{9}$$

This is a nonlinear ordinary differential equation due to the velocity squared term. The original methodology assumed the thrust was a summation of individual engines, but made no assumption that thrust varied with velocity. Unlike the original methodology, the developed program incorporated a quadratic thrust variation with velocity. Thus,

$$T = T_0 + T_1 V + T_2 V^2 \tag{10}$$

Note that the thrust is a function of the total velocity, *V*, not the horizontal velocity, *u*. This is unimportant during the ground roll where $V = u$, but becomes essential during the climb phase.

Four equations were developed for the climb phase. Two are the governing balance of force equations, one relates the velocity components to the climb angle, γ, and the fourth relates the total velocity, *V*, to the individual velocity components. The result is a system of nonlinear ordinary differential equations, shown as Equations 11, 12, 13 and 14.

$$\frac{du}{dt} = \frac{g}{W}\left[T\cos(\Lambda + \gamma) - 0.5\rho S(C_L \sin\gamma + C_D \cos\gamma)V^2\right] \tag{11}$$

$$\frac{dv}{dt} = \frac{g}{W}\left[T\sin(\Lambda + \gamma) + 0.5\rho S(C_L \cos\gamma - C_D \sin\gamma)V^2 - W\right] \tag{12}$$

$$\tan\gamma = \frac{v}{u} \tag{13}$$

$$V = \left(u^2 + v^2\right)^{1/2} \tag{14}$$

The equations were broken into a system of first order differential equations and then solved numerically using a time step, fourth order Runge-Kutta scheme developed by the author. The scheme uses an *a posteriori* error estimate between third and fourth order Runge-Kutta solutions and reduces the step size if the prescribed error tolerance is not met. The method is semi-adaptive because the step size is reduced only, rather than increased if the prescribed error tolerance is more than met, as in a true adaptive scheme. The reason for the semi-adaptive scheme was to allow the user to input the base step size and for the method to ensure the error tolerance was met between steps. This allows incremental output to be calculated much easier and reduces the number of interpolations necessary.

Originally, the Krenkel and Salzman method contained only a ground roll and climb phase, without a rotation phase. This caused the algorithm to consistently under predict most aircraft takeoff times, distances and velocities. As a first improvement to this problem, a constant velocity rotation phase was added, continuing for a user-defined number of seconds as proposed by Nicolai [6]. This created a considerable improvement in the program takeoff predictions. Further improvement was obtained by installing a rotation phase in which the ground roll equations of motion were solved for a user-defined number of seconds. This allowed the aircraft to accelerate while the rotation phase was occurring.

The ground and climb equations of motion, with quadratic thrust variation with velocity and a rotation phase included, were used to solve for both AEO and balanced field length OEI takeoff. The algorithm used to solve for AEO takeoff is similar to that used for the simplified Power's method and is as follows:

1) Calculate V_r from V_s from Eqs.1 and 2 given user input aircraft takeoff data.
2) Solve ground run Eq. 9 between V=0 and V=V_r for time, distance and velocity in the x-direction.
3) Solve ground run Eq. 9 between V=V_r and V=V_{lo}, simulating rotation.
4) Solve climb Eqs. 11, 12, 13 & 14 between V=V_{lo} and a user defined obstacle height.

The algorithm used for the OEI balance field length solution requires significantly longer run times than that used by the simplified Powers methodology. The extra run time is due to the fact that there is no closed form solution to the Krenkel and Salzman equations of motion. For each segment of takeoff the equations must be solved in time steps. In an iterative process, like the calculation of BFL, the time step method is slow. Because of the extra run time required, only summary data for balanced field length is output by the program, not incremental output. The algorithm of the final OEI BFL process for the Modified Krenkel and Salzman Methodology is defined below.

1) Given basic aircraft geometry and aerodynamics calculate V_s from Equation 1.
2) Calculate V_{lo} by applying user input, usually the FAR 25 limitations given in Equation 2.
3) Make two initial guesses at the critical velocity, both within the ground run:
    a) Solve for distance travelled for an OEI takeoff with the Runge Kutta method
        i) Solve for ground roll V=0 to V=V_r.
        ii) Solve for ground roll V=V_r to V=V_{lo}.
        iii) Solve for climb V=V_{lo} to user specified obstacle height.
    b) Solve for distance travelled for a takeoff aborted after engine failure.
        i) Solve for ground roll V=0 to V=V_1, the descision speed.
        ii) Solve for ground roll V=V_1 to V=0, stopping.
4) Compare the distances travelled for OEI takeoff and aborted takeoff for equality.
5) If the distances are not equal, a bisection method routine calculates the new estimated V_{crit}.
6) Iterate through 3, 4, and 5 until convergence is met with new values from the bisection method.

The modified Krenkel and Salzman methodology has several advantages over the simplified Powers' method. First, the quadratic thrust variation with velocity allows a more accurate engine representation. Second, the ability to vector the thrust during the takeoff run, allows application of this methodology to STOL aircraft and might be particularly applicable if thrust vectoring is used to help a HSCT rotate during takeoff. Third, the ability to enter ground run as well as climb aerodynamics allows the user much more control over the aircraft takeoff as well as being a better representation of true takeoff. Finally, like the simplified Powers' method, the modified Krenkel and Salzman method requires little data to produce results (25), is around 600 lines of FORTRAN code, and makes an attempt at having a user-friendly, self-contained input file. *[Note: Table 2 provides a list of the actual input required. The code for this approach is called TAKEOFF2].* There is only one potential disadvantage to the modified Krenkel and Salzman method. The use of numerical integration to solve the governing equations means that the program must take many steps to provide an accurate answer. This, however is not much of a problem, as full solutions for 0.5 second steps typically take under 2.5 minutes on a 386PC.

5.0 Aircraft Takeoff Comparisons

Three aircraft were selected to test the validity of the two developed codes. The first was a Boeing 747-200. In this case the developed codes were compared to Boeing flight test data [7]. The second aircraft was a DC-9 type craft. This aircraft was selected as it was one of the aircraft used to test NASA Langley's Flight Optimization System (FLOPS) takeoff module. Thus, the data was easily accessible and, after checking, produced accurate answers. The third test aircraft, an

HSCT, was selected to test the developed codes against higher order predictions for a "non-conventional" aircraft with greater aerodynamic requirements than conventional subsonic transports. The base for this model was a model included in the FLOPS takeoff module, modified to represent HSCT 2.4E. Tests included comparing as much critical takeoff output data as possible: V_r, V_{lo}, V_{obs}, X_r, X_{lo}, X_{obs}, BFL, T_r, T_{lo} , T_{obs} being the primary comparison points. An effort was made when creating input files from FLOPS files to avoid updating initial approximations. The goal was to simulate real conditions and to avoid purposely reducing error.

The results of the Boeing 747-200 comparison are shown in Figure 4. The run was made with an aircraft weighing 707,200 lbs with 10 degrees of flaps. The Boeing flight test results are presented at the bottom of the figure, while the relative error of the simplified Powers and modified Krenkel and Salzman methods are presented in the bar charts above. The simplified Powers methodology does not include a rotation phase and this data is unavailable. The results of the Powers method, though large in error compared to Krenkel and Salzman predictions, are applicable to the early stages of preliminary design where little input data is available and design changes will often change the aircraft 5% to 15%. The reason for the disparity between the Boeing data and the Powers predictions is that a rotation phase is not included. Thus, the V_{lo} of the Powers prediction is closer to the V_r of the Boeing data and of the Krenkel and Salzman data as well. The predictions of the modified Krenkel and Salzman method are quite pleasing. Relative errors between 2% and 3% for all values are very respectable and well within the limits used in preliminary design. The relative error is also within the time and distance accuracy of the Boeing data as noted in Figure 4. The results of the Krenkel and Salzman predictions are particularly pleasing because they compare well with experimental data provided by the aircraft manufacturer.

The FLOPS DC-9 aircraft was compared to the two developed methods and results are shown in Figure 5. Again the modified Krenkel and Salzman method outperformed the simplified Powers method, often by orders of magnitude. The error in the Powers method is again attributed to the lack of a rotation phase, which causes the program to under predict ground roll parameters, and due to the lack of specific climb aerodynamics, which also impacts the predictions. This time the simplified Powers method is probably on the edge of being applicable in the preliminary design phases due to its consistent 15+% relative error. The error in the Krenkel and Salzman method, while larger than with the 747, is consistently under 5%, very respectable for the number of input parameters used and for the fact that FLOPS takeoff is constrained by the FAR, while the developed codes use FAR requirements, but do not force the aircraft to fit within them.

The third, final and perhaps most important code validation is comparison of the developed codes with a HSCT. The base model used was included as a supersonic transport test for the FLOPS takeoff module [5]. The input data was updated to match a HSCT 2.4E with data obtained from a NASA Ames Aircraft Synthesis Program (ACSYNT) HSCT and from baseline data used by Hutchinson *et al.* [8]. The results of the comparison are shown in Fig. 6. The first thing to note is that both the simplified Powers and Krenkel and Salzman methodologies under-predicted all parameters. From Fig. 6 it is obvious that the simplified Powers method is not a good choice for unconventional aircraft. The relative errors are above 15% for most of the comparison parameters. The Powers method simply cannot represent a HSCT well enough with 13 input parameters. The modified Krenkel and Salzman method, on the other hand, while producing greater error than in the two previous test runs, is still applicable to preliminary design. Errors are always under 10% and usually around 5%.

While comparison runs are being discussed, it is important to note some of the techniques used to create the input files for the developed codes, particularly in the estimation of lift and drag coefficients. Lift and drag coefficients were always taken directly from C_L and C_D versus α data, whether from ACSYNT, FLOPS or flight test data. The code ground run, lift and drag coefficients were consistently taken at a zero angle of attack. The climb phase, lift and drag coefficients were taken at the estimated takeoff angle, usually between 6 and 15 degrees, producing lift coefficients which were around 80% to 85% of C_{Lmax}. In all cases care was taken to add extra drag terms to the base drag (ie. landing gear, windmill drag etc.). The above results were obtained by using these techniques. It must again be noted, however, that sensitivity to lift and drag inputs were found at times during the climb phase. This was particularly noticeable during BFL calculations.

6.0 Possible Improvements

This section is devoted to describing changes each of the developed programs which would improve their speed and accuracy. In some cases the modifications are relatively easy and in others the modifications would require extensive rewriting of the original code. The validity of some suggestions is not known, and these cases will be noted.

Two main improvements could be made to the simplified Powers method. One which would drastically improve the accuracy of the program is the addition of a rotation phase. It was noted in the comparison section that the Powers method consistently under-predicted ground roll distance, and consequently total distance due to the lack of a rotation phase. This improvement would be easy to implement. Just add a continuation of the ground roll for a user-specified amount of time as done in the Modified Krenkel and Salzman method.

The second improvement to the Powers' method would necessitate extensive rewriting of the original code. However, the addition of better climb-phase equations would vastly improve overall distance and time predictions. Implementing this change would require that climb equations of motion be developed, solved analytically and implemented into the program. Due to the amount of reworking necessary, and the natural limitations of the Powers method, this change is not recommended. The addition of a rotation phase, on the other hand, would be a simple and very beneficial implementation.

The Modified Krenkel and Salzman method, in its current form, works quite well. However, several possibilities exist for subtly improving the program. First, instead of single lift and drag coefficients for lift and climb phases, the program could be modified to put in a list of coefficients or a variation of coefficients with angle of attack (as with thrust variation with velocity) could be implemented. This would allow better aerodynamic representation of the aircraft and would probably improve program accuracy. A second possible improvement concerns the thrust vectoring capability of the aircraft. Currently the program has a single, user-defined thrust vectoring angle throughout the takeoff run. This could be improved to allow variable thrust vectoring during takeoff. This, as with the aerodynamics data, could be input into the program as a list or function with and would be relatively simple to implement.

7.0 Conclusions

The objective of this project was to develop personal computer-based codes which predict takeoff parameters for commercial jet aircraft. The developed codes calculated AEO as well as OEI balanced field length takeoff parameters. The takeoff definition used was based on the FAR Part 25, although the codes produced solutions which used FAR 25 requirements, but were not constrained within them.

Two codes were selected for development: the simplified method proposed by Powers, and a modified version of a method proposed by Krenkel and Salzman. The code developed around the simplified Powers method requires 13 input parameters and solves the governing equation analytically for takeoff times and distances. The method assumes constant thrust throughout the takeoff run and climb phase aerodynamics are the same as in the ground roll. The two major problems with the Powers methodology are the lack of a rotation phase and the use of ground roll equations of motion to predict the climb phase of takeoff. The lack of a rotation phase causes the method to under predict ground roll at times and the climb phase is often over predicted.

The modified Krenkel and Salzman method allows thrust vectoring and assumes thrust varying with velocity. A modification was made to assume thrust varied quadratically with velocity. Originally, the method solved the equations of motion, both nonlinear ordinary differential equations, parametrically. A modification was made to solve the equations of motion through a fourth order, semi-adaptive Runge-Kutta scheme developed by the author. Due to consistent under prediction of the ground roll, the method was also modified to include a rotation phase; a continuation of the ground roll for a user-defined amount of time. As with the simplified Powers method, the modified Krenkel and Salzman method iterated from an initial guess critical engine failure velocity to predict the BFL. Unlike the Powers' method, the Krenkel and Salzman method increased the engine out rotation velocity to allow the aircraft to take off with reduced thrust.

Both developed codes were compared and validated against other aircraft. Boeing 747-200 flight test data, NASA Langley's FLOPS programs predictions for a DC-9 and a modified FLOPS file for a HSCT 2.4E were used. The HSCT was an extreme case, as it is a relatively unconventional commercial aircraft. Under all test cases the Krenkel and Salzman method performed very well. Errors rarely went above 10%, and for a majority of the time were under 5%, well within the limits for preliminary design use. The simplified Powers method on the other hand had trouble accurately predicting takeoff parameters. For very "conventional" aircraft, the Powers method is adequate for preliminary design use, but unacceptable when higher T/W and W/S ratios are found.

Due to the accuracy with which the modified Krenkel and Salzman method predicted takeoff parameters, it is recommended for use in the preliminary design phase. The method uses only 25 input parameters, and has a "user-friendly" input file. It has the ability to perform thrust vectoring and solves for takeoff usually in under 30 seconds on a 386PC. The fact that it is easily available, unlike some of the higher order methods, as well as accurate makes it a good option.

References

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Table 1

Input data required for the simplified Powers method (Takeoff1)

1. Title card and case information
2. Density of air at takeoff (sl/ft^3)
3. Weight of aircraft (lbs)
4. Wing area of aircraft (ft^2)
5. Rolling aerodynamic drag coefficient CDroll *Note 0*
6. Rolling aerodynamic lift coefficient CLroll
7. Rolling friction coefficient MUroll *Note 1*
8. Braking friction coefficient MUbrake *Note 2*
9. CLmax of aircraft (in ground effect if possible)
10. Stall margin *Note 3*
11. Time between engine failure and braking (sec) *Note4*
12. Average constant thrust of all engines (lbs)
13. Average constant thrust with engine out (lbs)
14. Multiplier for velocity to cross obstacle *Note 5*
15. Integer for output device *Note 6*
16. Number of datapoints output for each curve *Note 7*

Note 0: CDroll is the drag coefficient during takeoff. It is typically Cd0+Cdi+Cdother, where: Cd0 is the zero lift drag coefficient, Cdi is the induced drag coefficient (CL)^2/(pi*E*AR), Cdother is the drag coefficient due to high lift devices & ground effect.

Note 1: Rolling friction coefficient is typically:

Dry concrete/Asphalt	- 0.02
Hard turf/Gravel	- 0.04
Short, dry grass	- 0.05
Long grass	- 0.10
Soft Ground	- 0.10 to 0.30

Note 2: Braking friction coefficient, typically: 0.20 to 0.40 with good assumptions being, 0.30 or 0.35

Note 3: Takeoff speed is usually defined as Vto = k * Vstall, where k is the stall margin and Vstall is the aircraft stall speed. k is usually defined as 1.1, although 1.2 is also used, (ie. the takeoff speed is 10% to 20% higher than stall speed).

Note 4: This is the time lag between engine failure and the decision to begin braking. (by MIL-M-007700B this is 3 sec after failure)

Note 5: Multiplied times Vto to get the velocity required to pass over a 35ft (11m) obstacle after liftoff Defined in the FAR as no less than 1.2, while common values are 1.25 and 1.3. An exception to the above values is given in the FAR for aircraft with four power units. The value of the multiplier then is 1.15

Note 6: Output destination is specified by the following integers:
6 - Sends output to the printer
7 - Sends output to a file
8 - Sends output to the screen

Note 7: This integer number specifies how many data points will be generated for each curve. For instance, 20 as an input will create 20 points between V=0 and V= Vto for normal takeoff.

Table 2

Input data required for the method of Krenkel and Salzman (Takeoff2)

1. Title Card with case information
2. Density of air at takeoff (sl/ft^3)
3. Weight of aircraft (lbs)
4. Wing area of aircraft (ft^2)
5. CLmax - max lift coefficient of the aircraft
6. CLgrd - lift coeff. for ground run takeoff segment
7. CLair - lift coeff. for climb takeoff segment
8. CDgrd - drag coeff. for ground run takeoff segment
9. CDair - drag coeff. for climb takeoff segment
10 MUgrd - rolling friction coefficient *Note 1*
11. MUbrk - braking friction coefficient *Note 2*
12. LAMBDA - thrust deflection angle, positive up (rad)
13. K - stall margin *Note 3*
14. TIME between engine failure and braking (sec) *Note 4*
15. OBSHT - height of obstacle (ft) (usu 35 or 50 ft)
16. PLOSS - fraction of power remaining when engine fails

17/18/19. 3 thrusts (lbs) *Note 5*

20/21/22 3 velocities (ft/s)

23. TSTEP - time step for incremental output (sec) *Note 6*
24. TROT - time required for rotation
25. Integer for output device *Note 7*

Note 1: Rolling friction coefficient is typically: Dry concrete/Asphalt- 0.02

Hard turf/Gravel -	0.04
Short, dry grass	- 0.05
Long grass	- 0.10
Soft Ground	- 0.10 to 0.30

Note 2: Braking friction coefficient, typically: 0.20 to 0.40 with good assumptions being, 0.30 or 0.35

Note 3: Takeoff speed is usually defined as Vto = k * Vstall, where k is the stall margin and Vstall is the aircraft stall speed. k is usually defined as 1.1, although 1.2 is also used, (ie. the takeoff speed is 10% to 20% higher than stall speed).

Note 4: This is the time lag between engine failure and the decision to begin braking.(by MIL-M-007700B this is 3 sec after failure)

Note 5: These three thrusts are used to calculate a quadratic thrust curve for the aircraft engine. Each thrust should correspond to the velocity below it. For cases with unknown thrust curves a constant thrust can be entered for three different velocities.

Note 6: This is the time step between incremental distance and velocity output points. From experience, 0.25, 0.5 or 1.0 second intervals work well. This time step is only for display purposes as the adaptive differential equation solver will often break the internal step size down to maintain solution integrity.

Note 7: Output destination is specified by the following integers:

6 - Sends output to the printer
7 - Sends output to a file
8 - Sends output to the screen

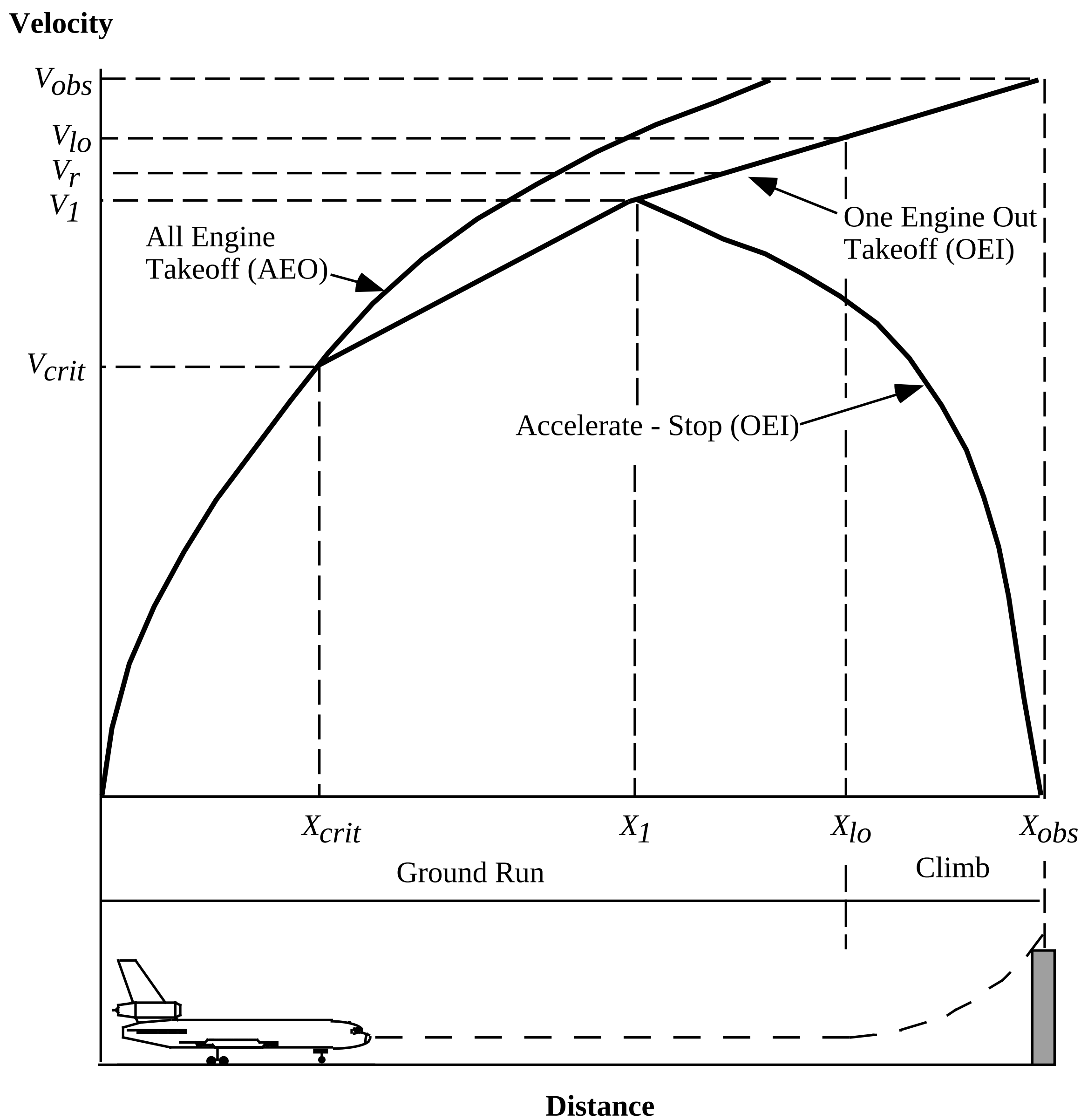


Figure 1. Definitions of takeoff parameters, both AEO and OEI.

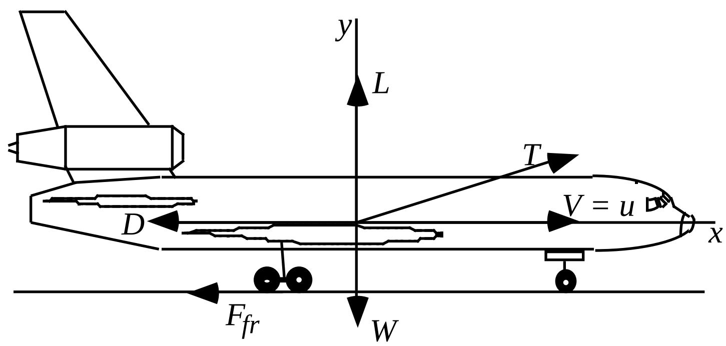


Figure 2. Aircraft force breakdown during ground roll.

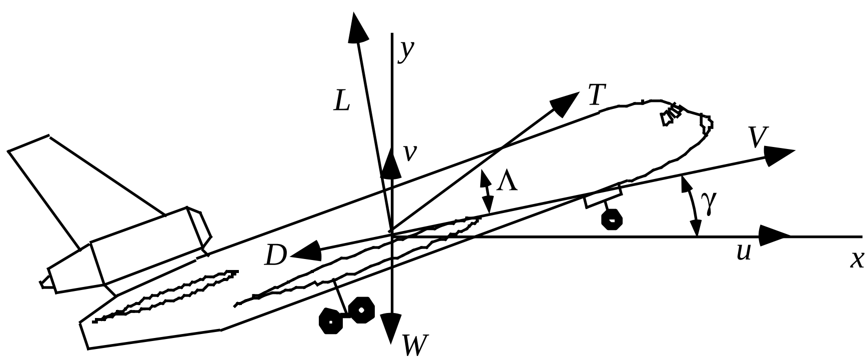
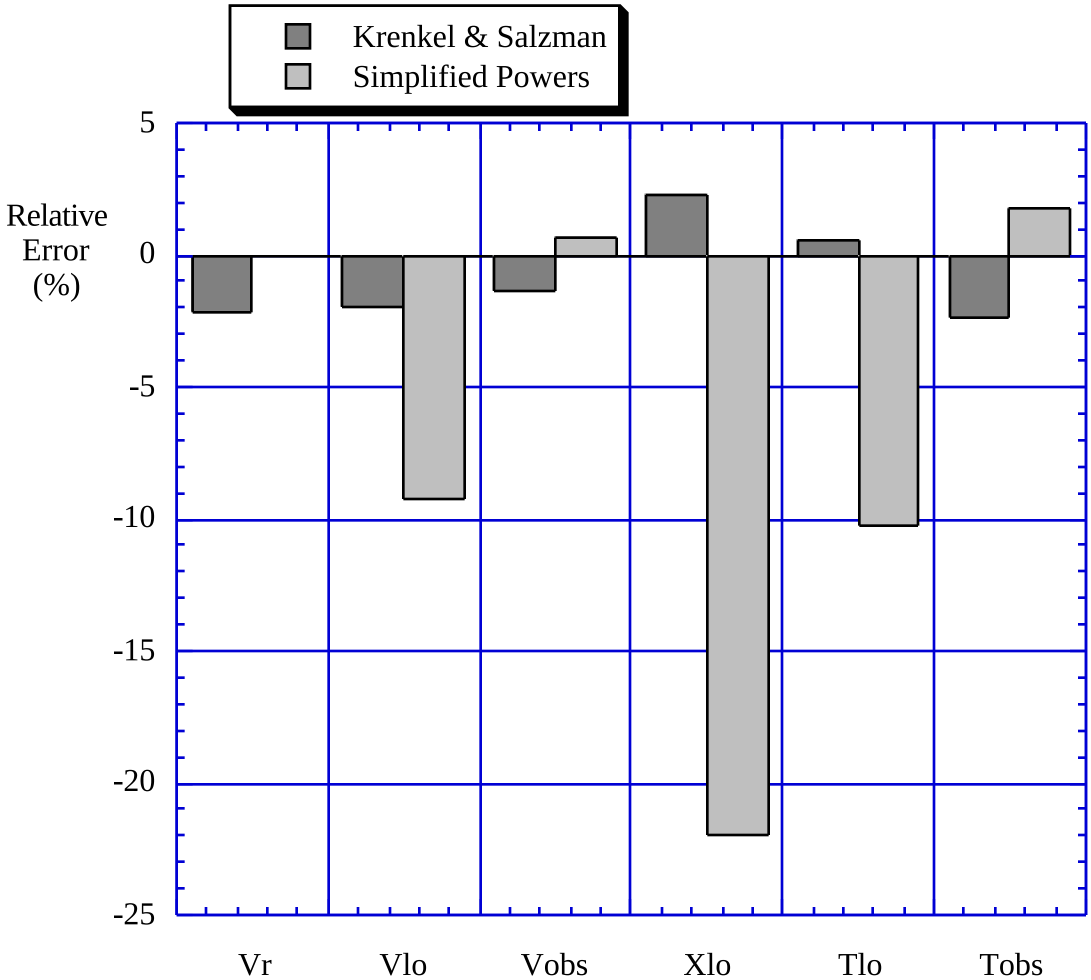


Figure 3. Aircraft force breakdown during climb.

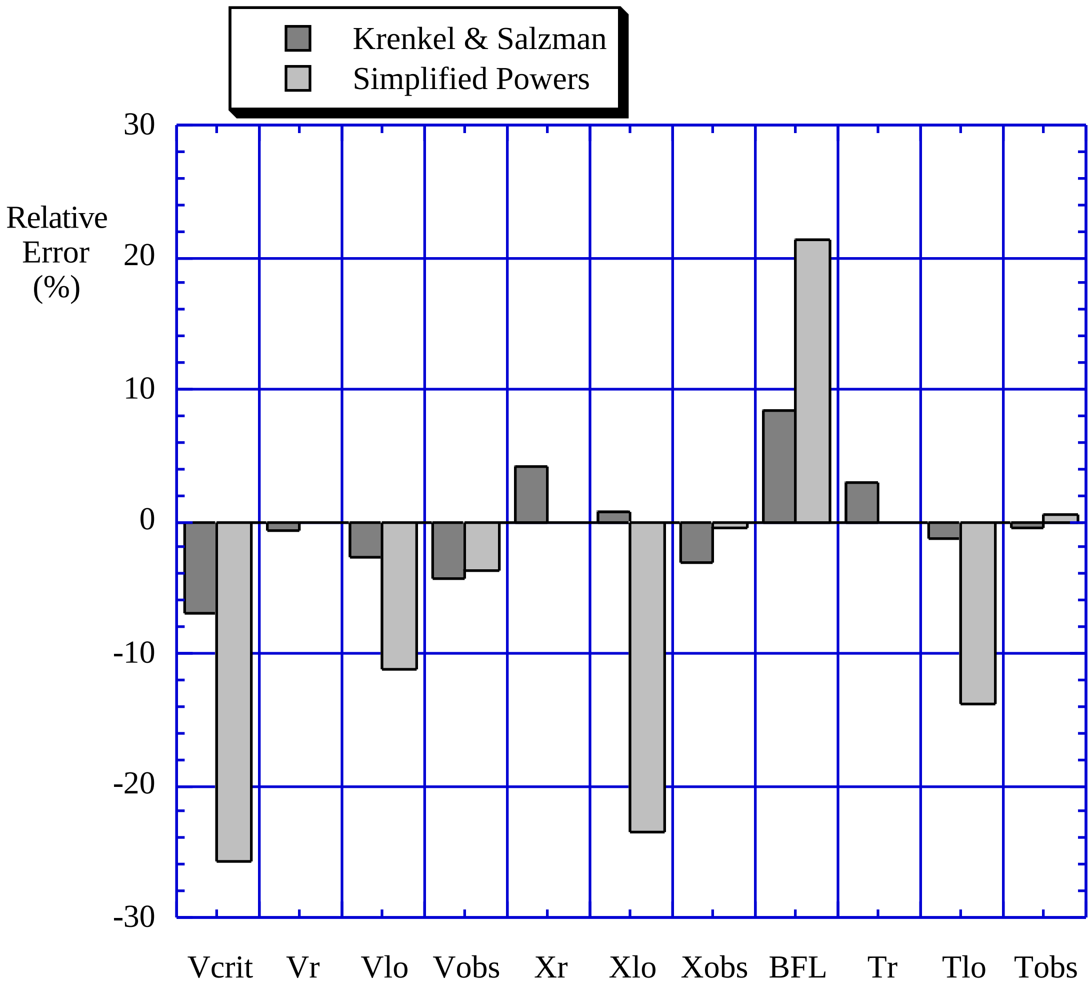


Baseline for comparison: Boeing data [7]

Vr	= 271.9 ft/s	Tr	= 47.5 s
Vlo	= 283.7 ft/s	Tlo	= 50 s
Vobs	= 288.8 ft/s	Tobs	= 54 s
Xlo	= 7500 ft		

Note: All times within 2s
All ditances within 100 ft.

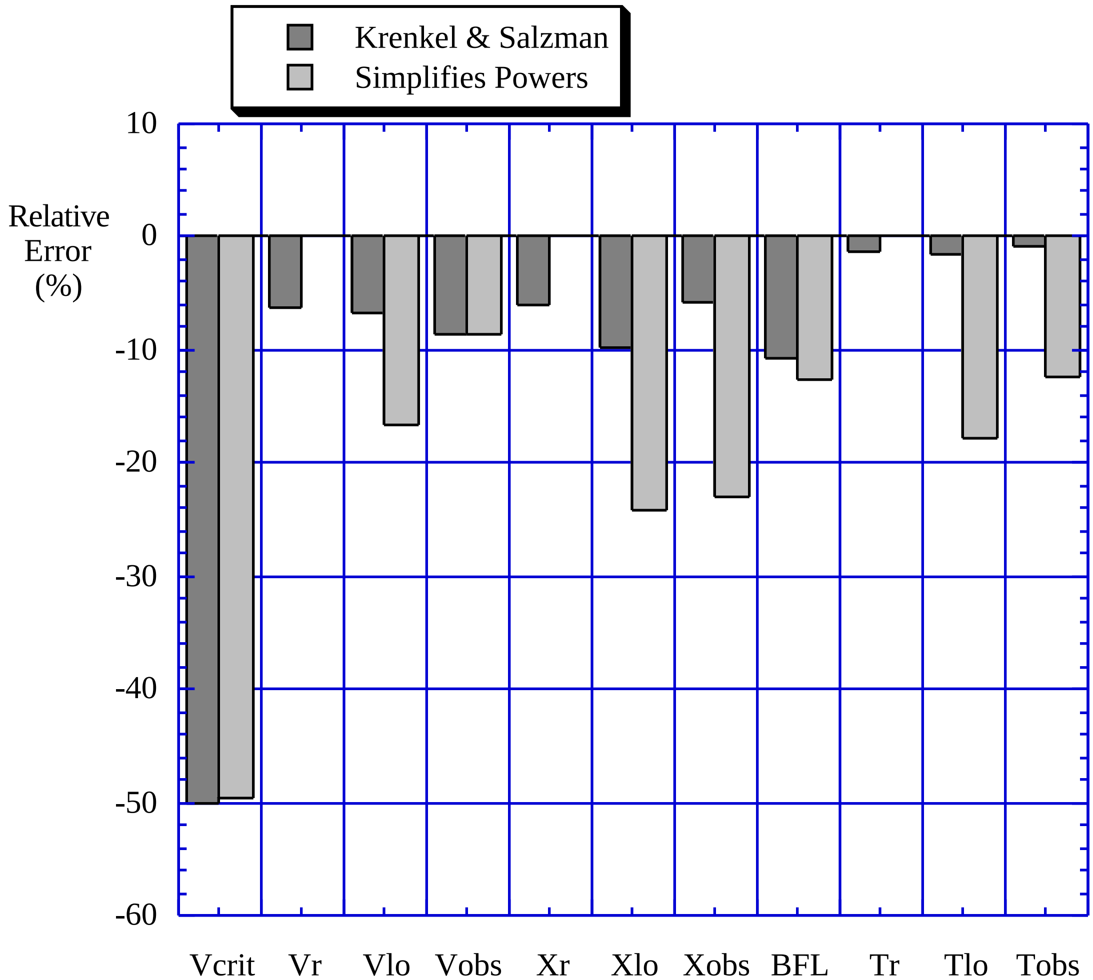
Figure 4. Comparison of relative error of the codes with flight test data for the 747-100.



Baseline for comparison: FLOPS predictions for DC-9 takeoff

Vcrit	= 199.8 ft/s	Xr	= 2749.5 ft	Tr	= 24.21 s
Vr	= 221.6 ft/s	Xlo	= 2537.2 ft	Tlo	= 27.55 s
Vlo	= 249.1 ft/s	Xobs	= 4397.7 ft	Tobs	= 30.87 s
Vobs	= 268.3 ft/s	BFL	= 5623.0 ft		

Figure 5. Comparison of relative error of the codes with FLOPS predictions for the DC-9 takeoff.



Baseline for comparison: FLOPS predictions for HSCT takeoff

Vcrit	= 229.3 ft/s	Xr	= 3665.1 ft	Tr	= 26.9 s
Vr	= 270.3 ft/s	Xlo	= 4840.4 ft	Tlo	= 31.0 s
Vlo	= 304.0 ft/s	Xobs	= 5987.6 ft	Tobs	= 34.7 s
Vobs	= 315.4 ft/s	BFL	= 6783.7 ft		

Figure 6. Comparison of relative error of the codes with FLOPS predictions for the HSCT takeoff.