

A Basic Aerodynamics Project for a simple wing-body configuration

The objective of this project is to compare theoretical/computational estimates with the results from a NASA wind tunnel test. I've picked the simplest case I could find.

The project uses Louis S. Stivers, Jr., "Effects of Fixing Boundary-Layer Transition for a Swept- and a Triangular-Wing and Body Combination at Mach Numbers from 0.60 to 1.40," NASA TN D-312, June 1960, as the basis for this project. You can download this report from the NASA Tech Report Server (the link is on the Configuration Aerodynamics web page).

You should use the swept wing configuration for this project.

Your results should be presented primarily as (good) plots. You can digitize the data in the report using Data Thief, WebPlotDigitizer or equivalent.

Establish Geometry

a) Find the wetted area (by component). Discuss the details you had to consider.

Find basic friction and wave drag

b) Estimate the minimum viscous drag (use FRICTION or equivalent).

- compare your estimates with WT data for the variation in Mach number (Fig. 6a)
- compare your estimates with WT data for the variation in Reynolds number (Fig. 6c)

c) Find the estimated wave drag for supersonic Mach numbers ($M = 1.1$ to 1.4). Use AWAVE (the Harris code). Combine this with the viscous drag for the comparison (Fig. 6a).

Find the basic subsonic aero characteristics

d) Find and compare your estimate of $C_{L\alpha}$ and C_m/C_l for subsonic Mach numbers. Use a VLM code. You might also include any other algebraic relations in the comparison.

e) Where is the aerodynamic center located in percent mac ?

Find the basic supersonic aero characteristics

f) Estimate the supersonic $C_{L\alpha}$ and C_m/C_l . What is the aerodynamic center shift from the subsonic value?

Drag Polars?

g) Estimate the drag polar for $M = 0.6$ and $M = 1.4$.

Performance

h) What is your estimate for L/D_{max} at $M = 0.6$ and 1.4 ? What C_L do these estimate correspond to? Consider both completely laminar and completely turbulent flow.

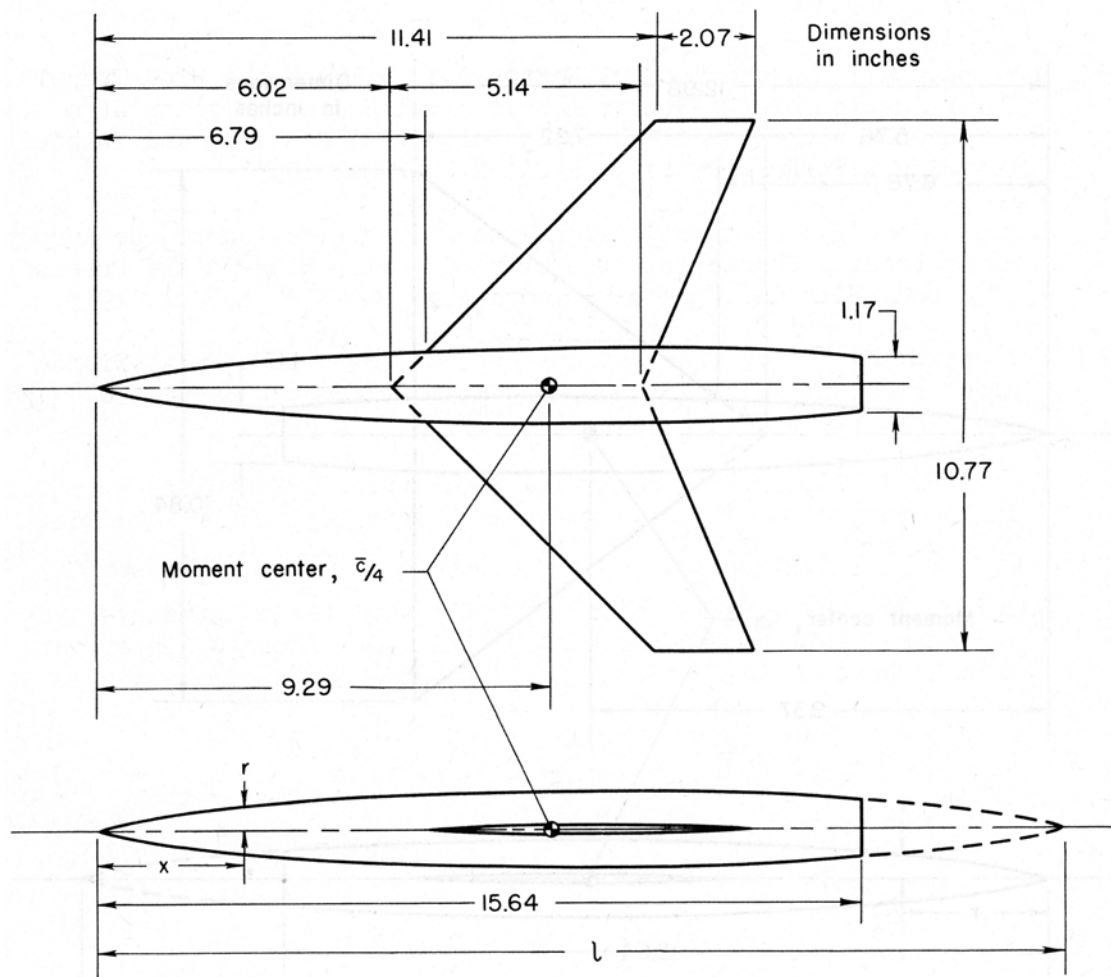
Conclusions

i) Discuss what you learned from this assignment?

Note: you should comment on the significance of your findings for each part as you present the results.

Establish Geometry

a) Find the wetted area (by component). Discuss the details you had to consider. Start with the geometry provided in the NASA TN:



Wing		Body
Aspect ratio	2.99	Ordinates given by:
Taper ratio	.40	$\frac{r}{r_0} = \left[1 - \left(1 - \frac{2x}{l} \right)^2 \right]^{\frac{3}{4}}$
Thickness-chord ratio	.03	Where:
Airfoil section	Biconvex	r = local radius
Area	38.83 sq. in.	$r_0 = r_{\text{maximum}} = 0.794$ in.
Mean aerodynamic chord	3.82 in.	x = longitudinal distance from nose
Leading-edge sweepback	45°	$l = 2(x \text{ for } r_0) = 19.833$ in.

Figure 1. Swept Wing-Body Model tested with results presented in NASA TN D-312. The surface area of the body can be found given the equation for the body. This is actually a Sears-Haack body truncated to allow for the model to be sting mounted. Unfortunately there doesn't seem to be a closed form expression for the surface area. However, it's easy to do a numerical integral.

Body of Revolution

$$r = r_0 \left[1 - \left(1 - \frac{2x}{l} \right)^2 \right]^{3/4}$$

$$r_0 = 0.794 \text{ in.}$$

$$l = 19.833 \text{ in}$$

Body runs back to 15.64

$$\text{Circumference, } C = 2\pi r$$

$$\text{Cross-sectional area, } X_{SEC} = \pi r^2$$

(needed later)

$$\text{Wetted Area, } S_{wet} = \int_0^{x_F} C dx = \int_0^{x_F} 2\pi r dx$$

$$S_{wet \text{ body}} = 2\pi r_0 \int_0^{x_F} \left[1 - \left(1 - 2\left(\frac{x}{l}\right) \right)^2 \right]^{3/4} dx$$

Do the integral numerically using the trapezoidal rule.

The next pages gives a short Fortran program, followed by the out put.

Fortran program used to estimate the surface area of the body

```
c
c   Sears-Haack surface area
c   wh mason July 20 2016/Oct 29, 2016

      dimension xt(251),rt(251),ct(251)
c
      pi = 3.14159265

      rzero = 0.794
      xl     = 19.833
      xbod   = 15.64

      imax = 91
      xmax = imax
      dx    = xbod/xmax

      write(6,200) rzero, xbod, xl
      do 10 i = 1,imax + 1
      x      = (i-1)*dx
      r      = rzero*(1.0 - (1.0 - 2.0*x/xl)**2)**0.75
      c      = 2.0*pi*r
      xsec   = pi*r**2

      xt(i) = x
      rt(i) = r
      ct(i) = c

      write(6,100) i, x, r, c, xsec
10  continue

      shsurf = 0.0
      do 20 i = 1,imax
      shsurf = shsurf + (ct(i) + ct(i+1))/2.0*dx
20  continue

      write(6,210) shsurf
100 format(2x,i4,2x,4f10.5)

200 format(/3x,'Sears-Haack Body and Surface Area'/
1  /4x,'rzero = ',f10.5,3x,'xbod = ',f10.5,4x,
2  'xl = ',f10.5//
3  5x,'i',8x,'x',9x,'r',9x,'c',5x,'xsection')
210 format(/3x,'S-H Surface Area = ',f10.5//)

      stop
      end
```

Output from the program:

Sears-Haack Body and Surface Area

rzero = 0.79400 xbod = 15.64000 xl = 19.83300

i	x	r	c	xsection
1	0.00000	0.00000	0.00000	0.00000
2	0.17187	0.06337	0.39817	0.01262
3	0.34374	0.10588	0.66524	0.03522
4	0.51560	0.14255	0.89570	0.06384
5	0.68747	0.17570	1.10396	0.09698
6	0.85934	0.20631	1.29628	0.13372
7	1.03121	0.23493	1.47612	0.17339
8	1.20308	0.26192	1.64566	0.21551
9	1.37495	0.28750	1.80641	0.25967
10	1.54681	0.31186	1.95945	0.30553
11	1.71868	0.33512	2.10560	0.35281
12	1.89055	0.35738	2.24551	0.40125
13	2.06242	0.37874	2.37970	0.45064
14	2.23429	0.39925	2.50858	0.50078
15	2.40615	0.41898	2.63251	0.55148
16	2.57802	0.43796	2.75178	0.60258
17	2.74989	0.45624	2.86665	0.65394
18	2.92176	0.47386	2.97733	0.70542
19	3.09363	0.49084	3.08403	0.75688
20	3.26549	0.50721	3.18689	0.80821
21	3.43736	0.52300	3.28608	0.85930
22	3.60923	0.53822	3.38172	0.91005
23	3.78110	0.55289	3.47393	0.96036
24	3.95297	0.56704	3.56282	1.01013
25	4.12484	0.58067	3.64848	1.05929
26	4.29670	0.59381	3.73100	1.10775
27	4.46857	0.60645	3.81045	1.15543
28	4.64044	0.61862	3.88692	1.20227
29	4.81231	0.63033	3.96046	1.24819
30	4.98418	0.64157	4.03113	1.29313
31	5.15604	0.65238	4.09899	1.33704
32	5.32791	0.66274	4.16410	1.37985
33	5.49978	0.67267	4.22649	1.42151
34	5.67165	0.68217	4.28621	1.46196
35	5.84352	0.69126	4.34330	1.50117
36	6.01538	0.69993	4.39780	1.53908
37	6.18725	0.70820	4.44974	1.57565
38	6.35912	0.71606	4.49915	1.61083
39	6.53099	0.72353	4.54606	1.64460
40	6.70286	0.73060	4.59050	1.67691
41	6.87473	0.73728	4.63249	1.70773
42	7.04659	0.74358	4.67206	1.73703
43	7.21846	0.74950	4.70923	1.76478
44	7.39033	0.75503	4.74401	1.79094
45	7.56220	0.76019	4.77643	1.81550
46	7.73407	0.76498	4.80650	1.83843
47	7.90593	0.76939	4.83424	1.85971
48	8.07780	0.77344	4.85965	1.87932
49	8.24967	0.77712	4.88276	1.89723
50	8.42154	0.78043	4.90357	1.91344
51	8.59341	0.78338	4.92209	1.92792
52	8.76528	0.78596	4.93834	1.94067

53	8.93714	0.78818	4.95231	1.95167
54	9.10901	0.79005	4.96402	1.96091
55	9.28088	0.79155	4.97347	1.96838
56	9.45275	0.79270	4.98066	1.97408
57	9.62462	0.79348	4.98561	1.97800
58	9.79648	0.79391	4.98830	1.98014
59	9.96835	0.79398	4.98875	1.98049
60	10.14022	0.79370	4.98694	1.97906
61	10.31209	0.79305	4.98289	1.97585
62	10.48396	0.79205	4.97659	1.97085
63	10.65582	0.79069	4.96804	1.96408
64	10.82769	0.78897	4.95722	1.95554
65	10.99956	0.78689	4.94415	1.94524
66	11.17143	0.78444	4.92881	1.93319
67	11.34330	0.78164	4.91119	1.91939
68	11.51516	0.77847	4.89129	1.90387
69	11.68703	0.77494	4.86909	1.88663
70	11.85890	0.77104	4.84459	1.86769
71	12.03077	0.76677	4.81778	1.84707
72	12.20264	0.76214	4.78864	1.82479
73	12.37451	0.75712	4.75715	1.80088
74	12.54637	0.75174	4.72331	1.77534
75	12.71824	0.74597	4.68709	1.74822
76	12.89011	0.73983	4.64847	1.71953
77	13.06198	0.73330	4.60745	1.68931
78	13.23385	0.72638	4.56398	1.65759
79	13.40571	0.71907	4.51805	1.62440
80	13.57758	0.71136	4.46963	1.58977
81	13.74945	0.70326	4.41870	1.55374
82	13.92132	0.69475	4.36522	1.51636
83	14.09319	0.68582	4.30916	1.47767
84	14.26505	0.67649	4.25049	1.43770
85	14.43692	0.66673	4.18916	1.39651
86	14.60879	0.65654	4.12514	1.35415
87	14.78066	0.64591	4.05838	1.31068
88	14.95253	0.63484	3.98883	1.26614
89	15.12440	0.62332	3.91643	1.22059
90	15.29626	0.61134	3.84113	1.17411
91	15.46813	0.59888	3.76287	1.12675
92	15.64000	0.58594	3.68158	1.07859

S-H Surface Area = 61.66352

This is the basic body surface area in square inches.

As a check, lets compare the results with a rule of thumb available on Ralph Carmichael's PDAS website. He made a study and says to use 72% of the surface area of a cylinder with the same length using the maximum diameter of the body. In this case we take the full length of the body, $l = 19.833$ inches, which produces a wetted area for this case of 71.24 in^2 from Ralph's Rule of Thumb. When I run the code using the full body length I get 71.122 in^2 . So I say the result given above is accurate (always do a sanity check!).

Next we need to subtract the surface area of the body that is covered by the wing root. This requires that we find the cross sectional area of the biconvex airfoil used in the wing.

Cross Sectional Area of Biconvex airfoil:

Need to estimate free surface area covered by wire root
- exclude from body wetted area

Airfoil: $y = 2 \cdot \left(\frac{t}{c}\right) \left(\frac{x}{c}\right) \left(1 - \frac{x}{c}\right)$

$$= 2 \left(\frac{t}{c}\right) \left[\frac{x}{c} - \left(\frac{x}{c}\right)^2 \right]$$

$$\frac{\text{Area}}{2} = \int_0^1 2 \left(\frac{t}{c}\right) \left[\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2 \right] d\left(\frac{x}{c}\right)$$

$$= 2 \cdot \left(\frac{t}{c}\right) \left[\int_0^1 \left(\frac{x}{c}\right) d\left(\frac{x}{c}\right) - \int_0^1 \left(\frac{x}{c}\right)^2 d\left(\frac{x}{c}\right) \right]$$

$$= 2 \left(\frac{t}{c}\right) \left[\left. \frac{1}{2} \left(\frac{x}{c}\right)^2 \right|_0^1 - \left. \frac{1}{3} \left(\frac{x}{c}\right)^3 \right|_0^1 \right]$$

$$= 2 \left(\frac{t}{c}\right) \left[\frac{1}{2} - \frac{1}{3} \right] = 2 \left(\frac{t}{c}\right) \left[\frac{1}{6} \right]$$

$$\frac{\text{Area}}{2} = \frac{1}{3} \left(\frac{t}{c}\right)$$

-7

$$\boxed{A = \frac{2}{3} \left(\frac{t}{c}\right)} \quad \begin{array}{l} \text{upper} \\ \text{and} \\ \text{lower} \end{array}$$

Cross section

$$\frac{A}{c^2} = \frac{2}{3} \cdot (0.03) = \underline{0.02}$$

This agrees with the numerical integration (see next)

Fuze covered by wing root:

$$C_{RT} = 4.713 \text{ in}$$

$$A = 0.02 \times 4.713^2 = .4452 \text{ in}^2 \text{ one side}$$

$$\text{Both sides } A_{\text{covered}} = 0.89 \text{ in}^2$$

Subtract this from the web area calc:

$$\boxed{S_{\text{web}} - S_{\text{Fuze}} = 61.67 - 0.89 = 60.78 \text{ in}^2}$$

Biconvex airfoil program to check analysis numerically.

```
c
c   Biconvex foil analysis
c   wh mason Oct 10 2016

      implicit real*8(a-h,o-z)
      dimension xc(251),yc(251),sa(251)
c
      pi = 3.14159265

      tcmax = 0.03

      imax = 200
      xcmax = imax
      dx = 1.0/xcmax

      write(6,200) tcmax, dx
      do 10 i = 1,imax + 1
      xc(i) = (i-1)*dx
      yc(i) = 2.0*tcmax*xc(i)*(1.0 - xc(i))

      write(6,100) i, xc(i), yc(i)

10 continue

      sarc = 0.0
      sarea = 0.0
      do 20 i = 1,imax

      da = (yc(i) + yc(i+1))/2.0*dx
      ds2 = dx**2 + (yc(i+1)-yc(i))**2
      ds = dsqrt(ds2)
      sarc = sarc + ds
      sarea = sarea + da

20 continue

      write(6,210) sarc,sarea

100 format(2x,i4,2x,5f10.5)

200 format(/3x,'Biconvex section arc and contour Area'/
1 /4x,'tcmax = ',f10.5,5x,'dx = ',f10.5//
2 5x,'i',8x,'x/c',7x,'y/c')

210 format(/3x,'Biconvex Airfoil '//
1 5x,'arc length = ',f10.5//
2 5x,'contour area = ',f10.5//)

      stop
      end
```

Output from biconvex airfoil code

(edited to reduce output length, $\Delta x = 0.005$ produced 201 station of output)

Biconvex section arc and contour Area

```
tcmax =    0.03000    dx =    0.00500

  i      x/c      y/c
  1      0.00000    0.00000
 11      0.05000    0.00285
 21      0.10000    0.00540
 31      0.15000    0.00765
 41      0.20000    0.00960
 42      0.20500    0.00978
 43      0.21000    0.00995
 51      0.25000    0.01125
 61      0.30000    0.01260
 71      0.35000    0.01365
 81      0.40000    0.01440
 91      0.45000    0.01485
101      0.50000    0.01500
111      0.55000    0.01485
121      0.60000    0.01440
131      0.65000    0.01365
141      0.70000    0.01260
151      0.75000    0.01125
161      0.80000    0.00960
171      0.85000    0.00765
181      0.90000    0.00540
191      0.95000    0.00285
201      1.00000    0.00000
```

Biconvex Airfoil

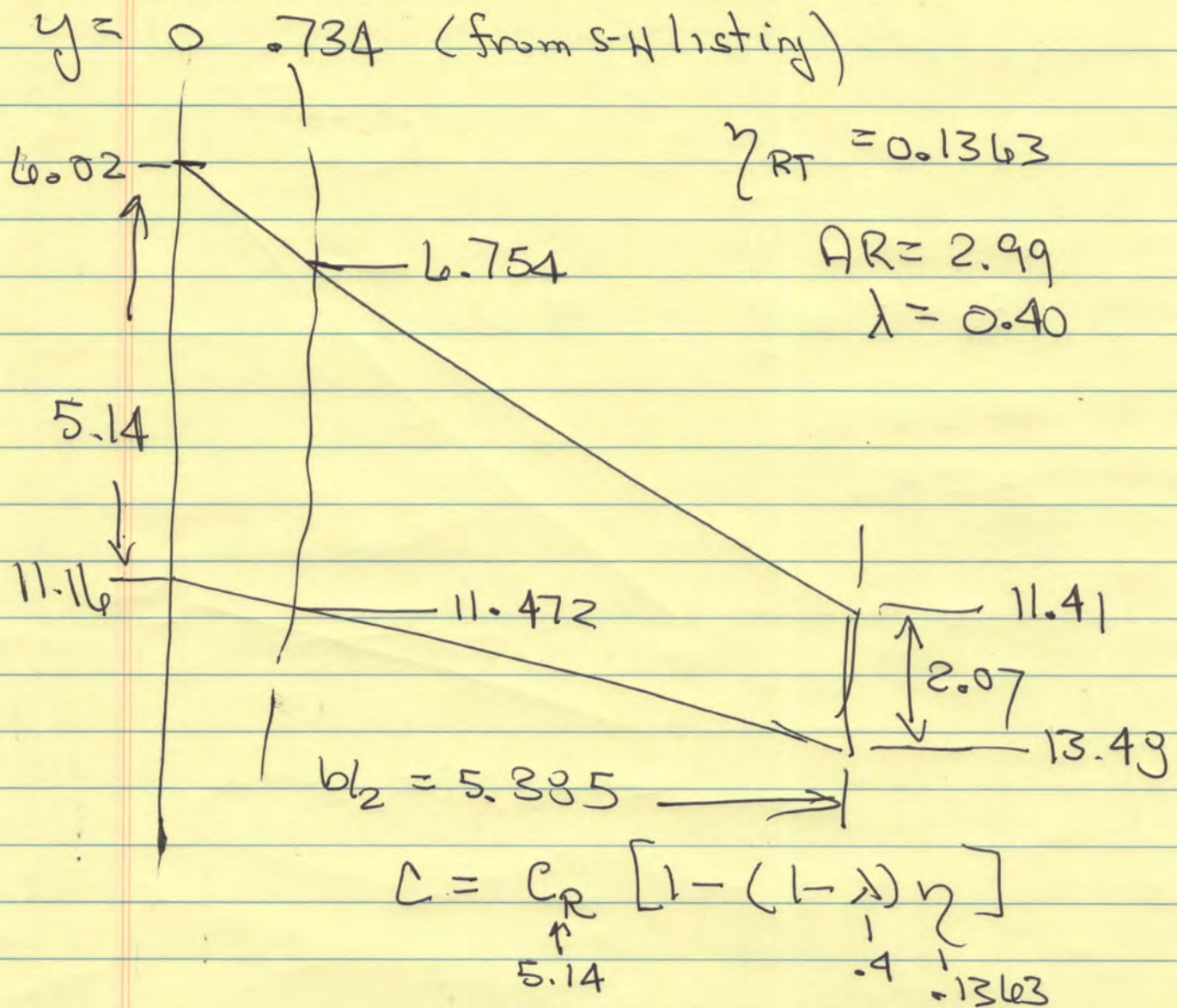
```
arc length   =    1.00060
```

```
contour area =    0.01000
```

This calculation verified the analytic results. In particular, the extremely small value of arc length led to making an analytic expression to check the numerical value. The results shown below when finding the wing wetted area illustrates the use of asymptotic analysis to find the expression (this was fun. I don't get to do this very often).

Now we need to find the wing wetted area. The first step is to find the exposed wing area.

Exposed Wing Surface Area



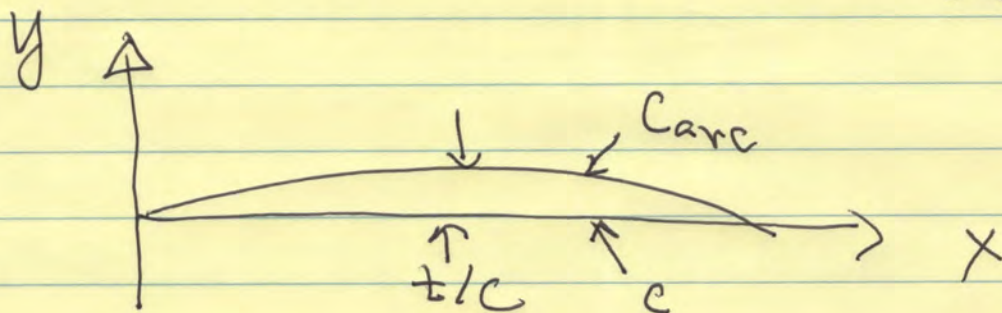
$S_{wet\ exposed} = \left(\frac{4.72 + 2.07}{2} \right) (5.385 - 0.734) \times 2 \times 2$
 $= 63.16 \text{ in}^2$

Top & Bottom
 \downarrow
 \uparrow
 Both sides

Neglecting the difference between chord & arc length

①
What is the arc length
of a biconvex airfoil

[Note: most times for thin airfoils
we ignore the difference
between c and C_{arc} . Let's
see if this is justified]



The airfoil is $\frac{y}{c} = 2 \cdot \left(\frac{t}{c}\right) \left(\frac{x}{c}\right) \left[1 - \left(\frac{x}{c}\right)\right]$

1st: try the analytic sol'n.

$$C_{arc} = \int_0^c \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\frac{d(y/c)}{d(x/c)} = 2 \cdot \left(\frac{t}{c}\right) \frac{d}{d(x/c)} \left[\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2 \right]$$

$$= 2 \left(\frac{t}{c}\right) \left[1 - 2 \left(\frac{x}{c}\right) \right]$$

(2)

$$\left[\frac{dy/c}{dx/c} \right]^2 = \left(2 \cdot \frac{t}{c} \right)^2 \left[1 - 4 \left(\frac{x}{c} \right) + 4 \left(\frac{x}{c} \right)^2 \right]$$

$$\begin{aligned} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} &= \sqrt{1 + \left\{ 2 \left(\frac{t}{c} \right) \right\}^2 \left[1 - 4 \left(\frac{x}{c} \right) + \left(\frac{x}{c} \right)^2 \right]} \\ &= \sqrt{1 + 2^2 \left(\frac{t}{c} \right)^2 - 2^2 \left(\frac{t}{c} \right)^2 4 \left(\frac{x}{c} \right) + \left(2 \frac{t}{c} \right)^2 4 \left(\frac{x}{c} \right)^2} \\ &= \sqrt{1 + 4 \left(\frac{t}{c} \right)^2 - 16 \left(\frac{t}{c} \right)^2 \left(\frac{x}{c} \right) + 16 \left(\frac{t}{c} \right)^2 \left(\frac{x}{c} \right)^2} \end{aligned}$$

Need the integral.

An examination of my integral suggests this will be a very messy integration. Let's try to use numerics.

Note, for small t/c there is probably an asymptotic formula.

(3)

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad \text{expansion}$$

$$(1 + \epsilon^2)^{1/2} =$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2$$

$$n = 1/2$$

$$\frac{1}{2} \quad \frac{1}{2} \cdot \frac{1}{2}$$

$$(1 + x)^{1/2} = 1 + \frac{1}{2}x + \frac{1}{8}x^2$$

$$\sqrt{1 - \left(\frac{dy}{dx}\right)^2} \approx 1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{8}\left(\frac{dy}{dx}\right)^4 + \dots$$

$$\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \approx \int_0^1 \left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{8}\left(\frac{dy}{dx}\right)^4 + \dots \right] dx$$

$$= \int_0^1 1 \frac{dx}{c} + \int_0^1 \frac{1}{2} \left(\frac{dy}{dx}\right)^2 \frac{dx}{c} + \dots$$

4

$$\frac{L_{arc}}{c} = 1 + \frac{1}{2} \int_0^1 \left[\left\{ 2 \left(\frac{t}{c} \right) \right\}^2 \left[1 - 4 \left(\frac{x}{c} \right) + 4 \left(\frac{x}{c} \right)^2 \right] \right] dx_c$$

$$= 1 + \underbrace{\frac{1}{2} \cdot 4 \left(\frac{t}{c} \right)^2 \int_0^1 dx_c}_{(1)} - \underbrace{\frac{1}{2} \cdot 4 \cdot \left(\frac{t}{c} \right)^2 \cdot 4 \int_0^1 \left(\frac{x}{c} \right) d \left(\frac{x}{c} \right)}_{(2)}$$

$$+ \frac{1}{2} \cdot 4 \left(\frac{t}{c} \right)^2 \cdot 4 \int_0^1 \left(\frac{x}{c} \right)^2 d \left(\frac{x}{c} \right)$$

$$= 1 + 2 \cdot \left(\frac{t}{c} \right)^2 - 2 \left(\frac{t}{c} \right)^2 \cdot 4 \cdot \frac{x}{2} + \frac{1}{2} \cdot 4 \left(\frac{t}{c} \right)^2 \cdot \frac{4}{3}$$

$$= 1 + 2 \left(\frac{t}{c} \right)^2 - 4 \left(\frac{t}{c} \right)^2 + \frac{16}{6} \left(\frac{t}{c} \right)^2$$

$$2 - 4 + \frac{12 - 24 + 16}{6} + \frac{23 - 21}{6} = \frac{4}{6}$$

$$= 1 + \frac{2}{3} \left(\frac{t}{c} \right)^2$$

$$t/c = 0.03$$

$$= \underline{\underline{0.0006}} \quad \text{--- same as output above}$$

this is what I got numerically

Thus:

For a biconvex airfoil with a small (k/c) , the ratio of the arc length to the chord is

$$\frac{C_{arc}}{c} = 1 + \frac{2}{3} \left(\frac{k}{c} \right)^2$$

Clearly this can be considered negligible

Summary of wetted area results:

Body

$$\begin{aligned}\text{Gross } S_{wet} &= 61.67 \text{ in}^2 \\ - \text{ area covered by wing root} &= \underline{0.89 \text{ in}^2} \\ \text{Net body wetted area} &= 60.78 \text{ in}^2 \\ &= 0.4221 \text{ ft}^2\end{aligned}$$

Wing

$$\begin{aligned}\text{Basic wing wetted area} &= 63.142 \text{ in}^2 \\ &= 0.4385 \text{ ft}^2 \\ \text{Adjusted for arc length:} \\ \quad \times 1.0006 &= 0.4387 \text{ ft}^2\end{aligned}$$

Curious that the wing and body wetted areas are so similar

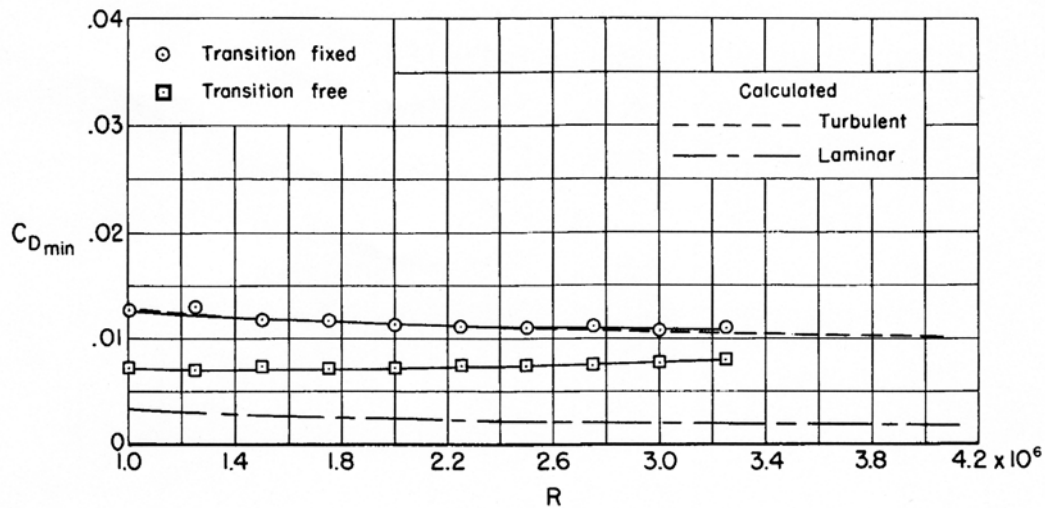
One measure often checked by aerodynamicists is the ratio of wetted area to reference area. In this case we get $S_w = 60.78 + 63.18 = 123.96 \text{ in}^2$. With a wing reference area of 38.83 in^2 , the ratio of $S_w/S_{ref} = 3.19$. For a real airplane with a separate control surface it's hard to get a ratio less than four.

Find basic friction and wave drag

b) Estimate the minimum viscous drag (use FRICTION or equivalent).

- compare your estimates with WT data for the variation in Mach number (Fig. 6a)
- compare your estimates with WT data for the variation in Reynolds number (Fig. 6c)

We'll do the Reynolds number sweep first. Figure 2 is Figure 6(c) from the TN. Transition fixed means the boundary layer trips are at 8.5% of the body length and 15% of the chord on the wing.



(c) $C_{D_{min}}$ vs. R ; swept wing; $M = 0.60$.

Figure 2. Wind tunnel test results for the swept wing body (TN D-312).

This is the input used for the FRICTION calculation. Note that we've converted inches to feet for this input. Also, the test results are given in terms of Reynolds number based on the mean aerodynamic chord. FRICTION requires a Reynolds number per unit length so that it can use the reference length of each component (I think this assumes that the length unit is feet).

NASA TN D-312 Wing Body - Turb Flow w Transition Fixed Re Sweep						
0.26965	1.	2.	1.0			
FUSELAGE		0.4221	1.303	0.05080	1.0	0.085
WING		0.4387	0.3183	0.03000	0.0	0.150
0.600	3.1417					
0.600	3.7700					
0.600	4.3984					
0.600	5.0267					
0.600	5.6550					
0.600	6.2834					
0.600	6.9117					
0.600	7.5400					
0.600	8.1684					
0.600	8.7967					
0.600	9.4251					
0.600	10.053					
0.600	10.682					
0.000	0.000					

Figure 3 shows the FRICTION estimates compared with the wind tunnel test results. The figure is complicated and worth studying.

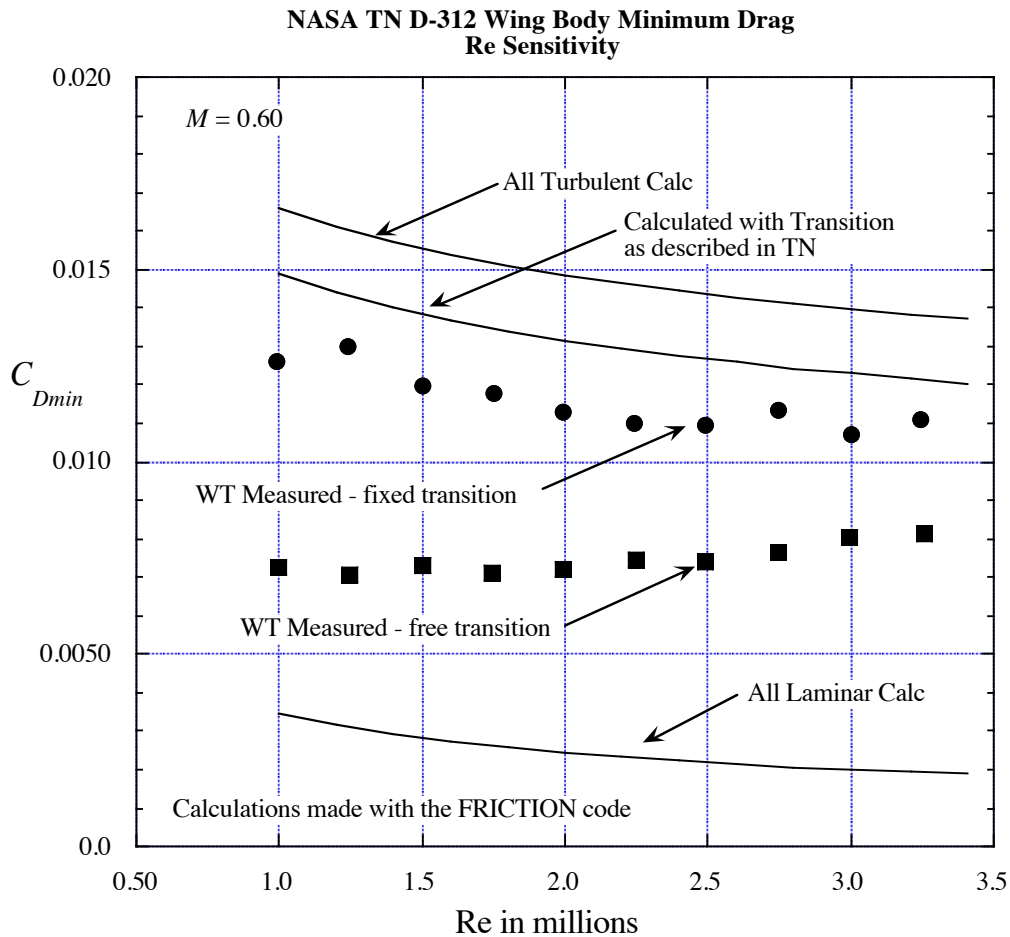


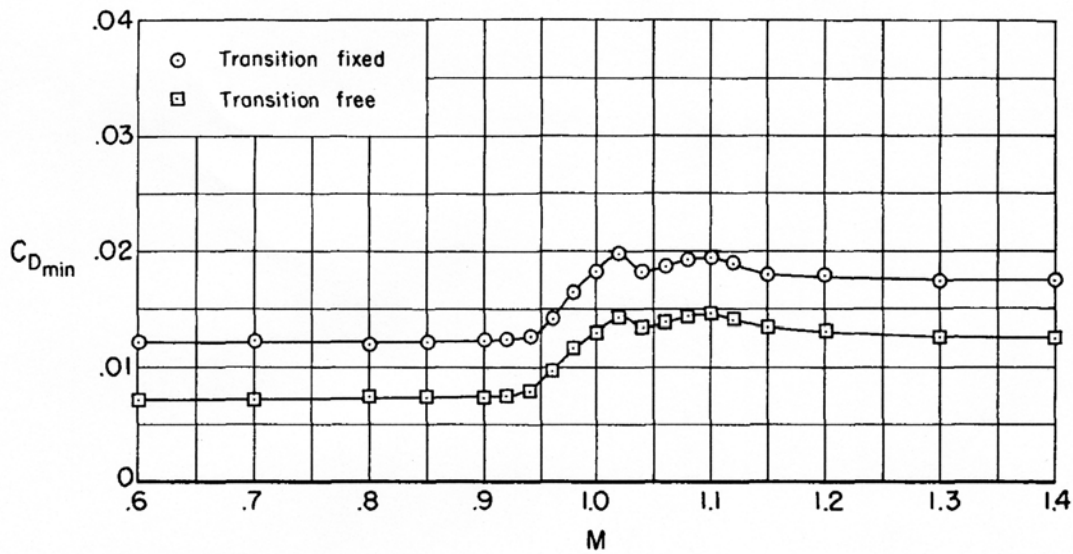
Figure 3. Comparison of FRICTION and test results over a range of Reynolds numbers.

We show both test results and three estimates from FRICTION. First, note that the fully turbulent and fully laminar estimates bound the wind tunnel values. Clearly without trip strips there is still a significant amount of turbulent flow, measured values are well above the purely laminar values. Also, the wind tunnel data shows that the difference between the two cases is larger at the lower Reynolds number and becomes smaller as the Reynolds number increases. This means that without trying to fix transition more of the flow is turbulent as the Reynolds number increases, as expected.

Next, note that assuming the trip strips fix the portion of laminar and turbulent flow the estimated friction values move down from the all turbulent case although the estimate is still higher than the measured value. It might be that the trips don't actually lead to turbulent flow exactly at their location. Also the trend with Reynolds number is very similar between the FRICTION estimate and the wind tunnel results.

Finally, this shows how today's computational methods make more demands on the wind tunnel test. It would have been very useful to know if the boundary layer was actually tripped at the location of the transition strips.

Figure 4 gives the Mach number sweep results for the wind tunnel test as presented in the TN.



(a) $C_{D_{min}}$ vs. M ; swept wing; $R = 1.5 \times 10^6$.

Figure 4. Wind tunnel test results for the swept wing body (TN D-312).

Below we give an example of the FRICTION input used to make the calculation.

```

NASA TN D-312 Wing Body - Turb Flow Mach Sweep w specified transition
0.26965  1.      2.      1.0
FUSELAGE      0.4221  1.303   0.05080  1.0    0.085
WING          0.4387  0.3183  0.03000  0.0    0.150

0.600  4.7130
0.700  4.7130
0.800  4.7130
0.850  4.7130
0.900  4.7130
0.920  4.7130
0.940  4.7130
0.960  4.7130
0.980  4.7130
1.000  4.7130
1.020  4.7130
1.040  4.7130
1.060  4.7130
1.080  4.7130
1.100  4.7130
1.125  4.7130
1.150  4.7130
1.175  4.7130
1.200  4.7130
1.300  4.7130
1.400  4.7130
0.000  0.000

```


Figure 5 contains both the transition fixed wind tunnel results shown in Figure 4 and the estimated viscous drag computed using FRICITION.

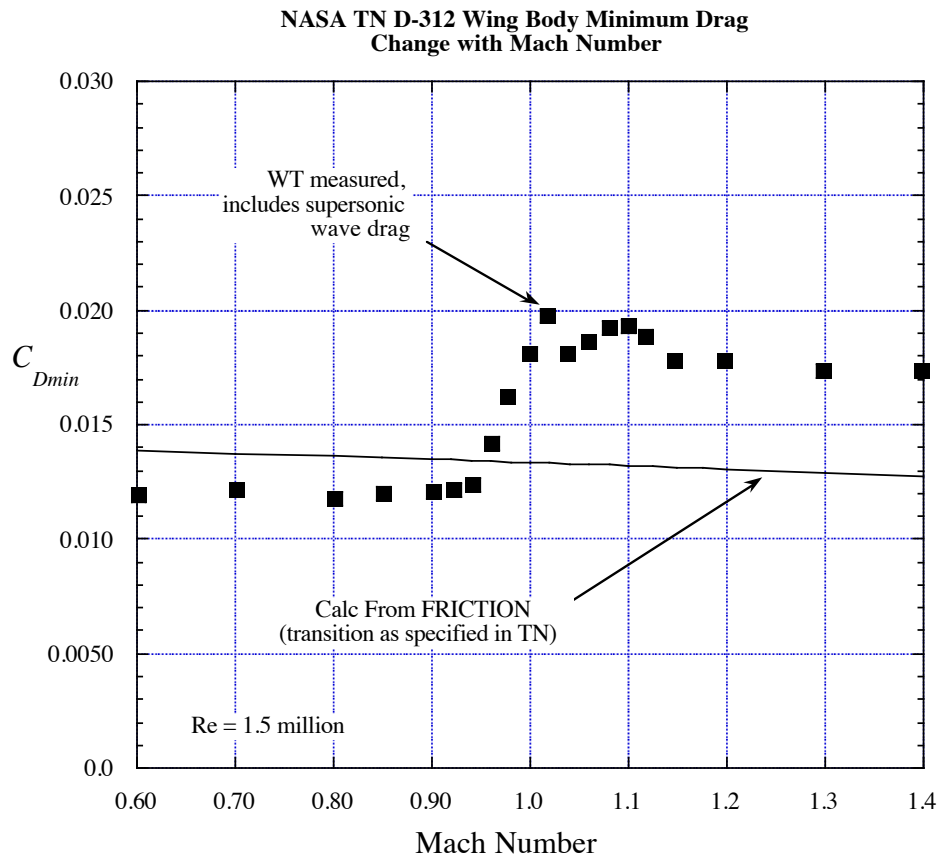


Figure 5. Comparison of wind tunnel and FRICITION results over a range of Mach numbers.

We see that FRICITION over estimates the measured values, as we saw in Figure 3. We also see the wind tunnel drag increases going from subsonic to supersonic flow. This is the wave drag component. As we expect, FRICITION doesn't capture this and we'll have to do a wave drag analysis and add it to the viscous flow to compare with the test results at supersonic speed. Although we see the slight trend of decreasing drag coefficient with Mach number from FRICITION, you can't see it in the wind tunnel data.

The isolated wing a-wave input

[illegible]

The complete wing-body awave input

[illegible]

I know these appear painful to construct, but it really didn't take that long. I suggest you create these files by using the sample case and editing the file.

The first step in the wave drag analysis is to look at the cross sectional area distribution as computed in awave. This is important to check that the input data doesn't have errors (easy to do, the plot will reveal any problems). Figure 6 shows the awave computed area distribution for both the body alone and wing-body case. The results are smooth so that the input is OK. Apparently because the objective of the test was to look at transition effects there was no area ruling of the model. I assume this made the model simpler. You can clearly see the "hump" due to the addition of the wing.

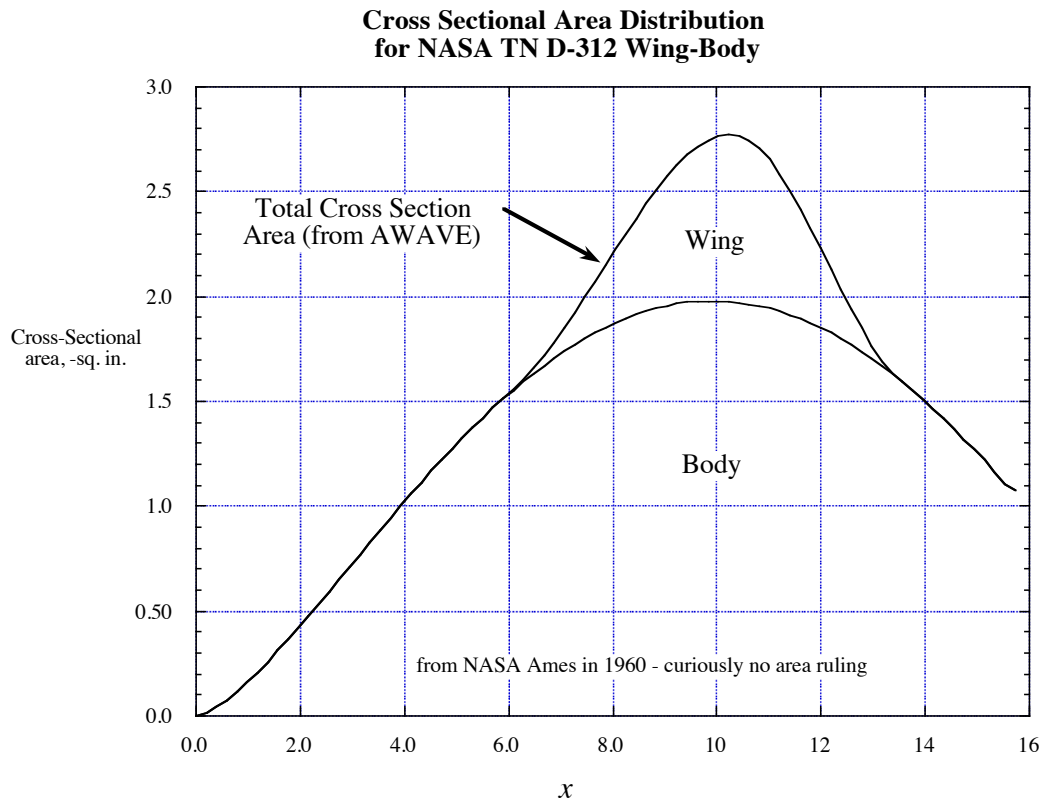


Figure 6. NASA TN D-312 cross-sectional area distribution.

Next, in Figure 7 we look at the results for the isolated body, the isolated wing, the complete wing-body configuration and the sum of the isolated body and isolated wing wave drag predictions.

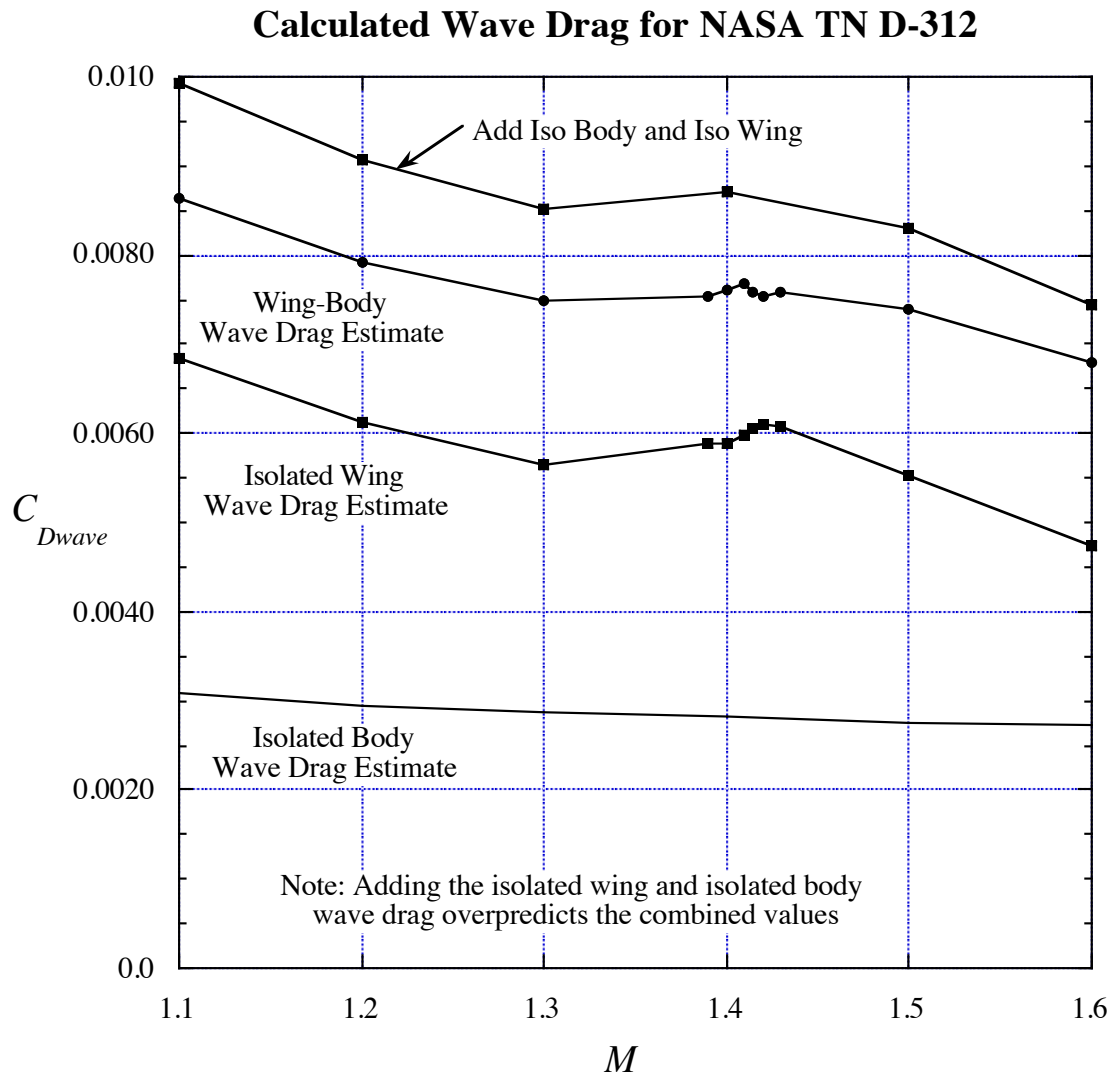


Figure 7. wave wave drag estimates for TN D-312.

A couple of aspects of the results are worth noting. While the isolated body wave drag varies smoothly with Mach number there is a “wobble” in the isolated wing and wing body calculations. If we recall that the leading edge sweep is 45° then we know that the leading edge is just sonic at $M = 1.414$. That explains the wiggle. Awave doesn’t handle this situation very well. The next thing to look at is the difference between the wing-body wave drag and the value we would get if we just added the isolated body and isolated wing results. When we do this we get a higher drag than the combined calculation. Clearly you can’t do a component build up for wave drag, you have to do the full combined analysis.

We are now ready to look at how the wave drag prediction compares to the wind tunnel test. For this to be meaningful we need to adjust the estimated friction down to agree with the wind tunnel data. I subtracted 18.8 counts from the friction estimate to match the wind tunnel values at $M = 0.80$. The resulting comparison is given in Figure 8. I was pleasantly surprised at the result.

Although the scale leaves a lot of blank space in the plot, it is the scale used in the wind tunnel report, see Figure 4.

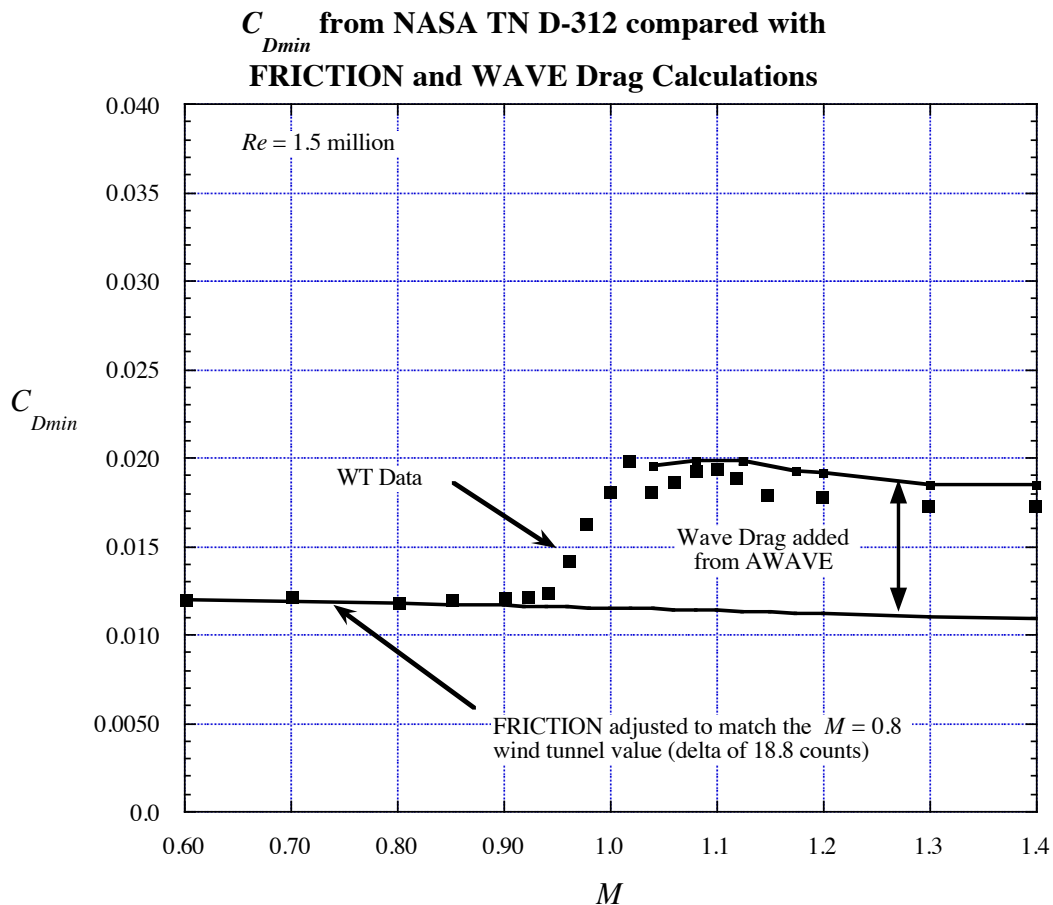


Figure 8. Comparison of awave wave drag estimate with wind tunnel measurements.

Some students might think that adjusting the friction estimate is cheating. It would be if the process had not been defined. The purpose of this activity is to illustrate how working engineers look at computations and tests (Please don't use the word "experiment" to describe testing. Experiments are what scientists do, not engineers. Engineers test.)

Find the basic subsonic aero characteristics

d) Find and compare your estimate of $C_{L\alpha}$ and C_m/C_l for subsonic Mach numbers. Use a VLM code. You might also include any other algebraic relations in the comparison.

e) Where is the aerodynamic center located in percent mac ?

First we review the wind tunnel test data. Figure 9 repeats the test data presented in Figure 4 of the NASA TN. In this case the lift curve slope has been found by “sloping” the test data. Since the slope is found indirectly there are no symbols, but rather a line. Also, they found that the lift curve slope was not exactly linear so that they got slightly different values for different C_L 's. It's interesting to see that the effect of fixing transition is most apparent for the curve labeled $C_L = 0$, and then primarily in the transonic region from $M = 0.85$ to 0.95 . The curves are presented across the Mach number range. In this section we concentrate on the subsonic values and will examine the supersonic values in the next section.

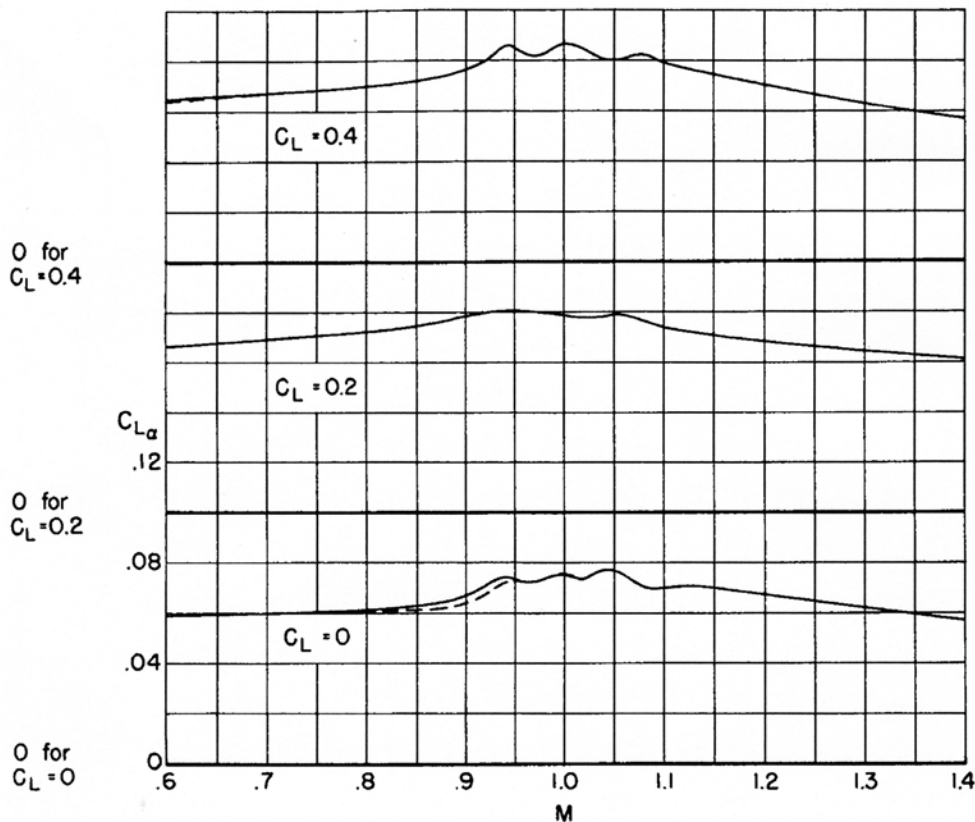


Figure 9. Wind tunnel derived lift curve slope from NASA TN D-312.

We will use the VLMpc code to estimate the lift curve slope and will also use the DATCOM algebraic formula (also used in Nicolai's book). We will find the lift curve slope using an isolated wing model, and then a model of the wing-body. The input data sets are given next.

Now we illustrate the VLMpc models used in the calculation. First we have the wing alone input file.

```
NASA TN D-312 isolated wing
1.      1.      3.62      38.83      -9.29
3.      0.      0.      0.0
-6.02    0.0    0.0    1.
-11.41   -5.385  0.0
-13.48   -5.385  0.    1.
-11.16    0.0
1.  9. 20.  .96  1.  0.  0.  0.      0.      0.
end of case
```

Next we show the wing body file used in VLMpc.

```
NASA TN D-312 Wing Body
2.      1.      3.62      38.83      -9.29
6.      0.      0.      0.0
0.0      0.0      0.0      1.
-4.0      -0.734      0.0      1.
-6.754     -0.734      0.0      1.
-11.41     -5.385      0.0
-13.48     -5.385      0.    1.
-11.472    -0.734      0.    1.
-11.472     0.0
3.      0.      0.      0.0
-11.472     0.0      0.    1.
-11.472    -0.734      0.    1.
-15.64     -0.734      0.    1.
-15.64     -0.      0.    1.
1.  9. 16.  .96  1.  0.  0.  0.      0.      1.
-1.5     -15.     -1.5     -15.000000
```

Someday I'd like to ask Rich Margason or John Lamar why the input to the code seems to be bass-akward. Anyway, that how it's input.

Figure 10 shows the comparison of the VLMpc results and the DATCOM formula with the wind tunnel test results.

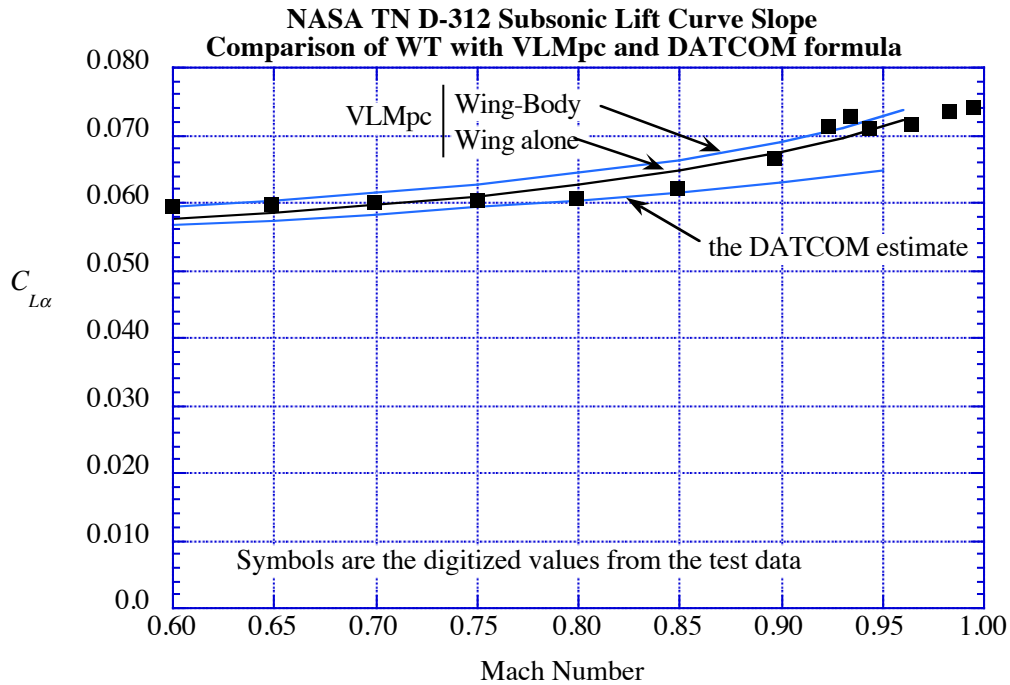


Figure 10. Lift curve slope comparison for TN D-3123 and VLMpc/DATCOM.

A few comments can be made about the results shown in the plot. First, all the estimates are relatively close to the test data. Next, there are only minor effects of including a body model on the lift curve slope. At the lowest Mach number (0.6) the wing body model seems to agree best with the test data, but only marginally. Curiously, the DATCOM formula, that I think can be traced back to the work of Eddie Polhamus, has Mach number effects that agree best with the data.

Next we look at the stability estimates, as represented by C_m/C_L . Figure 11 is a scan of Figure 4 in the NASA TN.

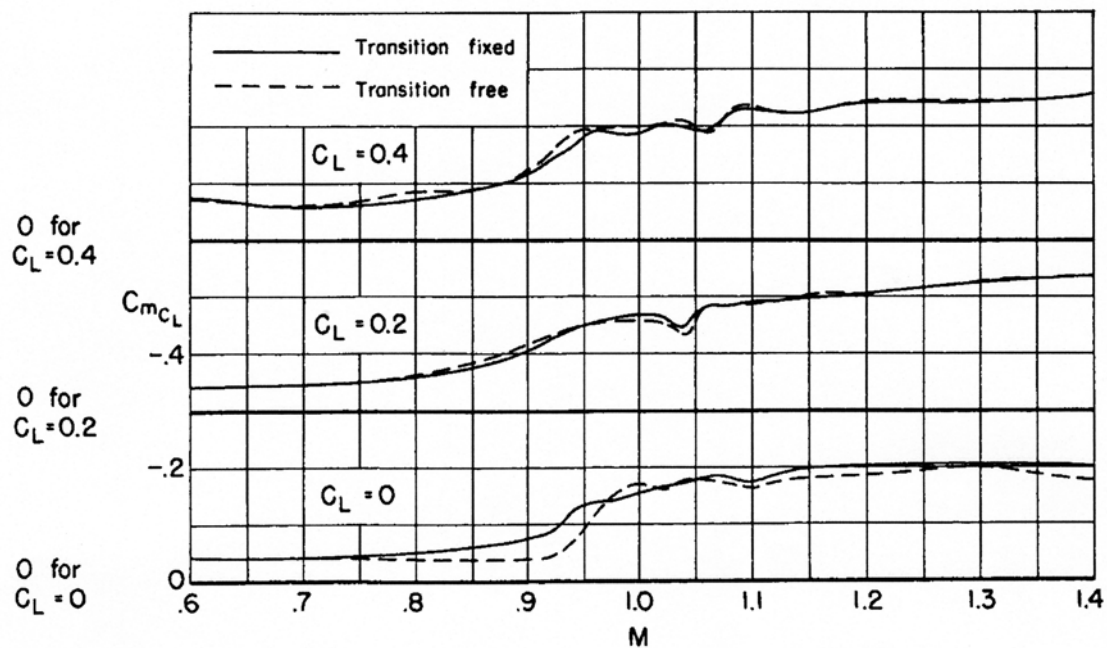


Figure 11. C_m/C_L from the wind tunnel test.

Here again the results are provided for three different C_L values. Looking closely the values are actually pretty similar. Figure 12 contains the results of comparing the stability from the test with VLMpc (there is no equivalent DATCOM formulas).

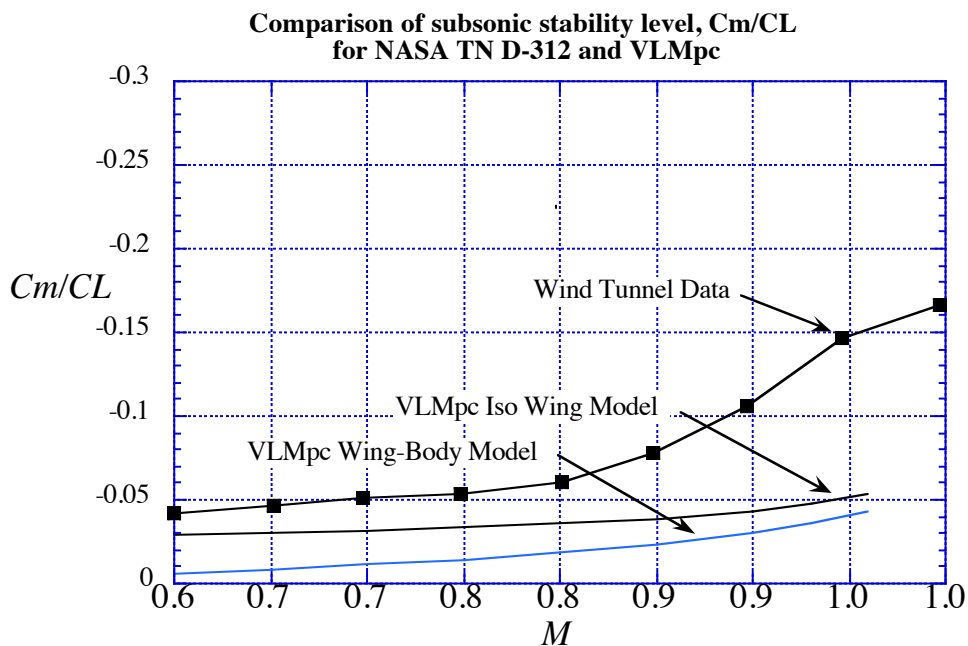


Figure 12. Stability comparison for TN D-3123 and VLMpc.

The apparently poor agreement between VLMpc and the wind tunnel data is one of the more surprising outcomes of this study. I expected the agreement to be better.

Let's look at the comparison in terms of the neutral point (aerodynamic center of the entire configuration). The moment reference used to compute pitching moment is the quarter chord of the *mac*. It is located at 9.29 inches aft of the nose, see Figure 1. You should verify that you understand how this value was obtained. I got 9.28 inches, however some of the values listed in Figure 1 have been rounded off, and that might account for the very slight difference in values. Now, recall that the neutral point or aerodynamic center can be found knowing the reference location used for the pitching moment and the stability value about that location:

$$\frac{x_{ac}}{c} = \frac{x_1}{c} - \frac{dC_{m_{x_1/c}}}{dC_L}$$

For this case x_1/c is the quarter chord, 0.25, with c being the mean aerodynamic chord (*mac*), 3.82 inches.

Therefore at $M = 0.6$, we have the following:

	C_m/C_L	the neutral point (% <i>mac</i>)
Wind Tunnel	-0.041	29.1
VLMpc Wing-body	-0.006	25.6
VLMpc wing alone	-0.029	27.9

The neutral point is one of the most important values when laying out the configuration. We see that adding the body to the wing calculation destabilizes the configuration as expected.

One thing to note is that the aerodynamic center for a wing is not exactly at the quarter chord. Students often assume that it is, although this is based on 2D thin airfoil theory. It is an approximate value. I have a chart illustrating this in my Config Aero subsonic wings presentation.

There could be an error in my analysis, although I don't see it. This would be a very good case to compare results from other codes. Considering the classic VLM method, I wrote my own code for an isolated wing and it agreed with the VLMpc results exactly for the Warren 12 planform (I expected to see at least some small difference and was surprised they produced exactly the same answers).

Find the basic supersonic aero characteristics

- f) Estimate the supersonic C_{L_α} and C_m/C_l . What is the aerodynamic center shift from the subsonic value?

We will include the entire Mach range in presenting these results. The data sets used for the supersonic calculations are given next.

The isolated wing case

```
TN D-312 Supersonic - wing alone
&WING rootle=6.02, rootte=11.16, rooty=0.0,
      tiple=11.41, tipte=13.48, tipy=5.385, rows=10, cols=10,
      OPT=0, tcroot=0.03, tctip=0.03, RNL=0, SECT=0,
      MACH=1.50, SREF=38.83, CBAR=3.82, REFMOM=9.29, OPT=0
&END
&AERO CASE=2, SREF=38.83, CBAR=3.82, REFMOM=9.29 &END
&INCR DELTA=2,&END
```

The wing body case

```
TN D-312 Supersonic - wing body
&BODY BCODE=-1, NTAB=27,
      X = 0.000, 0.516, 1.031, 1.547, 2.062, 2.578,
          3.094, 3.609, 4.125, 4.640, 5.328, 6.015,
          6.703, 7.390, 8.078, 8.765, 9.453, 10.140,
          10.828, 11.687, 12.546, 13.234, 13.921, 14.437,
          14.781, 15.296, 15.640,
      R = 0.0, 0.14255, 0.23493, 0.31186, 0.37874, 0.43796,
          0.49084, 0.53822, 0.58067, 0.61862, 0.66274, 0.69993,
          0.73060, 0.75503, 0.77344, 0.78596, 0.79270, 0.79370,
          0.78897, 0.77494, 0.75174, 0.72638, 0.69475, 0.66673,
          0.64591, 0.61134, 0.58584,
      xstart=6.754, NB=4, NROWS=10, NXBODY=31, OC=3,
      SREF=38.83, CBAR=3.82, REFMOM=9.29, MACH=1.30
&END
&WING rootle=6.754, rootte=11.472, rooty=0.734,
      tiple=11.41, tipte=13.48, tipy=5.385, rows=10, cols=10,
      OPT=0, tcroot=0.03, tctip=0.03, RNL=0, SECT=0,
      MACH=1.30, SREF=38.83, CBAR=3.82, REFMOM=9.29, OPT=0
&END
&AERO CASE=2, SREF=38.83, CBAR=3.82, REFMOM=9.29 &END
&INCR DELTA=2,&END
```

The Figure 13 shows the comparison for the lift curve slope.

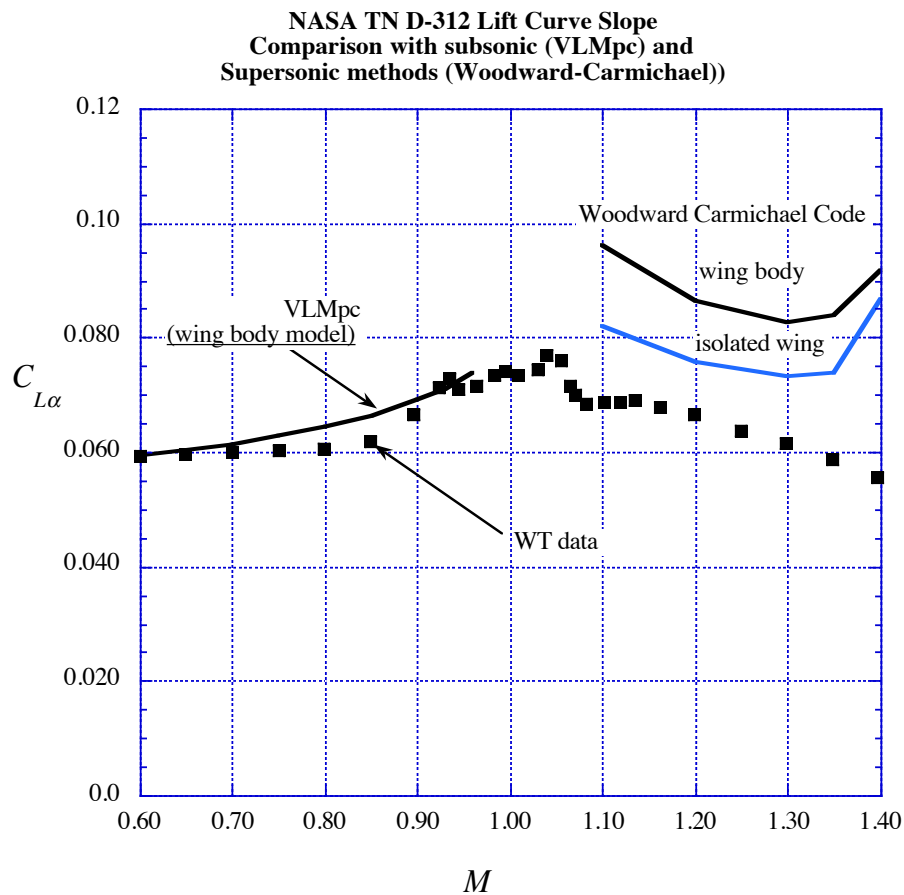


Figure 13. Comparison of wind tunnel and estimated lift curve slopes.

In this plot the subsonic estimates look pretty good, but the results from the Woodward-Carmichael code seem to be off significantly. One explanation is that I have almost no experience with this code. In the 1970s I used the Grumman version they bought from Frank Woodward a lot. But Ralph's code is considerably different, including very different input. Clearly further effort is required to investigate the discrepancy in the results. I only resorted to this program when I couldn't get the code that we developed at Grumman combining a number of Woodward codes into a single code with additional options for NASA (NASA CR 3676) to run on a modern computer.

Figure 14 shows the related comparisons for the stability estimates, C_m/C_L . In this case the supersonic results look better than the subsonic estimates (that were discussed above).

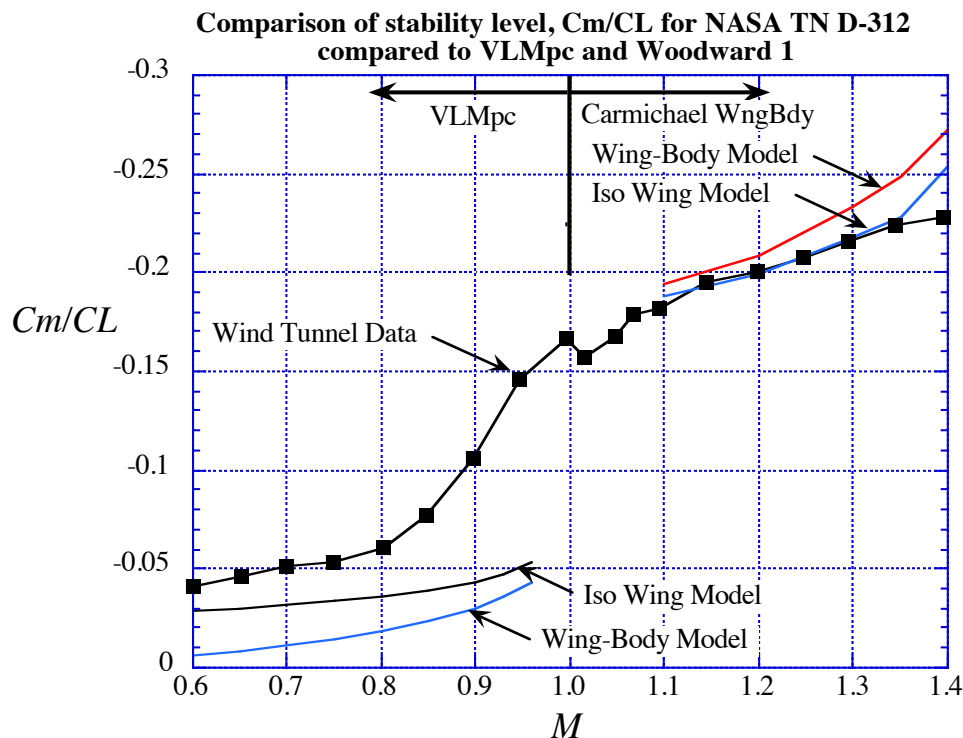


Figure 14. Stability comparison for TN D-312, VLMpc and program wing-body.

It's very curious that the lift curve slope calculations for the supersonic cases are worse than the stability values. Is this frustrating? Yes. But it's not too unusual. That's why aerodynamicists have to be constantly looking for test cases. You need to develop some intuition before making important design decisions based on numerical calculations.

Aerodynamic center shift

We examined the neutral point location for various situations at subsonic speed above. Now let's finish this part of the analysis up by looking at the ac shift. We'll just consider the wind tunnel values. We saw that the neutral point at $M = 0.6$ was 29.1% mac . At $M = 1.4$ the C_m/C_L is 0.228, so that the neutral point is at 47.8% of the mac . The shift from subsonic to supersonic is 18.7% of the mac . Once again we see that the classic values students learn from 2D thin airfoil theory are approximate. We don't get the classic 25% shift and the aerodynamic center does not move all the way back to 50% of the chord. Still these are handy rules of thumb.

Drag Polars

g) Estimate the drag polar for $M = 0.6$ and $M = 1.4$.

Here we will make some assumptions and see how the results compare with the wind tunnel data. The main assumption results from the choice of a biconvex airfoil section for the model. Remember they were mainly looking at transition effects and a biconvex untwisted, uncambered wing is useful at zero angle of attack. However the sharp leading edge is not going to be an aerodynamically efficient shape. This means we should compare the wind tunnel results with the 0% leading edge suction polar. Often known as “barn door lift,” I think it would be more accurate to describe it as “barn door drag.” If there is no suction (axial force zero), the drag is simply related to the normal force. Thus the drag due to lift is just $C_L^2/C_{L\alpha}$ (students should be able to derive this). This is also known as the $1/C_{L\alpha}$ polar (remember that here $C_{L\alpha}$ is per radian). We will use the wind tunnel value of the lift curve slope and the wind tunnel value of C_{Dmin} . For comparison we include the ideal drag polar, where we divide C_L^2 by πAR . Figure 15 shows what we get from this approach.

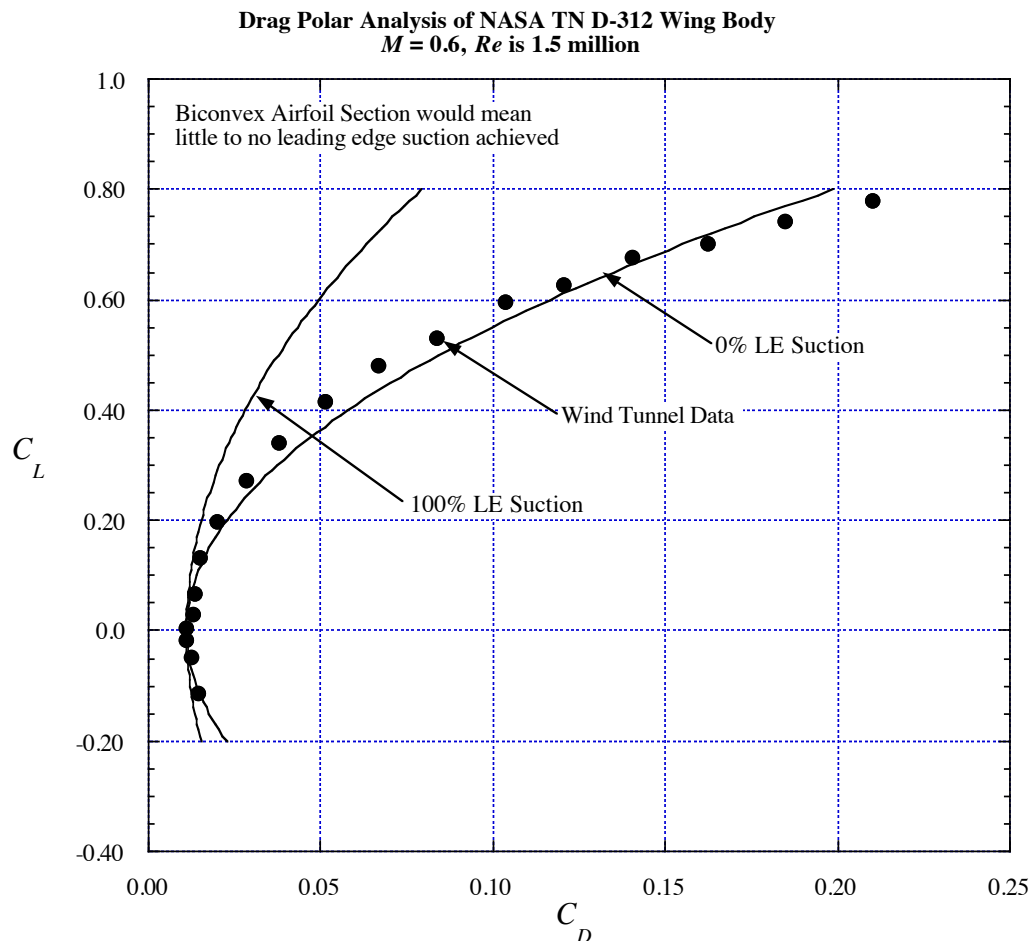


Figure 15. Subsonic wind tunnel drag polar compared to reference estimates.

We see that the wind tunnel results are indeed very close to the 0% suction estimates. Although some suction is developed initially, the data fall over on the 0% suction curve. These curves

should always be added to a drag polar to understand the quality of the wing aerodynamics. Without doing this it is impossible to tell how well your design is doing. Aerodynamicists often describe the wing performance in term of % leading edge suction. This means that at the design C_L you know how well you have done in terms of the bounding curves.

Next we use the same procedure for the supersonic case. The results are shown in Figure 16. Here we haven't yet added the 100% leading edge suction or optimum drag curve. That awaits developing some confidence in the supersonic wing body program (linear theory can find the minimum drag and corresponding camber shapes). The subsonic ideal polar shown above is not valid at supersonic speed.

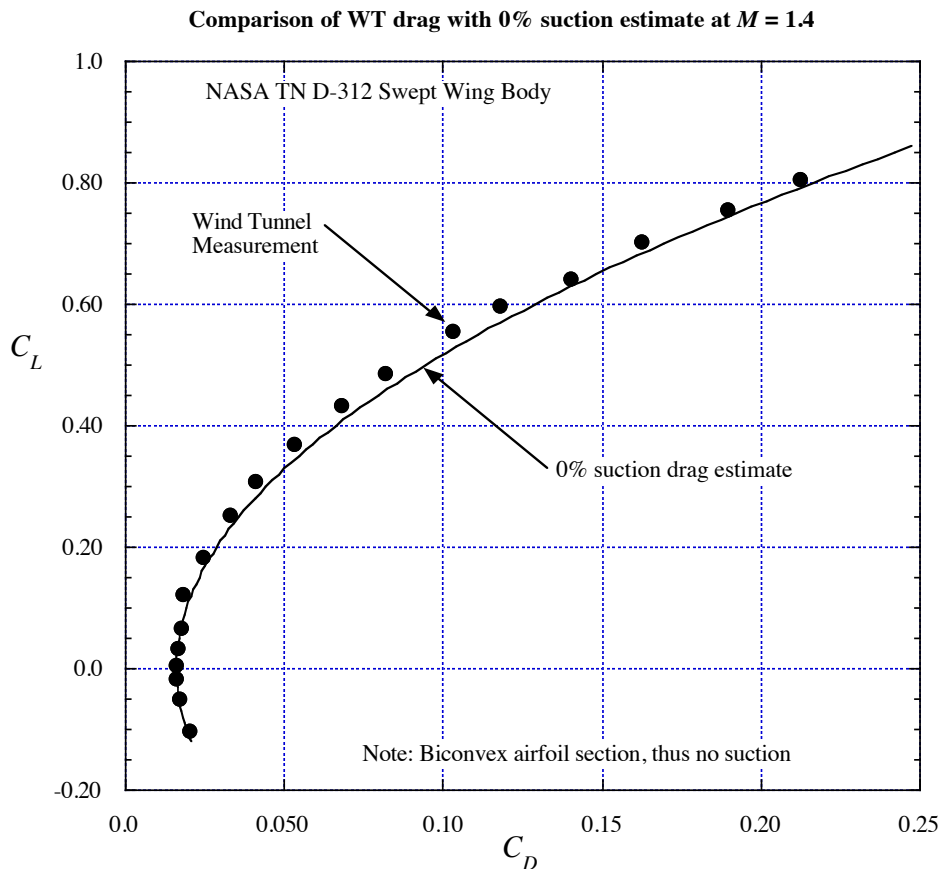


Figure 16. Supersonic wind tunnel drag polar compared to “barn door drag.”

As expected the 0% suction polar shape corresponds pretty well to the wind tunnel results. Thus we can estimate the drag polar of these types of configurations with fairly good confidence. However, it's not an efficient wing. Some CFD types have come into my office excited about how well they predicted the polar. They didn't appreciate that I could do just as well on the back of an envelope. That was probably not a good idea on my part, they didn't want to work with me again after that.

Performance

h) What is your estimate for L/D_{\max} at $M = 0.6$ and 1.4 ? What C_L do these estimate correspond to? Consider both completely laminar and completely turbulent flow.

Now we use the drag polars presented above to look at the L/D of the configuration. Figure 17 shows the L/D for both the subsonic ($M = 0.6$) and supersonic ($M = 1.4$) cases. For simplicity I'm plotting the 0% suction polar estimates (that were in reasonably good agreement with the test data). At $M = 0.6$ we find an L/D_{\max} of 8.78 at a C_L of 0.160. At $M = 1.4$ the L/D_{\max} is 7.04 at a C_L of 0.23. This illustrates two aspects of configuration aerodynamics. First, the increased minimum drag due to wave drag shows up directly as a reduction in L/D_{\max} . The second issue is that to get to the L/D_{\max} requires flying at a higher lift coefficient. This can mean a high altitude, and typically the total system design means there isn't enough thrust to fly at this lift coefficient and this leads to a special emphasis on obtaining a low value of $C_{D\min}$.

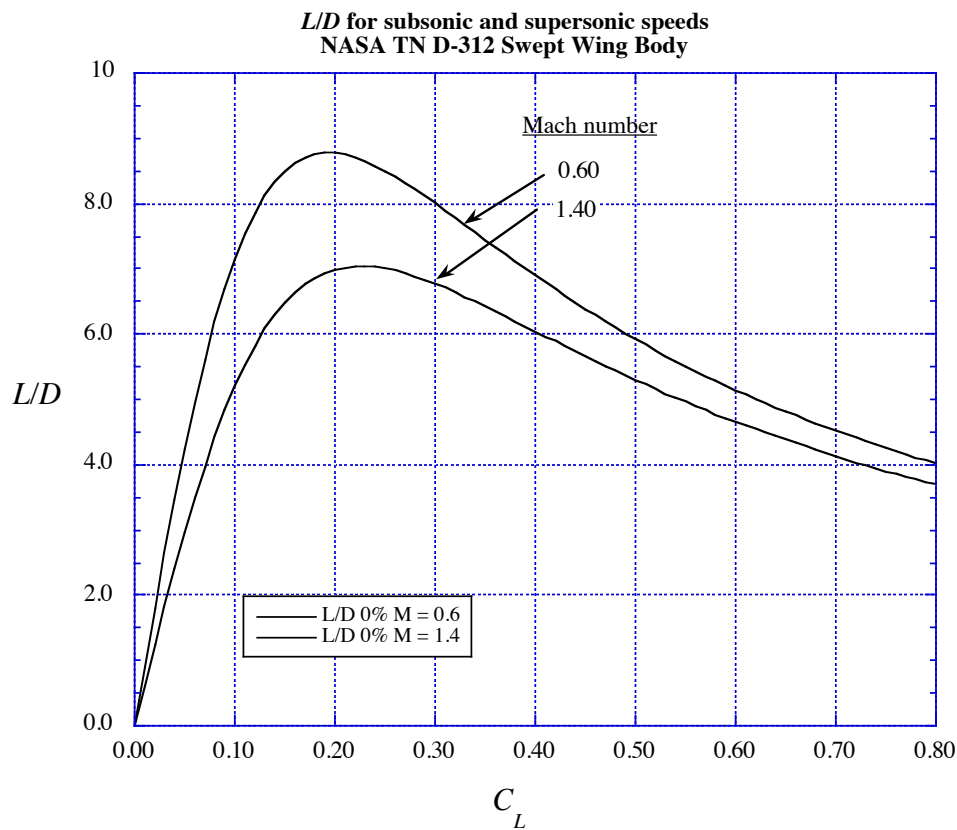


Figure 17. L/D for the subsonic and supersonic cases for the model in NASA TN D-312.

Now let's play some games to illustrate the importance of aerodynamic design technology. First, Figure 18 shows the dramatic effect of achieving a good aerodynamic wing design on the L/D . Here, for $M = 0.6$ we look at the difference in L/D depending on whether the wing achieves 0% or 100% leading edge suction. This is just a plot of how the drag polars in Figure 15 impact aerodynamic performance. The difference in L/D_{\max} goes from 8.77 to 14.57, a 66% increase.

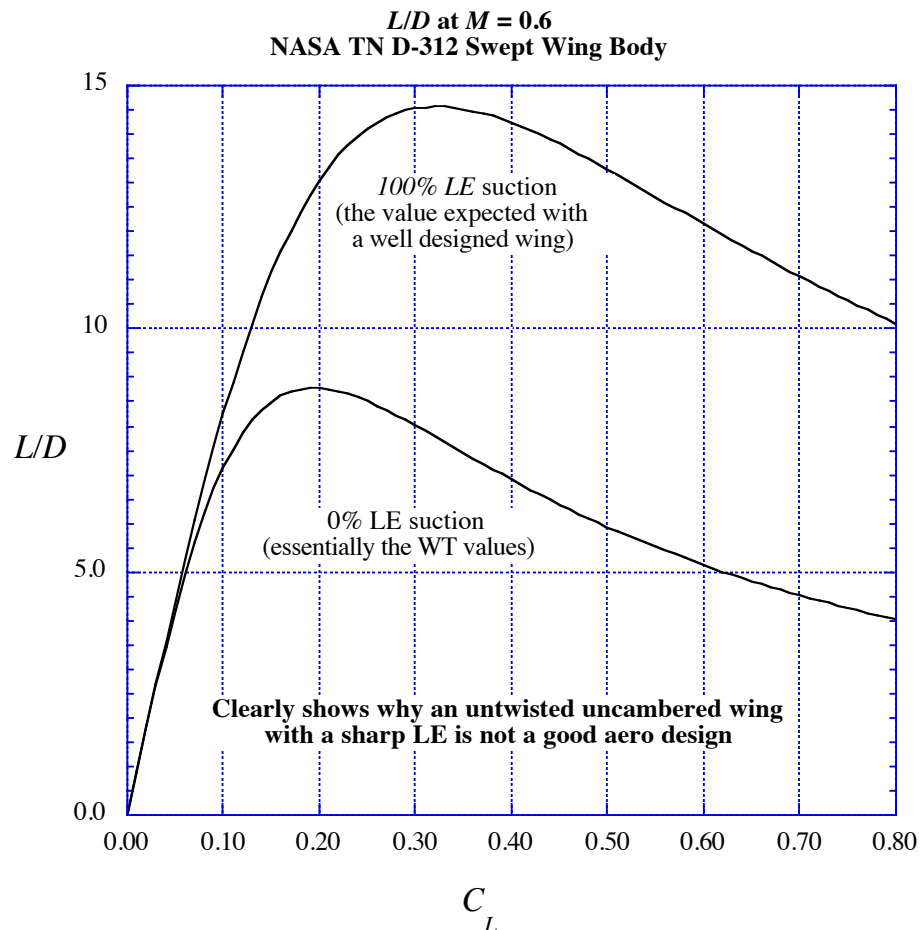


Figure 18. Subsonic L/D performance differences due to the wing design.

Now see what happens if we assume magic and the flow is completely laminar compared to completely turbulent. Figure 19 shows this (hypothetical) case. The huge increase in L/D_{\max} shows why aerodynamicists have been interested in obtaining laminar flow.

In this last section we've seen the increases in aerodynamic efficiency that result from the two primary contributors to the aerodynamic performance. Drag due to lift is reduced through wing and airfoil design. Zero lift drag is reduced not only by streamlining, but also achieving a significant run of laminar flow.

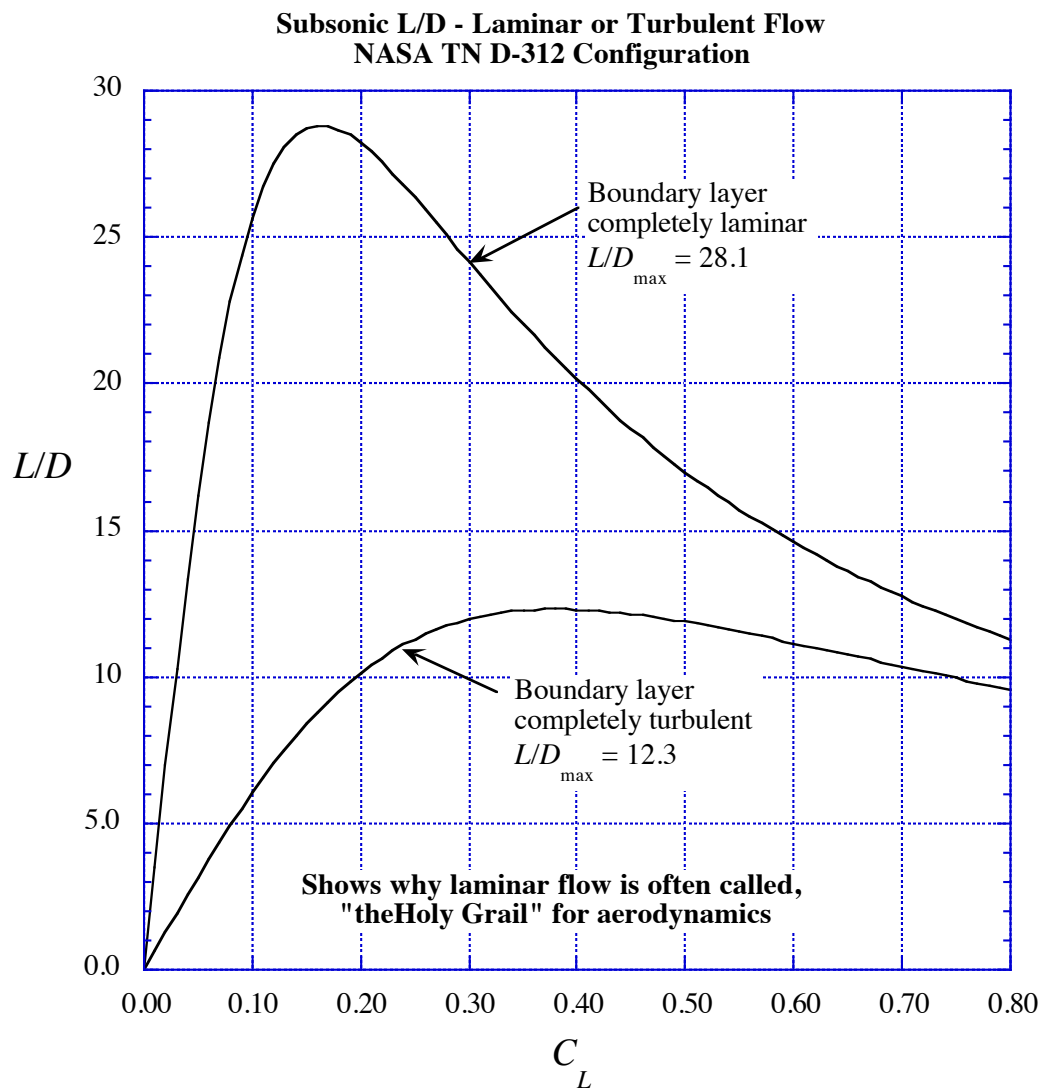


Figure 19. The impact of achieving a completely laminar boundary layer.

Note that to find the true effect of aero technology on the design requires a complete system design (MDO) where the planform area and shape would be found to best exploit technology advances. This is beyond the scope of this purely aerodynamic exercise.

What we learned and what we should do in the future.

1. We have seen that by far the largest part of the work of an applied aerodynamicist entails developing geometry. The tasks are pretty representative of the work of an aerodynamicist in conceptual/preliminary design.
2. This is the simplest possible case I can find. Think of the complications of a real airplane.
3. We got “reasonable” agreement with the wind tunnel data in most respects.
4. The effect of fixing transition is very small except at zero angle of attack. As soon as a small angle of attack is reached apparently the boundary layer transitions anyway, even if no transition strips are on the model.
5. We could break the wing into several panels in FRICTION and see the effect. My experience is that the effect on the results would be very small. Nevertheless it might be interesting to do this.
6. We need to check the supersonic work by using AERO2S and WNGDES (I can’t find out if AERO2S only works at subsonic speed).
7. I should go ahead and run the wingbody program to get the minimum drag for supersonic Mach numbers (an available option). I need to validate the code more thoroughly before this is done.
8. The simple codes used here (with instantaneous computing times) were the workhorses for aerodynamicists up through the 1970s and were used for many successful airplane designs. At that time it would take a day to get your results after submission!
9. We saw why airplanes don’t use biconvex airfoils, the drag (L/D) is terrible (0% suction).

Note: I can provide text files of the data sets contained in this write up to anyone interested in using the codes (most of which are either on my software web site or Ralph Carnichael’s PDAS web pages). You would learn more constructing the data sets for yourself.

W.H. Mason
November 2016