Elaboration on the Class Overheads for the use of fuel fractions to find the equations for fuel weight and empty weight with a weight discontinuity in the mission.

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Following Nicolai,[1] Consider the *TOGW*, called here W_{TO}, to be:

$$W_{TO} = W_{fuel} + W_{fixed} + W_{empty} \tag{1}$$

where the fixed weight includes a non-expendable part, which consists of the crew and equipment, and an expendable part, which consists of the bombs, and missiles, which might be dropped, as well as the passengers and baggage or freight. W_{empty} includes all weights except the fixed weight and the fuel. In particular, we are interested in including the effect of dropping something during the mission, or using extra fuel to perform a combat. Now, the difference between the takeoff and landing weight is due to fuel used (the mission fuel) and the bombs dropped, missiles fired and combat fuel used. Thus we can write:

$$W_{TO} - W_{landing} = W_{\substack{mission\\fuel}} + W_{dropped} \tag{2}$$

and solving for the fuel weight in Eq.(2):

$$W_{\substack{mission\\fuel}} = W_{TO} - W_{dropped} - W_{landing}\,. \tag{3}$$

The actual fuel weight includes trapped fuel that can't be used and any reserve fuel not included in the mission weight fractions. This additional fuel is usually given as a fraction of the takeoff weight, W_{TO}. Thus the fuel weight is

$$W_{fuel} = (1 + \underbrace{\frac{W_{reserve}}{W_{TO}}}_{\substack{\text{reserve fuel}\\\text{5\% typical}\\\text{for military}}} + \underbrace{\frac{W_{trapped}}{W_{TO}}}_{\substack{\text{trapped fuel}\\\text{1\% typical}}})W_{\substack{mission\\fuel}}\,. \tag{4}$$

Now, substitute into Eq.(4) for the mission fuel given by Eq.(3):

$$W_{fuel} = \left(1 + \frac{W_{reserve}}{W_{TO}} + \frac{W_{trapped}}{W_{TO}}\right)\left(W_{TO} - W_{dropped} - W_{landing}\right). \tag{5}$$

Factor out W_{TO} in Eq.(5) to get the dropped and landing weight as a fraction of the takeoff weight:

$$W_{fuel} = \left(1 + \frac{W_{reserve}}{W_{TO}} + \frac{W_{trapped}}{W_{TO}}\right)\left(1 - \frac{W_{dropped}}{W_{TO}} - \frac{W_{landing}}{W_{TO}}\right)W_{TO}\,. \tag{6}$$

[1] Nicolai, L.M., *Fundamentals of Aircraft Design*, METS, Inc., San Jose, CA.

For a specified mission we need to find the landing weight for a given takeoff weight. To keep track of the mission segments while finding the landing weight, we will switch to numbers to define the points along the mission as shown in Figure 1. This definition is fairly general, and is used in the computer program. If a particular mission doesn't include one of the segments, then the fuel used during that segment will be zero, and the airplane weight will not change (the weight ratio will be unity). Thus, $W_{TO} = W_1$, and $W_{landing} = W_8$.

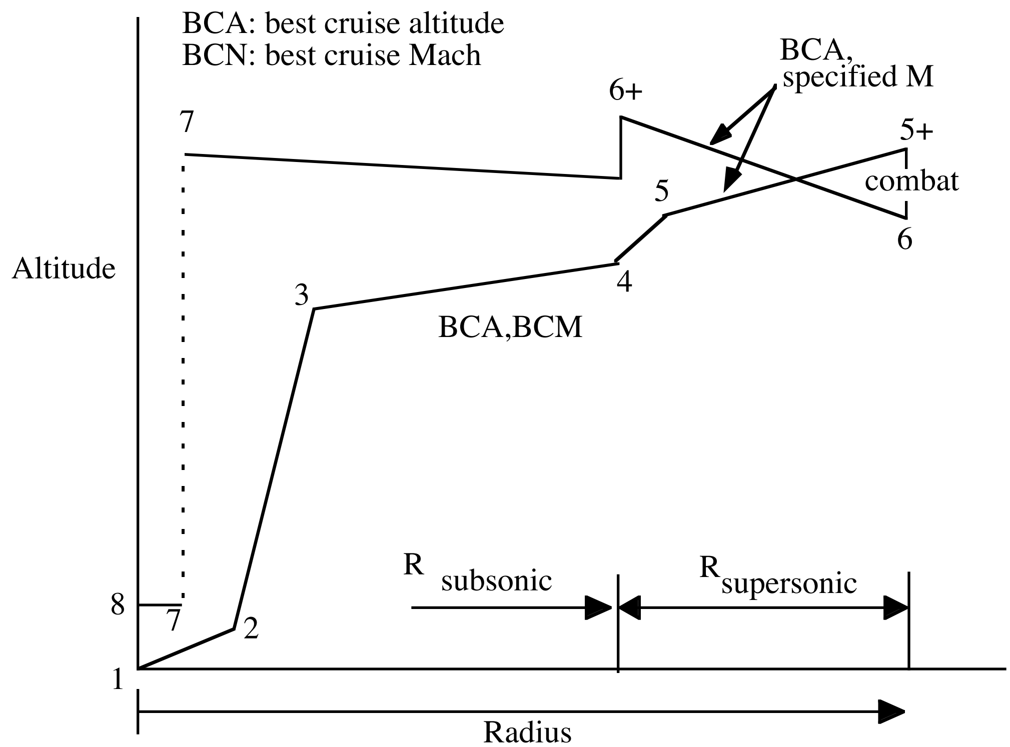

Figure 1. Definition of mission segments

Now we consider the details for finding the weight fraction W_8/W_1 including the combat discontinuity. We can write the weight fractions using our mission segment definition as:

$$\frac{W_8}{W_1} = \underbrace{\frac{W_8}{W_7}\cdot\frac{W_7}{W_{6^+}}\cdot\frac{W_{6^+}}{W_6}}_{W_{\text{return ratio}} = \left(\frac{W_8}{W_6}\right)}\cdot\underbrace{\frac{W_6}{W_{5^+}}}_{\text{combat}}\cdot\underbrace{\frac{W_{5^+}}{W_5}\cdot\frac{W_5}{W_4}\cdot\frac{W_4}{W_3}\cdot\frac{W_3}{W_2}\cdot\frac{W_2}{W_1}}_{W_{\text{outbound ratio}} = \left(\frac{W_{5^+}}{W_1}\right)} \tag{7}$$

or:

$$\frac{W_8}{W_1} = \left(\frac{W_8}{W_6}\right)\cdot\underbrace{\frac{W_6}{W_{5^+}}}_{\text{weight }\Delta}\cdot\left(\frac{W_{5^+}}{W_1}\right) \tag{8}$$

The outbound and return weight ratios are found from computing the fuel used. Typically, as shown in the class overheads, the range and endurance equations are used. Now we have to consider the step change in weight due to combat at segment W_6/W_{5+}. Write out the value at 6:

$$W_6 = W_{5^+} - (W_{\substack{combat\\fuel\,used}} + W_{\substack{bombs\\dropped}}) \,. \tag{9}$$

and divide by W_{5+}:

$$\frac{W_6}{W_{5^+}} = \left[1 - \frac{1}{W_{5^+}}(W_{\substack{\text{combat}\\\text{fuel used}}} + W_{\substack{\text{bombs}\\\text{dropped}}})\right]. \tag{10}$$

We get the expression we want by substituting Eq.(10) into Eq.(8):

$$\frac{W_8}{W_1} = \frac{W_8}{W_6}\left[1 - \frac{1}{W_{5^+}}(W_{\substack{\text{combat}\\\text{fuel used}}} + W_{\substack{\text{bombs}\\\text{dropped}}})\right]\frac{W_{5^+}}{W_1} \tag{11}$$

and slightly rearranging:

$$\frac{W_8}{W_1} = \frac{W_8}{W_6}\left[\frac{W_{5^+}}{W_1} - \frac{(W_{\substack{\text{combat}\\\text{fuel used}}} + W_{\substack{\text{bombs}\\\text{dropped}}})}{W_1}\right] = \frac{W_{landing}}{W_{TO}} \tag{11}$$

we obtain the expression that can be used in Eq.(6) for the landing weight, and subsequently the fuel weight from Eq.(5) for an assumed W_{TO}, such that we finally find

$$W_{empty} = W_{TO} - W_{fuel} - W_{fixed} \,. \tag{12}$$

We call this the $W_{EmptyAvail}$ in the code. The value of W_{TO} that solves the problem is the one for which $W_{EmptyAvail}$ is equal to the value of $W_{EmptyReqd}$ which comes from the statistical representation for this class of aircraft. Normally, an iterative procedure is used to find this value. This is then a starting point for the design using more detailed analysis.