Program LIDRAG

This program computes the span *e* for a single planar lifting surface given the spanload. It uses the spanload to determine the "*e*" using a Fast Fourier Transform. Numerous other methods could be used. For reference, note that the "*e*" for an elliptic spanload is 1.0, and the "*e*" for a triangular spanload is about 0.72.

The method was originally developed by Glauert and associated with lifting line theory. In that context it used the terminology of δ. See the comments at the end of this manual. Following Glauert's use of a Fourier series to describe the circulation distribution on the wing, and assuming that the spanload is symmetrical, we can write the series as:

$$\frac{cc_l}{c_a} = \sum_{n=1}^{N} a_n \sin\left[(2n-1)\theta\right],$$

where the a_n's are given by

$$a_n = \frac{4}{\pi}\int_0^{\pi/2} \frac{cc_l}{c_a}(\theta)\sin\left[(2n-1)\theta\right]d\theta.$$

The spanwise location $\eta = y/(b/2)$ is related to θ by $\eta = \cos\theta$. Here we assume that the spanload is symmetrical, as a result the even values of the series are zero and hence not needed.

The span *e* is then computed from:

$$e = \frac{1}{\sum_{n=1}^{N}(2n-1)\left(\frac{a_n}{a_1}\right)^2}$$

and

$$C_{D_i} = \frac{C_L^2}{\pi A \mathrm{Re}}$$

An associated result is the lift coefficient:

$$C_L = \frac{\pi}{4}a_1$$

The code is in the file LIDRAG.F. The sample input is also on the web page and is called B2LIDRAG.INP. The program prompts the user for the name of the input file.

The program was written by Dave Ives, and entered the public domain through the code contained in AFFDL-TR-77-122, "An Automated Procedure for Computing the Three Dimensional Transonic Flow over Wing-Body Combinations, Including Viscous Effects," Feb. 1978.

The input is the spanload obtained from any method. The output is the Trefftz plane induced drag *e* and the integral of the spanload, which produces the C_L. This is the "span" *e*. You should include a point at $\eta = 0$ and at $\eta = 1$ you should include a point with zero spanload. See the sample input for an example.

The input instruction:

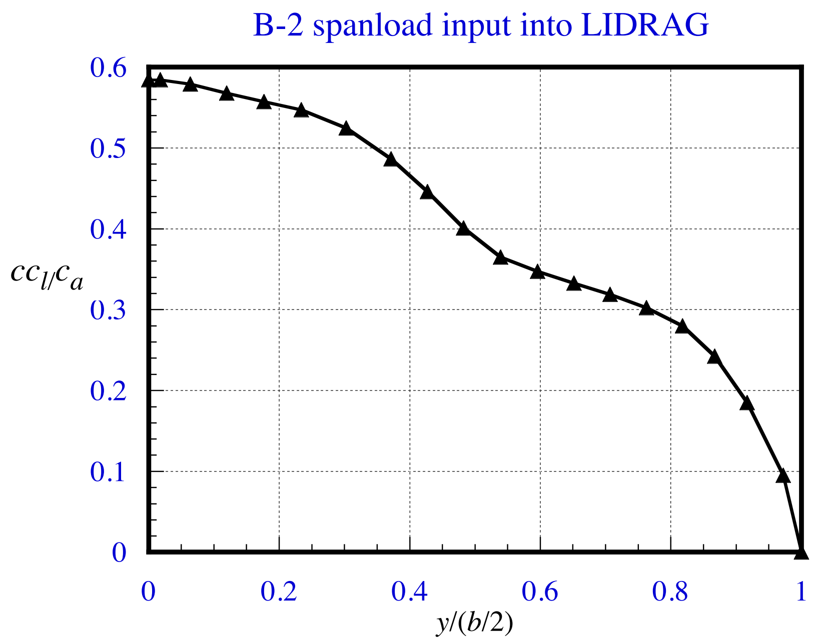
Card	Field	Columns	Variable	Description
1	1	1-10	FSPN	Number of spanwise stations of input
2	1	1-10	ETA	The spanwise location of input, $y/(b/2)$.
2	2	11-20	CCLCA	The spanload, *ccl*/*ca* (the local chord times the local lift coefficient divided by the average chord)

Note: Card 2 is repeated FSPN times

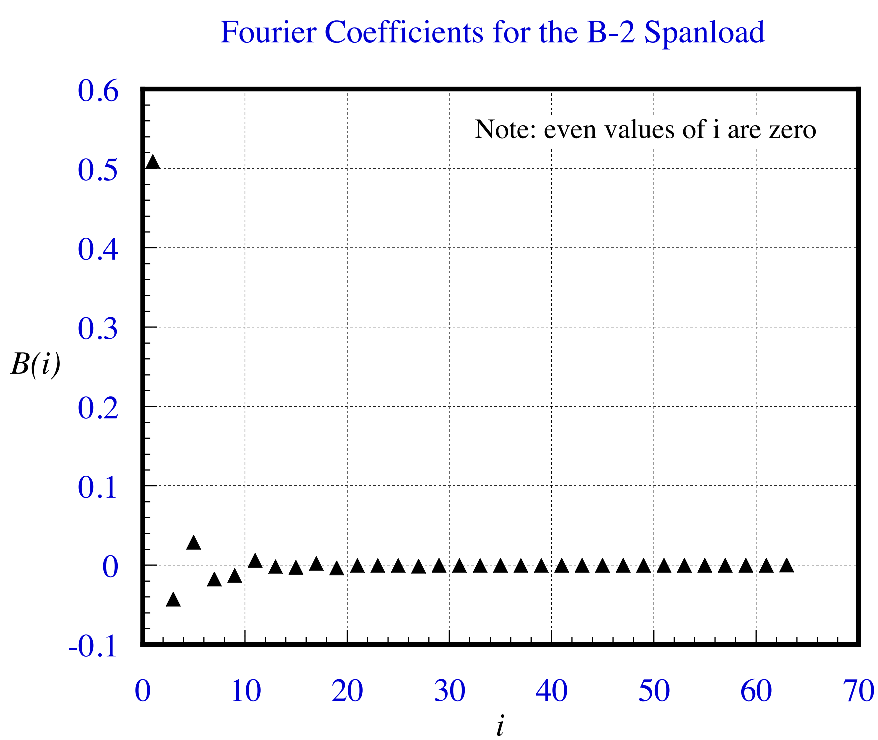
Sample input: (from the output of the **VLMpc** sample case for the B-2, and in the file b2ldg.inp on the web)

```
20.
 0.0       0.58435
 0.01805   0.58435
 0.06388   0.57919
 0.11943   0.56800
 0.17664   0.55739
 0.23385   0.54709
 0.30271   0.52459
 0.37158   0.48623
 0.42713   0.44590
 0.48269   0.40097
 0.53925   0.36490
 0.59581   0.34718
 0.65137   0.33280
 0.70693   0.31865
 0.76248   0.30225
 0.81804   0.27971
 0.86735   0.24229
 0.91667   0.18494
 0.97222   0.09480
 1.000     0.000
```

It is interesting to see a plot of the spanload and the related Fourier coefficients. The first plot shows the input spanload:



The related Fourier Series coefficients are illustrated in the following plot:



Here we see that coefficients quickly approach zero after the first few values.

Sample output:

```
Program LIDRAG

 enter name of input data file
b2ldg.inp

LIDRAG - LIFT INDUCED DRAG ANALYSIS

     INPUT SPANLOAD

        N     Y/(B/2)      CCLCA
        1     0.00000     0.58435
        2     0.01805     0.58435
        3     0.06388     0.57919
        4     0.11943     0.56800
        5     0.17664     0.55739
        6     0.23385     0.54709
        7     0.30271     0.52459
        8     0.37158     0.48623
        9     0.42713     0.44590
       10     0.48269     0.40097
       11     0.53925     0.36490
       12     0.59581     0.34718
       13     0.65137     0.33280
       14     0.70693     0.31865
       15     0.76248     0.30225
       16     0.81804     0.27971
       17     0.86735     0.24229
       18     0.91667     0.18494
       19     0.97222     0.09480
       20     1.00000     0.00000

    Span e =    0.94708     CL =    0.399

STOP
```

Note: this method originated with Glauert's lifting line theory method. See H. Glauert, *The Elements of Aerofoil and Airscrew Theory*, Cambridge University Press, 1st Edition, 1926, American Edition 1944. Subsequently it was found to apply to the trace of the spanload far downstream in the Trefftz plane. When it is used with lifting line theory δ is used. The relation between δ and e is:

$$e = \frac{1}{1+\delta}$$

See Bertin and Cummings, *Aerodynamic for Engineers*, Pearson Prentice Hall, 2009, Section 7.3.5 (pages 319and 320 in the 5th edition) for details. Since e is related purely to the shape of the spanload, for a twisted, cambered wing the e will vary with the lift coefficient. For the complete airplane the value of E is used, and includes viscous and fuselage effects. We conclude by noting that aerodynamicists always want an elliptical spanload, but because the induced drag is related to the wingspan by $1/b^2$, it is almost always better to increase the wingspan while holding the wing root bending moment fixed (a surrogate for the wing weight). The increased span reduces the drag much faster than the penalty from a nonelliptic value of e.