Stability and Control Derivative Estimation and Engine-Out Analysis

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Nomenclature

Symbols

A	aspect ratio
b	wing span
b_{vtail}	vertical tail span
$C_{D_{ewm}}$	drag coefficient due to windmilling of failed engine
C_L	lift coefficient
$C_{L_{\alpha_{htail}}}$	lift curve slope of the horizontal tail
$C_{l_{\alpha_{vtail}}}$	section lift curve slope of vertical tail
$C_{l_{\alpha_{vtail_{eff}}}}$	effective lift curve slope of vertical tail
$C_{L_{\alpha_{wb}}}$	lift curve slope of the wing and body
$C_{n_{avail}}$	available yawing moment coefficient at the engine-out flight condition
$C_{n_{req}}$	required yawing moment coefficient at the engine-out flight condition
C_{y_β}	variation of sideforce coefficient with yaw angle
C_{l_β}	variation of rolling moment coefficient with yaw angle
C_{n_β}	variation of yawing moment coefficient with yaw angle
D_{ewm}	drag due to windmilling of failed engine
d_{fuse}	maximum fuselage diameter
$d_{fuse_{vtail}}$	depth of the fuselage at the vertical tail quarter-chord position
d_i	engine inlet diameter
$d_{nacelle}$	nacelle diameter
l	horizontal distance between CG and vertical surface
l_e	buttline of outboard engine
L_{ext}	external rolling moment
$k_{C_{y_{\beta_v}}}$	empirical factor for vertical tail sideslip derivative estimation
K'	empirical correction factor for large control deflections
K_b	flap span factor
K_H	factor accounting for the relative size of the horizontal and vertical tails
K_{M_Γ}	compressibility correction to dihedral
K_N	empirical factor for body and body + wing effects
K_{R_l}	Reynold's number factor for the fuselage
K_{M_Λ}	compressibility correction to sweep
K_{wb}	factor for fuselage loss in the lift curve slope
K_{wbi}	wing-body interference factor
l_{tv}	horizontal distance between CG and engine nozzle
l_{vtail}	horizontal distance between CG and aerodynamic center of vertical tail

M	Mach number
$N_{engines}$	number of engines
N_{req}	required yawing moment
N_{max}	maximum attainable yawing moment
q_{eo}	dynamic pressure at the engine-out flight condition
S_{htail}	horizontal tail area
S_o	cross-sectional area of fuselage
S_{ref}	wing reference area
S_{vtail}	vertical tail area
T	maximum available thrust at given mach and altitude
T_o	static thrust at sea level
$\frac{V_n}{V}$	ratio of mean nozzle exit velocity to freestream velocity
Y_{ext}	external sideforce
z_{tv}	vertical distance between CG and engine nozzle
z_{vtail}	vertical distance between CG and aerodynamic center of vertical tail
$\Delta C_{L_{cc}}$	change in vertical tail C_L due to circulation control
α	angle of attack (rad)
β	sideslip angle (positive with relative wind from right)
β_M	compressibility factor = $\sqrt{1 - M^2}$
δ_a	aileron deflection (positive for right up, left down)
δ_r	rudder deflection (positive right)
η_{htail}	dynamic pressure ratio at the horizontal tail
φ	bank angle (positive right roll)
Γ	dihedral angle (deg)
κ	ratio of actual lift curve slope to 2π
$\Lambda_{c/2}$	half-chord sweep angle
$\Lambda_{c/4}$	quarter-chord sweep angle
σ	ratio of density at a given altitude to density at sea level

Subscripts

avail	available
bs	body side
cc	circulation control
eff	effective
fuse	fuselage
htail	horizontal tail
req	required

tv	thrust vectoring
vtail	vertical tail
wb	wing-body
wing	wing

The FORTRAN code variable names and definitions are given in the Appendix.

1. Introduction

This report describes the estimation of stability and control derivatives using the method of Reference [1] (which is essentially DATCOM [2]), and the establishment of the engine-out constraint based on the required yawing moment coefficient. The use of thrust vectoring and circulation control to provide additional yawing moment is also described.

1.1. Control Surface Sign Conventions

The control surface sign conventions are defined such that a positive control deflection generates a positive roll or yaw moment according to the right hand rule with a conventional body axis coordinate system, as shown in Figure 1-1. A positive aileron deflection is defined with the right aileron up and the left aileron down. The aileron deflection is the average deflection of the two surfaces from the neutral position. A positive rudder deflection is defined with the trailing edge to the right, as viewed from above.

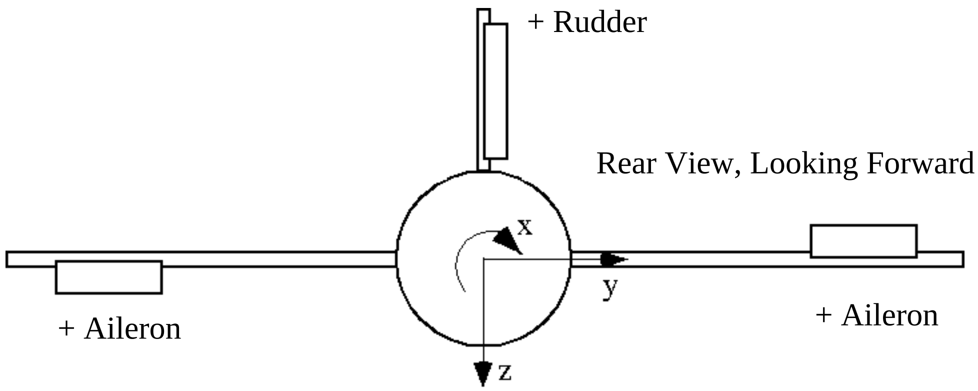

Figure 1-1: Control surface sign conventions

2. Engine-out Methodology

The engine-out constraint is established by constraining the maximum available yawing moment coefficient ($C_{n_{avail}}$) to be greater than the required yawing moment coefficient ($C_{n_{req}}$) for the engine-out flight condition:

$$C_{n_{avail}} \geq C_{n_{req}} \tag{2-1}$$

2.1. Required Yawing Moment Coefficient

The required yawing moment coefficient is the yawing moment coefficient required to maintain steady flight with one failed outboard engine at 1.2 times the stall speed, as specified by FAR 25.149. The remaining outboard engine must be at the maximum available thrust, and the bank angle cannot be larger than 5˚.

Figure 2-1 shows the engine-out geometry for a twin-engine configuration. The yawing moment coefficient required to maintain steady flight with an inoperative engine is given by:

$$C_{n_{req}} = \frac{(T + D_{ewm})l_e}{qS_{ref}b} \tag{2-2}$$

where T is the maximum available thrust at the given Mach number and altitude, and D_{ewm} is the drag due to the windmilling of the failed engine.

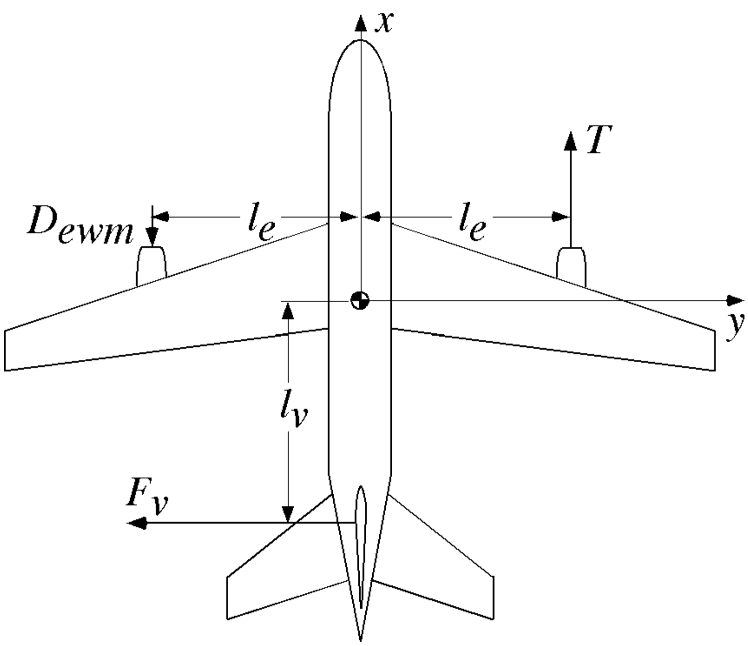

Figure 2-1: Engine-out geometry

The drag due to the windmilling of the failed engine is calculated using the method described in Appendix G-8 of Torenbeek [3].

$$D_{ewm} = qS_{ref}C_{D_{ewm}} \tag{2-3}$$

$$C_{D_{ewm}} = \frac{0.0785 d_i^2 + \dfrac{2}{1 + 0.16 M^2} \dfrac{\pi}{4} d_i^2 \dfrac{V_n}{V}\left(1 - \dfrac{V_n}{V}\right)}{S_{ref}} \tag{2-4}$$

where:

d_i is the engine inlet diameter

M is the Mach number

V_n is the nozzle exit velocity

$\frac{V_n}{V}$ 0.92 for high bypass ratio engines

S_{ref} is the wing reference area

Torenbeek's windmilling drag equation was validated against the flight test data of the 747. As shown in Figure 2-2, Torenbeek's equation shows relatively good agreement with the flight test data over a range of Mach numbers.

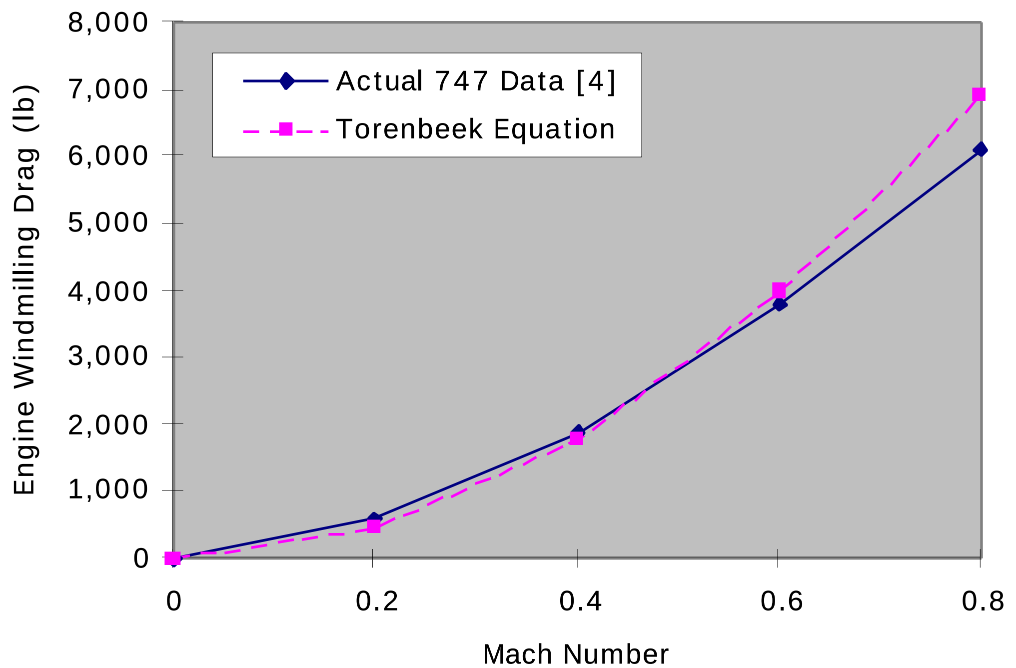

Figure 2-2: Engine windmilling drag validation

2.2. Maximum Available Yawing Moment Coefficient

The maximum available yawing moment coefficient is obtained at an equilibrium flight condition with a given bank angle (φ) and a given maximum rudder deflection (δ_r). The bank angle is limited to a maximum of 5˚ by FAR 25.149, and the aircraft is allowed to have some sideslip (β).

The sideslip angle is found by summing the forces along the *y*-axis:

Sideforce Equation:

$$C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r + C_{y_\beta}\beta + C_L \sin\varphi - \frac{T\sin\varepsilon}{qS_{ref}} - \Delta C_{L_{cc}}\frac{S_{vtail}}{S_{ref}} = -\frac{Y_{ext}}{qS_{ref}} \qquad (2\text{-}5)$$

In a conventional control system, the vertical tail is the dominant controller for generating a yawing moment. However, thrust vectoring and circulation control can be used to generate additional yawing moments. Since the engine-out condition is a critical constraint for a truss-braced wing with tip-mounted engines, the capability to model thrust vectoring and circulation control on the vertical tail was added to the code. The fifth term in the equation above ($\frac{T\sin\varepsilon}{qS_{ref}}$) is due to the thrust being vectored at an angle ε to the centerline, and the sixth term ($\Delta C_{L_{cc}}\frac{S_{vtail}}{S_{ref}}$) is due to the change in C_L at the vertical tail due to circulation control. Since the external sideforce (Y_{ext}) is zero, and $C_{y_{\delta_a}}$ is assumed to be zero, this equation can be simplified and solved for the sideslip angle:

$$\beta = \frac{-C_{y_{\delta_r}}\delta_r - C_L \sin\varphi + \frac{T\sin\varepsilon}{qS_{ref}} + \Delta C_{L_{cc}}\frac{S_{vtail}}{S_{ref}}}{C_{y_\beta}} \qquad (2\text{-}6)$$

The aileron deflection required to maintain equilibrium flight is obtained by summing the rolling moments about the *x*-axis:

Rolling Moment Equation:

$$C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r + C_{l_\beta}\beta - \frac{T\sin\varepsilon}{qS_{ref}}\frac{z_{tv}}{b} - \Delta C_{L_{cc}}\frac{S_{vtail}}{S_{ref}}\frac{z_{vtail}}{b} = -\frac{L_{ext}}{qS_{ref}b} \qquad (2\text{-}7)$$

By setting the external rolling moment (L_{ext}) equal to zero, this equation can be solved for the aileron deflection:

$$\delta_a = \frac{-C_{l_{\delta_r}}\delta_r - C_{l_\beta}\beta + \frac{T\sin\varepsilon}{qS_{ref}}\frac{z_{tv}}{b} + \Delta C_{L_{cc}}\frac{S_{vtail}}{S_{ref}}\frac{z_{vtail}}{b}}{C_{l_{\delta_a}}} \qquad (2\text{-}8)$$

The rudder deflection is initially set to the given maximum allowable steady-state value, and the sideslip angle and aileron deflection for equilibrium flight are determined by Eqs. (2-6) and (2-8). The maximum allowable steady-state deflection is typically 20˚-25˚. This allows for an additional 5˚ of deflection for maneuvering. A warning statement is printed if the calculated deflection exceeds the maximum allowable deflection.

The maximum available yawing moment is found by summing the contributions due to the ailerons, rudder, and sideslip:

Yawing Moment Equation:

$$C_{n_{avail}} = C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r + C_{n_\beta}\beta + \frac{T \sin\varepsilon}{qS_{ref}}\frac{l_{tv}}{b} + \Delta C_{L_{cc}}\frac{S_{vtail}}{S_{ref}}\frac{l_{vtail}}{b} \tag{2-9}$$

This value of the available yawing moment coefficient is then constrained in the optimization problem to be greater than the required yawing moment coefficient, as shown in Eq. (2-1).

2.3. Why can't the vertical tail achieve its maximum lift coefficient?

The Output section shows the results of the above methodology for a 747 with no thrust vectoring and no circulation control. The maximum available yawing moment is achieved with a bank angle of 5˚ and a sideslip angle of 3˚. This orientation would be used for a failure of the left engine. The pilot or automatic flight control system would roll the aircraft 5˚ in the direction of the operating engine and yaw slightly away from it. Note that in this flight condition, the vertical tail is only flying at an angle of attack of 3˚, which is far below the angle of attack corresponding to the maximum lift coefficient of a typical vertical tail. One might expect that the maximum available yawing moment is obtained when the vertical tail is flying at its maximum lift coefficient, but this is not true because the equilibrium equations above must always be satisfied for steady flight. To illustrate this point, Eq. (2-5) has been solved for the bank angle with no thrust vectoring and no circulation control:

$$\varphi = \sin^{-1}\left[-\frac{\left(C_{y_{\delta_r}}\delta_r + C_{y_\beta}\beta\right)}{C_L}\right] \tag{2-10}$$

According to Reference [5], the angle of attack corresponding to the maximum lift coefficient for a NACA 66(215)-216 airfoil section with 15˚ of flap deflection is 15˚. Therefore if the vertical tail in the 747 example mentioned above were flying at the maximum lift coefficient, the rudder deflection (δ_r) would be 15˚, and the vertical tail angle of attack (β) would be at least 15˚ (3D effects would require an even larger angle).

If these values are plugged into Eq. (2-10) with a C_L of 1.11 and the 747 values for the stability and control derivatives (as given in Nelson [6]), the bank angle required to maintain equilibrium flight is 15.5˚. Since this bank angle is much larger than the maximum allowable bank angle of 5˚ specified in FAR 25.149, the vertical tail cannot fly at the maximum lift coefficient and maintain equilibrium flight.

This brief analysis shows the need for circulation control or thrust vectoring. Since both of these mechanisms can generate a larger side force at the vertical tail without requiring a change in β, they can create a larger yawing moment coefficient at the same flight condition.

3. Stability and Control Derivative Estimation

The stability and control derivatives are estimated using the method of Roskam [1], which was adapted from the USAF Stability and Control DATCOM [2].

MacMillin [7] used a similar approach for the High-Speed Civil Transport. In MacMillin's work, however, the baseline stability and control derivatives were estimated using a vortex-lattice method, and the DATCOM method was only used to augment these baseline values with the effects due to changing the geometry of the vertical tail.

The Fortran source code for the stability subroutine is shown in the Appendix.

3.1. Angle of Sideslip Derivatives

3.1.1. Sideforce Coefficient

The variation of sideforce coefficient with sideslip angle has contributions from the wing, fuselage, and vertical tail. Note that all of the stability and control derivatives have units of rad^{-1}.

$$C_{y_\beta} = C_{y_{\beta_{wing}}} + C_{y_{\beta_{fuse}}} + C_{y_{\beta_{vtail}}} \tag{3-1}$$

The wing contribution is a function of the dihedral angle (in deg).

$$C_{y_{\beta_{wing}}} = -0.0001 \left| \Gamma \right| \frac{180}{\pi} \tag{3-2}$$

The fuselage and nacelle contributions are estimated by:

$$C_{y_{\beta_{fuse}}} = -2K_{wbi} \frac{S_o}{S_{ref}} \tag{3-3}$$

where:

K_{wbi} is the wing-body interference factor, which is determined from a curve fit to Figure 7.1 in Roskam:

$$K_{wbi} = 0.85\frac{-z_{wing}}{d_{fuse}/2} + 1 \qquad for\ \frac{z_{wing}}{d_{fuse}/2} < 0 \tag{3-4}$$

$$K_{wbi} = 0.5\frac{z_{wing}}{d_{fuse}/2} + 1 \qquad for\ \frac{z_{wing}}{d_{fuse}/2} > 0 \tag{3-5}$$

and

$$S_o \quad \pi\left(\frac{d_{fuse}}{2}\right)^2 + N_{engines}\pi\left(\frac{d_{nacelle}}{2}\right)^2 \tag{3-6}$$

The contribution of a vertical tail in the plane of symmetry is found from:

$$C_{y_{\beta_{vtail}}} = -k_{C_{y_{\beta_v}}} C_{l_{\alpha_{vtail_{eff}}}}\left(1 + \frac{d\sigma}{d\beta}\right)\eta_v \frac{S_{vtail}}{S_{ref}} \tag{3-7}$$

where:

$k_{C_{y_{\beta_v}}}$ is determined from a curve fit to Figure 7.3 in Roskam:

$$k_{C_{y_{\beta_v}}} = 0.75 \qquad for\ \frac{b_{vtail}}{d_{fuse_{vtail}}} < 2 \tag{3-8}$$

$$k_{C_{y_{\beta_v}}} = \frac{1}{6}\frac{b_{vtail}}{d_{fuse_{vtail}}} + \frac{5}{12} \qquad for\ \ 2 < \frac{b_{vtail}}{d_{fuse_{vtail}}} < 3.5 \tag{3-9}$$

$$k_{C_{y_{\beta_v}}} = 1 \qquad for\ \ \frac{b_{vtail}}{d_{fuse_{vtail}}} > 3.5 \tag{3-10}$$

$$C_{l_{\alpha_{vtail_{eff}}}} = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta_M^2}{\kappa^2}\left(1 + \frac{\tan^2 \Lambda_{c/2}}{\beta_M^2}\right) + 4}} \tag{3-11}$$

$$\kappa = \frac{C_{l_{\alpha_{vtail}}}}{2\pi} \tag{3-12}$$

$C_{l_{\alpha_{vtail}}}$ is assumed to have a value of 2π.

$$\beta_M = \sqrt{1 - M^2} \tag{3-13}$$

$$\left(1 + \frac{d\sigma}{d\beta}\right)\eta_v = 0.724 + 3.06\,\frac{\left(\frac{S_{vtail}}{S_{ref}}\right)}{1 + \cos\Lambda_{c/4}} + 0.4\,\frac{z_w}{d} + 0.009A \tag{3-14}$$

Note that the effective aspect ratio of the vertical tail must be used in place of A in Eqs. (3-11) and (3-14).

$$A_{vtail_{eff}} = \frac{A_{V(B)}}{A_V} A_{vtail}\left[1 + K_H\left(\frac{A_{V(HB)}}{A_{V(B)}} - 1\right)\right] \tag{3-15}$$

where:

$\frac{A_{V(B)}}{A_V}$ is the ratio of the aspect ratio of the vertical tail in the presence of the body to that of the isolated panel, which is determined from the following curve fit to Figure 7.5 in Roskam, with the taper ratio assumed to be less than or equal to 0.6:

$$\frac{A_{V(B)}}{A_V} = 0.002\left(\frac{b_{vtail}}{d_{fuse_{vtail}}}\right)^5 - 0.0464\left(\frac{b_{vtail}}{d_{fuse_{vtail}}}\right)^4 + 0.404\left(\frac{b_{vtail}}{d_{fuse_{vtail}}}\right)^3 - 1.6217\left(\frac{b_{vtail}}{d_{fuse_{vtail}}}\right)^2 + 2.7519\left(\frac{b_{vtail}}{d_{fuse_{vtail}}}\right) + 0.0408 \tag{3-16}$$

$\frac{A_{V(HB)}}{A_{V(B)}}$ is the ratio of the vertical tail aspect ratio in the presence of the horizontal tail and body to that of the tail in the presence of the body alone. It is assumed to have a value of 1.1, based on Figure 7.6 in Roskam. This is valid for the 747 and 777 tail geometries.

K_H is a factor accounting for the relative size of the horizontal and vertical tails, which is determined from the following curve fit to Figure 7.7 in Roskam:

$$K_H = -0.0328\left(\frac{S_{htail}}{S_{vtail}}\right)^4 + 0.2885\left(\frac{S_{htail}}{S_{vtail}}\right)^3 - 0.9888\left(\frac{S_{htail}}{S_{vtail}}\right)^2 + 1.6554\left(\frac{S_{htail}}{S_{vtail}}\right) - 0.0067 \tag{3-17}$$

3.1.2. Rolling Moment Coefficient

The variation of rolling moment coefficient with sideslip angle has contributions from the wing-body, horizontal tail, and vertical tail.

$$C_{l_\beta} = C_{l_{\beta_{wb}}} + C_{y_{\beta_{htail}}} + C_{y_{\beta_{vtail}}} \tag{3-18}$$

The contribution from the wing-body is estimated by:

$$C_{l_{\beta_{wb}}} = \left[C_L \left(\left(\frac{C_{l_\beta}}{C_L} \right)_{\Lambda_{c/2}} K_{M_\Lambda} K_f + \left(\frac{C_{l_\beta}}{C_L} \right)_A \right) + \Gamma \left(\frac{C_{l_\beta}}{\Gamma} K_{M_\Gamma} + \frac{\Delta C_{l_\beta}}{\Gamma} \right) + \left(\Delta C_{l_\beta} \right)_{Z_w} \right] \frac{180}{\pi} \quad (3\text{-}19)$$

where:

$\left(\frac{C_{l_\beta}}{C_L} \right)_{\Lambda_{c/2}}$ is the wing sweep contribution, obtained from the following curve fit to Figure 7.11 in Roskam for λ = 0.5:

$$\left(\frac{C_{l_\beta}}{C_L} \right)_{\Lambda_{c/2}} = \frac{-0.004\Lambda_{c/2}}{45} \frac{180}{\pi} \quad (3\text{-}20)$$

K_{M_Λ} is the compressibility correction to sweep, assumed to have a value of 1.0, based on Figure 7.12 in Roskam. This is valid for the 747 and 777 geometries at low Mach numbers.

K_f is the fuselage correction factor, assumed to have a value of 0.85, based on Figure 7.13 in Roskam. This is valid for the 747 and 777 geometries.

$\left(\frac{C_{l_\beta}}{C_L} \right)_A$ is the aspect ratio contribution, assumed to have a value of 0, based on Figure 7.14 in Roskam for λ = 0.5 and a high aspect ratio. This is valid for the 747 and 777 geometries.

$\frac{C_{l_\beta}}{\Gamma}$ is the wing dihedral effect, obtained from a curve fit to Figure 7.15 in Roskam for λ = 0.5, low sweep, and a high aspect ratio. Note that for extremely high aspect ratios, the curve fit is an extrapolation from the plot in Roskam.

K_{M_Γ} is the compressibility correction to dihedral, assumed to have a value of 1.0, based on Figure 7.16 in Roskam. This is valid for the 747 and 777 geometries at low Mach numbers.

$$\frac{\Delta C_{l_\beta}}{\Gamma} = -0.0005\sqrt{A}\left(\frac{d}{b}\right)^2 \quad (3\text{-}21)$$

$$d = \sqrt{\frac{\pi\left(\frac{d_{fuse}}{2}\right)^2}{0.7854}} \tag{3-22}$$

$$\left(\Delta C_{l_\beta}\right)_{Z_w} = \frac{-1.2\sqrt{A}}{180/\pi}\frac{z_w}{b}\frac{2d}{b} \tag{3-23}$$

The contribution from the horizontal tail is approximately zero, since it has a small lift coefficient, small dihedral, and small area relative to the wing.

$$C_{l_{\beta_{htail}}} = 0 \tag{3-24}$$

The contribution from the vertical tail is estimated by:

$$C_{l_{\beta_{vtail}}} = C_{y_{\beta_{vtail}}}\frac{(z_{vtail}\cos\alpha - l_{vtail}\sin\alpha)}{b} \tag{3-25}$$

The fuselage angle of attack is the ratio of the lift coefficient to the lift curve slope minus the effective wing incidence angle. The effective wing incidence angle with 20˚ of flap deflection is approximately 5˚.

$$\alpha = \frac{C_L}{C_{L_\alpha}} - 5.0\frac{\pi}{180} \tag{3-26}$$

The aircraft lift curve slope is calculated by:

$$C_{L_\alpha} = C_{L_{\alpha_{wb}}} + C_{L_{\alpha_{htail}}}\eta_{htail}\frac{S_{htail}}{S_{ref}} \tag{3-27}$$

where:

$$C_{L_{\alpha_{wb}}} = K_{wb}C_{L_{\alpha_w}} \tag{3-28}$$

$C_{L_{\alpha_w}}$ and $C_{L_{\alpha_{htail}}}$ are found using the following equation with the appropriate values ofaspect ratio and sweep.

$$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta_M^2}{\kappa^2}\left(1 + \frac{\tan^2\Lambda_{c/2}}{\beta_M^2}\right) + 4}} \tag{3-29}$$

$$\beta_M = \sqrt{1 - M^2} \tag{3-30}$$

The dynamic pressure ratio at the horizontal tail is assumed to be 0.95.

$$\eta_{htail} = 0.95 \tag{3-31}$$

3.1.3. Yawing Moment Coefficient

The variation of yawing moment coefficient with sideslip angle has contributions from the wing, fuselage, and vertical tail.

$$C_{n_\beta} = C_{n_{\beta_{wing}}} + C_{n_{\beta_{fuse}}} + C_{n_{\beta_{vtail}}} \tag{3-32}$$

The wing contribution to the yawing moment coefficient is negligible for small angles of attack.

$$C_{n_{\beta_{wing}}} \quad 0 \tag{3-33}$$

The fuselage contribution to the yawing moment coefficient is determined by:

$$C_{n_{\beta_{fuse}}} = -K_N K_{R_l} \frac{S_{bs}}{S_{ref}} \frac{l_{fuse}}{b} \frac{180}{\pi} \tag{3-34}$$

where:

K_N is an empirical factor for body and body + wing effects, assumed to have a value of 0.0011, based on Figure 7.19 in Roskam. This is valid for the 747 and 777 geometries.

K_{R_l} is a Reynolds number factor for the fuselage, obtained from a curve fit to Figure 7.20 in Roskam, based on the calculated fuselage Reynolds number.

The fuselage side area is approximated as 83% of the fuselage length times diameter. This is a good approximation for the 747 and 777 geometries.

$$S_{bs} = 0.83 l_{fuse} d_{fuse} \tag{3-35}$$

The contribution from the vertical tail is estimated by the following equation, where α is defined in Eq. (3-26).

$$C_{n_{\beta_{vtail}}} = -C_{y_{\beta_{vtail}}} \frac{(l_{vtail} \cos\alpha + z_{vtail} \sin\alpha)}{b} \tag{3-36}$$

3.2. Lateral Control Derivatives

3.2.1. Sideforce Coefficient

The variation of sideforce coefficient with aileron deflection is assumed to be zero.

$$C_{y_{\delta_a}} = 0 \tag{3-37}$$

3.2.2. Rolling Moment Coefficient

The first step in the estimation of the rolling moment coefficient is to estimate the rolling moment effectiveness parameter $\left(\beta C'_{l_\delta}/\kappa\right)$ from Figure 11.1 in Roskam. For 747 and 777-like configurations with λ = 0.5 and M = 0.25, it is approximately 0.18.

The rolling effectiveness of two full-chord controls is estimated by:

$$C'_{l_\delta} = \frac{\kappa}{\beta_M}\left(\frac{\beta C'_{l_\delta}}{\kappa}\right) \tag{3-38}$$

where the section lift curve slope is assumed to be $2\pi/\beta_M$, and κ is the ratio of the actual section lift curve slope to $2\pi/\beta_M$.

The aileron lift effectiveness is estimated from Roskam's Figures 10.5 and 10.6 with c_f/c = 0.20 and t/c = 0.08. These assumptions result in a value of 3.5 from Figure 10.5, and a value of 1.0 from Figure 10.6 The aileron effectiveness is given by:

$$C_{l_\delta} = \left(\frac{C_{l_\delta}}{C_{l_{\delta_{Theory}}}}\right) C_{l_{\delta_{Theory}}} \tag{3-39}$$

$$\alpha_\delta = \frac{C_{l_\delta}}{C_{l_\alpha}} \tag{3-40}$$

The rolling effectiveness of the partial-chord controls is estimated by:

$$C_{l_\delta} = \alpha_\delta C'_{l_\delta} \tag{3-41}$$

The δ in the equation above refers to the sum of the left and right aileron deflections. Since we define the aileron deflection (δ_a) as one half of the sum of the deflections, the variation of rolling moment coefficient with aileron deflection is given by:

$$C_{l_{\delta_a}} = \frac{C_{l_\delta}}{2} \tag{3-42}$$

3.2.3. Yawing Moment Coefficient

The variation of yawing moment coefficient with aileron deflection is given by:

$$C_{n_{\delta_a}} = KC_L C_{l_{\delta_a}} \tag{3-43}$$

where K is estimated from Figure 11.3 in Roskam with $\lambda = 0.5$, A = 8, and $\eta_i = 0.74$.

3.3. Directional Control Derivatives

3.3.1. Sideforce Coefficient

The variation of sideforce coefficient with rudder deflection is given by:

$$C_{y_{\delta_r}} = C_{l_{\alpha_{vtaileff}}} \frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{C_l}} K' K_b \frac{S_{vtail}}{S_{ref}} \tag{3-44}$$

where:

$\frac{(\alpha_\delta)_{C_L}}{(\alpha_\delta)_{C_l}}$ is the ratio of the 3D flap-effectiveness parameter to the 2D flap-effectiveness parameter. It is estimated with a piecewise curve fit to Figure 10.2 in Roskam with an assumed value of $c_f/c = 0.33$.

K_b is the flap span factor, which is estimated to be 0.95 from Figure 10.3 in Roskam with $\Delta\eta = 0.85$.

K' is an empirical correction factor for large control deflections. It is estimated with a curve fit to Figure 10.7 in Roskam with $c_f/c = 0.3$.

3.3.2. Rolling Moment Coefficient

The variation of rolling moment coefficient with rudder deflection is given by:

$$C_{l_{\delta_r}} = C_{y_{\delta_r}} \left(\frac{z_{vtail} \cos\alpha - l_{vtail} \sin\alpha}{b} \right) \tag{3-45}$$

3.3.3. Yawing Moment Coefficient

The variation of yawing moment coefficient with rudder deflection is given by:

$$C_{n_{\delta_r}} = -C_{y_{\delta_r}} \frac{(l_{vtail} \cos\alpha + z_{vtail} \sin\alpha)}{b} \tag{3-46}$$

4. Validation

4.1. Boeing 747-100

The stability and control derivatives were validated with the 747-100. Table 4-1 shows a comparison of the predicted stability and control derivatives with the flight test derivatives presented in Nelson [6]. Note that the sign differences in the last three values are due to a different sign convention for the rudder deflection.

Table 4-1: Comparison of stability and control derivatives for 747-100

Derivative	Flight Test	Prediction	Error
C_{y_β}	-0.96	-0.6824	0.2776
C_{l_β}	-0.221	-0.2988	0.0778
C_{n_β}	0.150	0.0562	0.0938
$C_{l_{\delta_a}}$	0.0461	0.0501	0.0040
$C_{n_{\delta_a}}$	0.0064	0.0070	0.0006
$C_{y_{\delta_r}}$	0.175	-0.2854	0.1104
$C_{l_{\delta_r}}$	0.007	-0.0185	0.0115
$C_{n_{\delta_r}}$	-0.109	0.1496	0.0406

A correction factor was applied to each of the derivatives to increase their accuracy. Each correction factor shown in Table 4-2 is the ratio of the actual value to the predicted value for the 747-100 for the M = 0.25 flight condition given in NASA CR-2144 [8]. These correction factors may have to be recalibrated if the configuration is significantly different from the 747.

Table 4-2: Stability and control derivative correction factors

Derivative	Correction Factor
C_{y_β}	1.4068
C_{l_β}	0.7396
C_{n_β}	2.6690
$C_{l_{\delta_a}}$	0.9202
$C_{n_{\delta_a}}$	0.9143
$C_{y_{\delta_r}}$	0.6132
$C_{l_{\delta_r}}$	0.3784
$C_{n_{\delta_r}}$	0.7286

5. Input

The following listing is a sample input file for the Boeing 747-100. The input variables are given in the Appendix. This set of inputs was used to create the correction factors shown in the Validation section.

```
input file for stab
boeing747
1
7.0        dihedral_wing (deg)
6.2        z_wing (ft)
23.0       dia_fuse (ft)
5500.      sref (ft^2)
33.5       hspan_vtail (ft)
14.4       depth_fuse_vtail (ft)
36.4       c_vtail_root (ft)
11.5       c_vtail_tip (ft)
0.25       mach_eo
45.        sweep_vtail_1_4 (deg)
33.5       sweep_wing_1_2 (deg)
97.8       hspan_wing (ft)
36.4       hspan_htail (ft)
31.16      sweep_htail_1_2 (deg)
1.11       cl
26.        z_vtail (ft)
100.       l_vtail (ft)
225.2      length_fuse (ft)
4          new
0          nef
8.4        dia_nacelle (ft)
1467.      sh (ft^2)
2.3769e-3  rho_eo (slug/ft^3)
1116.4     a_eo (ft/s)
3.7372e-7  mu_eo (slug/(ft-s))
15.        dr_max (deg)
25.        da_max (deg)
0.         thrust_tv (lb)
0.         angle_tv (deg)
122.       l_tv (ft)
7.         z_tv (ft)
0.0        cl_circ_ctrl
```

6. Output

The following listing is the output file for the Boeing 747-100. The definitions of the variables are given in the Appendix. Note that the stability and control derivatives in this file represent the corrected values for the calibration case shown above.

```
stab output file
boeing747

Input

          1 = write_flag
     7.0000 = dihedral_wing (deg)
     6.2000 = z_wing (ft)
    23.0000 = dia_fuse (ft)
  5500.0000 = sref (ft^2)
    33.5000 = hspan_vtail (ft)
    14.4000 = depth_fuse_vtail (ft)
    36.4000 = c_vtail_root (ft)
    11.5000 = c_vtail_tip (ft)
     0.2500 = mach_eo
    45.0000 = sweep_vtail_1_4 (deg)
    33.5000 = sweep_wing_1_2 (deg)
    97.8000 = hspan_wing (ft)
    36.4000 = hspan_htail (ft)
    31.1600 = sweep_htail_1_2 (deg)
     1.1100 = cl
    26.0000 = z_vtail (ft)
   100.0000 = l_vtail (ft)
   225.2000 = length_fuse (ft)
          4 = new
          0 = nef
     8.4000 = dia_nacelle (ft)
  1467.0000 = sh (ft^2)
     0.0024 = rho_eo (slug/ft^3)
  1116.4000 = a_eo (ft/s)
 0.3737E-06 = mu_eo (slug/(ft-s))
    15.0000 = dr_max (deg)
    25.0000 = da_max (deg)
     0.0000 = thrust_tv (lb)
     0.0000 = angle_tv (deg)
   122.0000 = l_tv (ft)
     7.0000 = z_tv (ft)
     0.0000 = cl_circ_ctrl

Output

    -0.9601 = cy_beta (rad-1)
    -0.2210 = cl_beta (rad-1)
     0.1500 = cn_beta (rad-1)

     0.0000 = cy_da (rad-1)
     0.0461 = cl_da (rad-1)
     0.0064 = cn_da (rad-1)

    -0.1750 = cy_dr (rad-1)
    -0.0070 = cl_dr (rad-1)
     0.1090 = cn_dr (rad-1)

     3.0396 = beta (deg)
     5.0000 = phi (deg)
    16.8350 = da (deg)
    15.0000 = dr (deg)
     2.3776 = ar_vtail_eff
     0.0384 = cn_avail
```

References

[1] Roskam, J., *Methods for Estimating Stability and Control Derivatives of Conventional Subsonic Airplanes*, Roskam Aviation and Engineering Corporation, Lawrence, Kansas, 1971.

[2] Hoak, D.E. et al., *USAF Stability and Control DATCOM*, Flight Control Division, Air Force Flight Dynamics Laboratory, WPAFB, Ohio, 1978.

[3] Torenbeek, E., *Synthesis of Subsonic Airplane Design*, Delft Univ. Press, Delft, The Netherlands, 1982.

[4] Hanke, C.R., “The Simulation of a Large Jet Transport Aircraft, Vol. I: Mathematical Model,” NASA CR-1756, March 1971.

[5] Abbott, I.H., and von Doenhoff, A.E., *Theory of Wing Sections*, Dover, New York, 1959.

[6] Nelson, R. C., *Flight Stability and Automatic Control*, McGraw-Hill Co., New York, 1989.

[7] MacMillin, P.E., Golovidov, O.B., Mason, W.H., Grossman, B., and Haftka, R.T., *Trim, Control, and Performance Effects in Variable-Complexity High-Speed Civil Transport Design*, MAD 96-07-01, July, 1996.

[8] Heffley, R.K., and Jewell, W.F., *Aircraft Handling Qualities Data*, NASA CR-2144, December, 1972.

Appendix: Code Listing for Stability Subroutine (stab.f)

```
c///////////////////////////////////////////////////////////////////////
c
c  subroutine stab
c
c    This subroutine calculates the maximum available yawing moment
c  coefficient of a given aircraft configuration at a given flight
c  condition.  Note that right rudder deflection is defined as
c  positive, and right aileron up, left aileron down is defined as
c  positive.  Both of these control deflections generate positive
c  moments about their respective axes.  This is the convention used
c  by Roskam.  The thrust vectoring angle (angle_tv) is also defined
c  as positive for a right deflection.
c
c  Inputs
c
c  outfile              output filename
c  title                title of aircraft configuration
c  write_flag           write flag (0 = no output file, 1 = output file written)
c  dihedral_wing        wing dihedral angle (deg)
c  z_wing               distance from body centerline to quarter-chord point of
c                        exposed wing root chord, positive for the quarter-chord
c                        point below the body centerline (ft)
c  dia_fuse             fuselage diameter (ft)
c  sref                 wing reference area (ft^2)
c  hspan_vtail          vertical tail span (ft)
c  depth_fuse_vtail     fuselage depth at the fuselage station of the
c                        quarter-chord of the vertical tail (ft)
c  c_vtail_root         root chord of vertical tail
c  c_vtail_tip          tip chord of vertical tail
c  mach_eo              mach number
c  sweep_vtail_1_4_deg  vertical tail quarter-chord sweep angle (deg)
c  sweep_wing_1_2_deg   average wing half-chord sweep angle (deg)
c  hspan_wing           wing half-span (ft)
c  hspan_htail          horizontal tail half-span (ft)
c  sweep_htail_1_2_deg  horizontal tail half-chord sweep angle (deg)
c  cl                   lift coefficient
c  z_vtail              vertical distance from CG to AC of vertical tail (ft)
c  l_vtail              horizontal distance from CG to AC of vertical tail (ft)
c  length_fuse          fuselage length (ft)
c  new                  number of engines on the wing
c  nef                  number of engines on the fuselage
c  dia_nacelle          nacelle diameter (ft)
c  rho_eo               density at engine-out flight condition (slug/ft^3)
c  a_eo                 speed of sound at engine-out flight condition (ft/s)
c  mu_eo                viscosity at engine-out flight condition (slug/(ft-s))
c  dr_max               maximum allowable steady-state rudder deflection (deg)
c  da_max               maximum allowable steady-state aileron deflection (deg)
c  thrust_tv            maximum available thrust of the aft engine (lb)
c  angle_tv             horizontal angle between the fuselage centerline and the
c                        effective thrust vector (deg, positive to the right)
c  l_tv                 horizontal distance between CG and thrust vectoring
c                        nozzle (ft)
```

```
c  z_tv                 vertical distance between CG and thrust vectoring
c                        nozzle (ft)
c  cl_circ_ctrl         change in lift coefficient due to circulation control
c                        (nondimensionalized by q and the vertical tail area)
c
c  Outputs
c
c  ar_vtail_eff         effective aspect ratio of vertical tail
c  cn_avail             maximum available yawing moment coefficient
c
c  Internal Variables
c
c  alpha                angle of attack (rad)
c  alpha_d              section lift effectiveness
c  alpha_d_cl           section flap effectiveness (from Figure 10.2)
c  ar                   wing aspect ratio
c  ar_vtail             actual aspect ratio of vertical tail
c  ar_htail             actual aspect ratio of horizontal tail
c  avb_av               ratio of the aspect ratio of the vertical panel in the
c                        presence of the body to that of the isolated panel
c                        (from Figure 7.5)
c  avhb_avb             ratio of the vertical panel aspect ratio in the
c                        presence of the horizontal tail and body to that of
c                        the panel in the presence of the body alone (from
c                        Figure 7.6)
c  bcld_kappa           rolling moment effectiveness parameter (from Figure
c                        11.1)
c  beta                 sideslip angle, positive from the right (rad)
c  beta_m               square root of (1 - mach_eo)**2
c  cf_c                 ratio of flap chord to wing or tail chord
c  cf_factor            flap chord factor (from Figure 10.2)
c  cl_alpha             lift-curve slope of entire aircraft (rad^-1)
c  cl_alpha_2d          2-dimensional lift-curve slope at MAC (rad^-1)
c  cl_alpha_h           lift-curve slope of horizontal tail (rad^-1)
c  cl_alpha_vtail       original lift-curve slope of vertical tail (rad^-1)
c  cl_alpha_vtail_eff   effective lift-curve slope of vertical tail (rad^-1)
c  cl_alpha_w           lift-curve slope of wing (rad^-1)
c  cl_alpha_wb          lift-curve slope of wing-body combination (rad^-1)
c  cl_beta              variation of rolling moment coefficient with sideslip
c                        angle
c  cl_beta_cor          corrected value of cl_beta
c  cl_beta_htail        horizontal tail contribution to cl_beta
c  cl_beta_vtail        vertical tail contribution to cl_beta
c  cl_beta_wingbody     wing-body contribution to cl_beta
c  cl_d                 rolling effectiveness of partial-chord controls
c  cl_da                variation of rolling moment coefficient with aileron
c                        deflection
c  cl_da_cor            corrected value of cl_da
c  cl_dr                variation of rolling moment coefficient with rudder
c                        deflection
c  cl_dr_cor            corrected value of cl_dr
c  clb_cl_a             aspect ratio contribution to cl_beta_wingbody (from
c                        Figure 7.14)
c  clb_cl_lambda        wing sweep contribution to cl_beta_wingbody (from
c                        Figure 7.11)
c  clb_gamma            dihedral effect on cl_beta (from Figure 7.15)
c  cld_prime            rolling effectiveness of two full-chord ailerons
c                        (Equation 11.2)
c  cld_ratio            empirical correction for plain TE flaps (Fig. 10.6)
c  cld_theory           theoretical lift effectiveness of plain TE flaps
c                        (Fig. 10.5)
```

```
c  cn_beta              variation of yawing moment coefficient with sideslip
c                        angle
c  cn_beta_cor          corrected value of cn_beta
c  cn_beta_fuse         fuselage contribution to cn_beta
c  cn_beta_vtail        vertical tail contribution to cn_beta
c  cn_beta_wing         wing contribution to cn_beta
c  cn_da                variation of yawing moment coefficient with aileron
c                        deflection
c  cn_da_cor            corrected value of cn_da
c  cn_dr                variation of yawing moment coefficient with rudder
c                        deflection
c  cn_dr_cor            corrected value of cn_dr
c  cy_beta              variation of side force coefficient with sideslip angle
c  cy_beta_cor          corrected value of cy_beta
c  cy_beta_fuse         fuselage contribution to cy_beta
c  cy_beta_vtail        vertical tail contribution to cy_beta
c  cy_beta_wing         wing contribution to cy_beta
c  cy_da                variation of side force coefficient with aileron
c                        deflection
c  cy_dr                variation of side force coefficient with rudder
c                        deflection
c  cy_dr_cor            corrected value of cy_dr
c  d                    d in Equation 7.10 (estimated from Equation 7.11)
c  da                   aileron deflection, positive for right aileron up, left
c                        aileron down (rad)
c  dclb_gamma           body-induced effect on wing height (from Equation 7.10)
c  dclb_zw              another body-induced effect on wing height (from
c                        Equation 7.12)
c  debug_flag           printing flag for debugging output (0 = no debugging
c                        info printed, 1 = debugging info printed)
c  dr                   rudder deflection, positive for right deflection (rad)
c  eff_vtail            vertical tail effectiveness factor estimated by
c                        Equation 7.5
c  eta_h                dynamic pressure ratio at the horizontal tail
c  f_cy_beta            correction factor for cy_beta
c  f_cl_beta            correction factor for cl_beta
c  f_cn_beta            correction factor for cn_beta
c  f_cl_da              correction factor for cl_da
c  f_cn_da              correction factor for cn_da
c  f_cy_dr              correction factor for cy_dr
c  f_cl_dr              correction factor for cl_dr
c  f_cn_dr              correction factor for cn_dr
c  flap_eff_ratio       flap effectiveness ratio (from Figure 10.2)
c  i                    index
c  k                    empirical factor for estimating the variation of yawing
c                        moment coefficient with aileron deflection
c  k_b                  span factor for plain flap (from Figure 10.3)
c  k_cy_beta_v          empirical factor from Figure 7.3
c  k_f                  fuselage correction factor (from Figure 7.13)
c  k_h                  factor accounting for relative size of horizontal and
c                        vertical tails (from Figure 7.7)
c  k_m_lambda           compressibility correction to wing sweep (from Figure
c                        7.12)
c  k_m_gamma            compressibility correction to dihedral effect (from
c                        Figure 7.16)
c  k_n                  factor for body and body + wing effects (from Figure
c                        7.19)
c  k_prime              empirical correction for lift effectiveness of plain
c                        flaps at high flap deflections (from Figure 10.7)
c  k_r_l                Reynold's number factor for the fuselage (from Figure
c                        7.20)
```

```
c  k_wbi               wing-body interference factor from Figure 7.1
c  k_wb                factor for loss in lift curve due to body
c  kappa               ratio of the actual lift-curve slope to 2*pi
c  phi                 bank angle, positive to the right (rad)
c  q                   dynamic pressure (lb/ft^2)
c  re_fuse             fuselage Reynolds number
c  sbs                 body side area (ft^2)
c  sh                  area of horizontal tail (ft^2)
c  sv                  area of vertical tail (ft^2)
c  sweep_htail_1_2     horizontal tail half-chord sweep angle (rad)
c  sweep_vtail_1_2     vertical tail half-chord sweep angle (rad)
c  sweep_vtail_1_4     vertical tail half-chord sweep angle (rad)
c  sweep_wing_1_2      average wing half-chord sweep angle (rad)
c  x                   temporary variable for curve fits
c
c  Created by:  Joel Grasmeyer
c  Last Modified:  03/01/98
c
c////////////////////////////////////////////////////////////////////////

      subroutine stab(outfile,title,write_flag,dihedral_wing,z_wing,
     & dia_fuse,sref,hspan_vtail,depth_fuse_vtail,c_vtail_root,
     & c_vtail_tip,mach_eo,sweep_vtail_1_4_deg,sweep_wing_1_2_deg,
     & hspan_wing,hspan_htail,sweep_htail_1_2_deg,cl,z_vtail,l_vtail,
     & length_fuse,new,nef,dia_nacelle,sh,rho_eo,a_eo,mu_eo,dr_max,
     & da_max,thrust_tv,angle_tv,l_tv,z_tv,cl_circ_ctrl,ar_vtail_eff,
     & cn_avail)

      implicit none

      character*72 outfile, title
      integer i, write_flag, unit_out, new, nef, debug_flag
      real pi, dihedral_wing, z_wing, dia_fuse, sref, hspan_vtail, ar,
     & depth_fuse_vtail, c_vtail_root, c_vtail_tip, mach_eo, sv, sh,
     & sweep_wing_1_2, hspan_wing, cy_beta, ar_vtail, k, thrust_tv,
     & cy_beta_wing, cy_beta_fuse, cy_beta_vtail, ar_vtail_eff, alpha,
     & cl_alpha_vtail, beta_m, eff_vtail, kappa, cl_alpha_vtail_eff, q,
     & cl_beta, cl_beta_htail, cl_beta_vtail, cl_beta_wingbody, sbs,
     & cl, k_wbi, avb_av, avhb_avb, k_h, clb_cl_lambda, cn_da, l_tv,
     & k_m_lambda, k_f, clb_cl_a, clb_gamma, k_m_gamma, dclb_gamma,
     & d, dclb_zw, cl_alpha, z_vtail, l_vtail, cn_beta_fuse, da,
     & cn_beta_vtail, cn_beta, cn_beta_wing, k_n, k_r_l, phi, angle_tv,
     & length_fuse, re_fuse, cl_da, cy_da, bcld_kappa, cld_prime, cl_d,
     & alpha_d, cld_theory, cld_ratio, cl_alpha_2d, cl_dr, dr, cn_dr,
     & cy_dr, cf_factor, k_prime, alpha_d_cl, flap_eff_ratio, k_b,
     & cf_c, beta, rho_eo, a_eo, mu_eo, cn_avail, k_cy_beta_v, da_max,
     & dia_nacelle, dr_max, f_cy_beta, f_cl_beta, z_tv, cl_circ_ctrl,
     & f_cn_beta, f_cl_da, f_cn_da, f_cy_dr, f_cl_dr, f_cn_dr, x,
     & sweep_vtail_1_4, sweep_vtail_1_2, sweep_wing_1_2_deg, k_wb,
     & sweep_vtail_1_4_deg, cy_beta_cor, cl_beta_cor, cn_beta_cor,
     & cl_da_cor, cn_da_cor, cy_dr_cor, cl_dr_cor, cn_dr_cor,
     & hspan_htail, sweep_htail_1_2_deg, sweep_htail_1_2, eta_h,
     & ar_htail, cl_alpha_h, cl_alpha_w, cl_alpha_wb

      pi = acos(-1.)

c Initialize value of debug_flag
      debug_flag = 0

c Convert sweep angles from degrees to radians
      sweep_wing_1_2 = sweep_wing_1_2_deg*pi/180.
```

```
      sweep_htail_1_2 = sweep_htail_1_2_deg*pi/180.
      sweep_vtail_1_4 = sweep_vtail_1_4_deg*pi/180.

c Append extension to idrag output filename
      i = 1
      do while (outfile(i:i) .ne. '.')
        i = i + 1
      end do
      outfile(i+1:i+5) = 'stab'
      outfile(i+6:) = ''

c Write input data to output file for confirmation
      if (write_flag .eq. 1) then
        unit_out = 171
        open(unit_out,file=outfile)
        write(unit_out,"('stab output file')")
        write(unit_out,"(a72)") title
        write(unit_out,*)
        write(unit_out,"(a5)") 'Input'
        write(unit_out,*)
        write(unit_out,101) write_flag, '= write_flag'
        write(unit_out,100) dihedral_wing, '= dihedral_wing (deg)'
        write(unit_out,100) z_wing, '= z_wing (ft)'
        write(unit_out,100) dia_fuse, '= dia_fuse (ft)'
        write(unit_out,100) sref, '= sref (ft^2)'
        write(unit_out,100) hspan_vtail, '= hspan_vtail (ft)'
        write(unit_out,100) depth_fuse_vtail, '= depth_fuse_vtail (ft)'
        write(unit_out,100) c_vtail_root, '= c_vtail_root (ft)'
        write(unit_out,100) c_vtail_tip, '= c_vtail_tip (ft)'
        write(unit_out,100) mach_eo, '= mach_eo'
        write(unit_out,100) sweep_vtail_1_4*180./pi,
     & '= sweep_vtail_1_4 (deg)'
        write(unit_out,100) sweep_wing_1_2*180./pi,
     & '= sweep_wing_1_2 (deg)'
        write(unit_out,100) hspan_wing, '= hspan_wing (ft)'
        write(unit_out,100) hspan_htail, '= hspan_htail (ft)'
        write(unit_out,100) sweep_htail_1_2*180./pi,
     & '= sweep_htail_1_2 (deg)'
        write(unit_out,100) cl, '= cl'
        write(unit_out,100) z_vtail, '= z_vtail (ft)'
        write(unit_out,100) l_vtail, '= l_vtail (ft)'
        write(unit_out,100) length_fuse, '= length_fuse (ft)'
        write(unit_out,101) new, '= new'
        write(unit_out,101) nef, '= nef'
        write(unit_out,100) dia_nacelle, '= dia_nacelle (ft)'
        write(unit_out,100) sh, '= sh (ft^2)'
        write(unit_out,100) rho_eo, '= rho_eo (slug/ft^3)'
        write(unit_out,100) a_eo, '= a_eo (ft/s)'
        write(unit_out,103) mu_eo, '= mu_eo (slug/(ft-s))'
        write(unit_out,100) dr_max, '= dr_max (deg)'
        write(unit_out,100) da_max, '= da_max (deg)'
        write(unit_out,100) thrust_tv, '= thrust_tv (lb)'
        write(unit_out,100) angle_tv, '= angle_tv (deg)'
        write(unit_out,100) l_tv, '= l_tv (ft)'
        write(unit_out,100) z_tv, '= z_tv (ft)'
        write(unit_out,100) cl_circ_ctrl, '= cl_circ_ctrl'
      end if

c Calculate stability and control derivatives via Roskam's methods

c Sideslip angle derivatives
```

```
      cy_beta_wing = -0.0001*abs(dihedral_wing)*180./pi

c Estimate k_wbi from Figure 7.1 (curve fit)
      if (z_wing/(dia_fuse/2.) .le. 0.) then
        k_wbi = 0.85*(-z_wing/(dia_fuse/2.)) + 1.
      elseif (z_wing/(dia_fuse/2.) .gt. 0.) then
        k_wbi = 0.5*z_wing/(dia_fuse/2.) + 1.
      end if

c Estimate the side force coefficient due to the fuselage and nacelles
      cy_beta_fuse = -2.*k_wbi*( pi*(dia_fuse/2.)**2 +
     &               (new + nef)*pi*(dia_nacelle/2.)**2 )/sref

c Estimate k_cy_beta_v from Figure 7.3 (curve fit)
      x = hspan_vtail/depth_fuse_vtail
      if (x .le. 2.) then
        k_cy_beta_v = 0.75
      elseif (x .gt. 2. .and. x .lt. 3.5) then
        k_cy_beta_v = x/6. + 5./12.
      elseif (x .ge. 3.5) then
        k_cy_beta_v = 1.
      end if

c Estimate avb_av from Figure 7.5 (curve fit for taper ratio <= 0.6)
      x = hspan_vtail/depth_fuse_vtail
      avb_av = 0.002*x**5 - 0.0464*x**4 + 0.404*x**3 - 1.6217*x**2 +
     &         2.7519*x + 0.0408

c Factor from Figure 7.6 is for zh/bv = 0.
      avhb_avb = 1.1

c Estimate k_h from Figure 7.7 (curve fit)
      sv      = hspan_vtail*(c_vtail_root + c_vtail_tip)/2.
      x       = sh/sv
      k_h = -0.0328*x**4 + 0.2885*x**3 - 0.9888*x**2 + 1.6554*x -
     &      0.0067

c Estimate the effective aspect ratio for the vertical tail
      ar_vtail     = hspan_vtail**2/sv
      ar_vtail_eff = avb_av*ar_vtail*(1. + k_h*(avhb_avb - 1.))

c Assume the section lift-curve slope is 2.*pi
      cl_alpha_vtail = 2.*pi

c Estimate the effective lift-curve slope for the vertical tail
      kappa             = cl_alpha_vtail/(2.*pi)
      beta_m            = sqrt( 1. - mach_eo**2 )
      sweep_vtail_1_2   = atan( (c_vtail_root/4. + hspan_vtail*
     &                    tan(sweep_vtail_1_4) + c_vtail_tip/4. -
     &                    c_vtail_root/2.)/hspan_vtail )
      cl_alpha_vtail_eff = 2.*pi*ar_vtail_eff/( 2. +
     &                    sqrt( ar_vtail_eff**2*beta_m**2/kappa**2*
     &                    ( 1. +  tan(sweep_vtail_1_2)**2/
     &                    beta_m**2 ) + 4. ) )

c Estimate the third term in eqn. 7.4 from eqn. 7.5
      eff_vtail     = 0.724 + 3.06*sv/sref/(1. +
     &                cos(sweep_vtail_1_4)) + 0.4*z_wing/dia_fuse +
     &                0.009*ar_vtail_eff
      cy_beta_vtail = -k_cy_beta_v*cl_alpha_vtail_eff*eff_vtail*sv/sref
```

```
c Calculate total variation of side force coefficient with sideslip angle
      cy_beta = cy_beta_wing + cy_beta_fuse + cy_beta_vtail

c Factor from Figure 7.11 is approximated by a curve fit for lambda = 0.5
      clb_cl_lambda = -0.004/45*sweep_wing_1_2*180./pi

c Factor from Figure 7.12 is approximated for 747 and 777 configurations
c at low Mach numbers
      k_m_lambda = 1.0

c Factor from Figure 7.13 is approximated for 747 and 777 configurations
      k_f = 0.85

c Factor from Figure 7.14 is approximated for lambda = 0.5 and high AR
      clb_cl_a = 0.000

c Factor from Figure 7.15 is approximated by a linear curve fit for
c lambda equal to 0.5, low sweep, and high AR
      ar        = (2.*hspan_wing)**2/sref
      clb_gamma = -0.00012 - 0.00013/10*ar

c Factor from Figure 7.16 is approximated for 747 and 777 configurations
c at low Mach numbers
      k_m_gamma = 1.0

c Estimate body-induced effect on wing height from eqns. 7.10, 7.11, and 7.12
      d          = sqrt(pi*(dia_fuse/2.)**2/0.7854)
      dclb_gamma = -0.0005*sqrt(ar)*(d/(2.*hspan_wing))**2
      dclb_zw    = -1.2*sqrt(ar)/(180./pi)*z_wing/(2.*hspan_wing)*
     &             2.*d/(2.*hspan_wing)

c Wing-body contribution to cl_beta (wing twist effect is neglected)
      cl_beta_wingbody = ( cl*(clb_cl_lambda*k_m_lambda*k_f +
     & clb_cl_a) + dihedral_wing*(clb_gamma*k_m_gamma + dclb_gamma) +
     & dclb_zw )*180./pi

c Since the horizontal tail has a small lift coefficient, small dihedral,
c  and small area relative to the wing, it is negligible.
      cl_beta_htail = 0.

c Calculate the lift curve loss factor due to the fuselage
      x = dia_fuse/(2.*hspan_wing)
      k_wb = 1 - 0.25*x**2 + 0.025*x

c Assume the 2D lift-curve slope is 2*pi/beta_m
      cl_alpha_2d = 2*pi/beta_m
      kappa       = cl_alpha_2d/(2.*pi/beta_m)

c Calculate the lift curve slope of the wing alone and wing-body combination
      cl_alpha_w = 2.*pi*ar/( 2. + sqrt( ar**2*beta_m**2/kappa**2*
     &             ( 1. +  tan(sweep_wing_1_2)**2/beta_m**2 ) + 4. ) )
      cl_alpha_wb = k_wb*cl_alpha_w

c Calculate the lift curve slope of the horizontal tail
      ar_htail = (2.*hspan_htail)**2/sh
      cl_alpha_h = 2.*pi*ar_htail/( 2. + sqrt( ar_htail**2*beta_m**2/
     &             kappa**2*( 1. +  tan(sweep_htail_1_2)**2/beta_m**2 )
     &             + 4. ) )

c Assume the dynamic pressure ratio at the horizontal tail is 0.95
      eta_h = 0.95
```

```
c Calculate the lift curve slope of the total aircraft
      cl_alpha = cl_alpha_wb + cl_alpha_h*eta_h*sh/sref

c Calculate the angle of attack of the fuselage centerline.  The wing
c incidence angle is assumed to be 5 deg.
      alpha    = cl/cl_alpha - 5.*pi/180.

c Estimate the vertical tail contribution to cl_beta
      cl_beta_vtail = cy_beta_vtail*( z_vtail*cos(alpha) - l_vtail*
     &                sin(alpha) )/(2.*hspan_wing)

c Calculate total variation of rolling moment coefficient with sideslip angle
      cl_beta = cl_beta_wingbody + cl_beta_htail + cl_beta_vtail

c Wing contribution to cn_beta is negligible for small angles of attack.
      cn_beta_wing = 0.

c Estimate empirical factor for body and body + wing effects from Figure 7.19
c  Constant value assumed for 747 and 777-like configurations
      k_n = 0.0011

c Calculate fuselage Reynolds number at the engine-out flight condition
      re_fuse = rho_eo*mach_eo*a_eo*length_fuse/mu_eo

c Estimate fuselage Reynolds number effect on wing-body from Figure 7.20
      k_r_l = 1. + 1.2/log(350.)*log(re_fuse/1000000.)

c Estimate fuselage contribution to cn_beta
      sbs          = 0.83*dia_fuse*length_fuse
      cn_beta_fuse = -180./pi*k_n*k_r_l*sbs/sref*
     &               length_fuse/(2.*hspan_wing)

c Estimate vertical tail contribution to cn_beta
      cn_beta_vtail = -cy_beta_vtail*( l_vtail*cos(alpha) +
     &                z_vtail*sin(alpha) )/(2.*hspan_wing)

c Calculate total variation of yawing moment coefficient with sideslip angle
      cn_beta = cn_beta_wing + cn_beta_fuse + cn_beta_vtail

c Assume variation of sideforce coefficient with aileron deflection is zero
      cy_da = 0.

c Estimate the rolling moment effectiveness parameter from Figure 11.1
c for lambda = 0.5, and for 747 and 777-like ailerons at mach 0.25
      bcld_kappa = 0.18

c Estimate the rolling effectiveness of two full-chord controls by Eqn. 11.2
      cld_prime = kappa/beta_m*bcld_kappa

c Estimate aileron effectiveness by assuming cf/c = 0.20 and t/c = 0.08
      cld_theory = 3.5
      cld_ratio  = 1.0
      cl_d       = cld_ratio*cld_theory
      alpha_d    = cl_d/cl_alpha_2d

c Determine the rolling effectiveness of the partial-chord controls by
c Eqn. 11.3.  Note that this is the change in cl with respect to a change
c in the sum of the left and right aileron deflections (d).
      cl_d = alpha_d*cld_prime
```

```
c Estimate variation of rolling moment coefficient with aileron deflection
c by neglecting differential control effects.  Since the aileron deflection
c (da) is defined as half of the sum of the left and right deflections, cl_d
c from the equation above must be divided by 2.
      cl_da = cl_d/2.

c The method in Roskam for estimating cn_da does not account for the
c effect of differential ailerons and the use of spoilers for roll control
c on the yaw moment.  Therefore, the factor k is estimated
c based on the ratio of cn_da to cl_da from the 747 flight test data
c presented in Nelson.  Note that the effect of cl is absorbed into
c the factor k.
      k = 0.0064/0.0461

c Estimate variation of yawing moment coefficient with aileron deflection
      cn_da = k*cl_da

c Estimate the flap chord factor from Figure 10.2 for cf/c = 0.33
c  The flap effectiveness ratio is estimated with a piecewise curve fit
      cf_c = 0.33
      alpha_d_cl = -sqrt( 1. - (1. - cf_c)**2 )
      if (alpha_d_cl .ge. -0.5) then
        flap_eff_ratio = 1.42 + 1.8*alpha_d_cl
      elseif (alpha_d_cl .ge. -0.6) then
        flap_eff_ratio = 1.32 + 1.6*alpha_d_cl
      elseif (alpha_d_cl .ge. -0.7) then
        flap_eff_ratio = 1.08 + 1.2*alpha_d_cl
      else
        flap_eff_ratio = 0.94 + alpha_d_cl
      end if
      flap_eff_ratio = 1. + flap_eff_ratio/( ar_vtail_eff -
     &                 0.5*(-alpha_d_cl - 2.1) )
      cf_factor = flap_eff_ratio*alpha_d_cl

c Estimate empirical correction for lift effectiveness of plan flaps at
c from Figure 10.7 for cf/c = 0.33.
      x = dr_max
      if (x .lt. 15.) then
        k_prime = 1.
      else
        k_prime = 4e-7*x**4 - 7e-5*x**3 + 0.0047*x**2 - 0.1453*x +
     &             2.3167
      end if

c Estimate span factor for plain flap from Figure 10.3 for delta eta = 0.85
      k_b = 0.95

c Estimate variation of sideforce coefficient with rudder deflection
      cy_dr = cl_alpha_vtail_eff*cf_factor*k_prime*k_b*sv/sref

c Estimate variation of rolling moment coefficient with rudder deflection
      cl_dr = cy_dr*( z_vtail*cos(alpha) - l_vtail*sin(alpha) )/
     &        (2.*hspan_wing)

c Estimate variation of yawing moment coefficient with rudder deflection
      cn_dr = -cy_dr*( l_vtail*cos(alpha) + z_vtail*sin(alpha) )/
     &        (2.*hspan_wing)

c Multiply empirical estimates by their respective correction factors
c The correction factors are the ratio of the actual 747 derivatives to
c the 747 derivatives predicted by the method above at the M=0.25 flight
```

```
c condition defined in NASA CR-2144 and Nelson.  The rudder deflection
c was 15 deg for this calibration.
      cy_beta_cor = 1.4068*cy_beta
      cl_beta_cor = 0.7396*cl_beta
      cn_beta_cor = 2.6690*cn_beta
      cl_da_cor   = 0.9202*cl_da
      cn_da_cor   = 0.9143*cn_da
      cy_dr_cor   = 0.6132*cy_dr
      cl_dr_cor   = 0.3784*cl_dr
      cn_dr_cor   = 0.7286*cn_dr

c Calculate the dynamic pressure
      q = 0.5*rho_eo*(mach_eo*a_eo)**2

c Set the rudder deflection to 20 deg, and the bank angle to 5 deg
      dr  = dr_max*pi/180.
      phi = 5.*pi/180.

c Solve for the sideslip angle and aileron deflection
      beta  = ( -cy_dr_cor*dr - cl*sin(phi) +
     &        sign( thrust_tv*sin(angle_tv*pi/180.)/(q*sref),
     &        angle_tv ) + cl_circ_ctrl*sv/sref )/cy_beta_cor
      da    = ( -cl_dr_cor*dr - cl_beta_cor*beta + sign( thrust_tv*
     &        sin(angle_tv*pi/180.)*z_tv/(q*sref*2.*hspan_wing),
     &        angle_tv ) + cl_circ_ctrl*z_vtail/(2.*hspan_wing)*
     &        sv/sref )/cl_da_cor

c Check if the aileron deflection is greater than the max allowable value
      if (da .gt. da_max) then
        print*,'Warning from stab.f: Required aileron deflection is ',
     &         'greater than the maximum allowable value.'
      end if

c Calculate the maximum available yawing moment coefficient
      cn_avail = cn_da_cor*da + cn_dr_cor*dr + cn_beta_cor*beta +
     &           sign( thrust_tv*sin(angle_tv*pi/180.)*l_tv/
     &           (q*sref*2.*hspan_wing), angle_tv ) +
     &           cl_circ_ctrl*l_vtail/(2.*hspan_wing)*sv/sref

c Write output data
      if (write_flag .eq. 1) then
        write(unit_out,*)
        write(unit_out,"(a6)") 'Output'
        write(unit_out,*)

c This section is normally commented out.  It can be used to print the
c uncorrected values of the derivatives for debugging purposes.
        if (debug_flag .eq. 1) then
          write(unit_out,100) cy_beta_wing,  '= cy_beta_wing (rad-1)'
          write(unit_out,100) cy_beta_fuse,  '= cy_beta_fuse (rad-1)'
          write(unit_out,100) cy_beta_vtail, '= cy_beta_vtail (rad-1)'
          write(unit_out,100) cy_beta, '= cy_beta (rad-1)'
          write(unit_out,*)
          write(unit_out,100) cl_beta_wingbody,
     & '= cl_beta_wingbody (rad-1)'
          write(unit_out,100) cl_beta_htail, '= cl_beta_htail (rad-1)'
          write(unit_out,100) cl_beta_vtail, '= cl_beta_vtail (rad-1)'
          write(unit_out,100) cl_beta,       '= cl_beta (rad-1)'
          write(unit_out,*)
          write(unit_out,100) cn_beta_wing,  '= cn_beta_wing (rad-1)'
          write(unit_out,100) cn_beta_fuse,  '= cn_beta_fuse (rad-1)'
```

```
          write(unit_out,100) cn_beta_vtail, '= cn_beta_vtail (rad-1)'
          write(unit_out,100) cn_beta,       '= cn_beta (rad-1)'
          write(unit_out,*)
          write(unit_out,100) cy_da,         '= cy_da (rad-1)'
          write(unit_out,100) cl_da,         '= cl_da (rad-1)'
          write(unit_out,100) cn_da,         '= cn_da (rad-1)'
          write(unit_out,*)
          write(unit_out,100) cy_dr,         '= cy_dr (rad-1)'
          write(unit_out,100) cl_dr,         '= cl_dr (rad-1)'
          write(unit_out,100) cn_dr,         '= cn_dr (rad-1)'
          write(unit_out,*)
        end if

c This section prints the corrected values of the derivatives
        write(unit_out,100) cy_beta_cor,  '= cy_beta (rad-1)'
        write(unit_out,100) cl_beta_cor,  '= cl_beta (rad-1)'
        write(unit_out,100) cn_beta_cor,  '= cn_beta (rad-1)'
        write(unit_out,*)
        write(unit_out,100) cy_da,        '= cy_da (rad-1)'
        write(unit_out,100) cl_da_cor,    '= cl_da (rad-1)'
        write(unit_out,100) cn_da_cor,    '= cn_da (rad-1)'
        write(unit_out,*)
        write(unit_out,100) cy_dr_cor,    '= cy_dr (rad-1)'
        write(unit_out,100) cl_dr_cor,    '= cl_dr (rad-1)'
        write(unit_out,100) cn_dr_cor,    '= cn_dr (rad-1)'
        write(unit_out,*)
        write(unit_out,100) beta*180./pi, '= beta (deg)'
        write(unit_out,100) phi*180./pi,  '= phi (deg)'
        write(unit_out,100) da*180./pi,   '= da (deg)'
        write(unit_out,100) dr*180./pi,   '= dr (deg)'
        write(unit_out,100) ar_vtail_eff, '= ar_vtail_eff'
        write(unit_out,100) cn_avail,     '= cn_avail'
        write(unit_out,*)
        close(unit_out)
      endif
  100 format(f11.4, 1x, a)
  101 format(7x, i4, 1x, a)
  102 format(f11.0, 1x, a)
  103 format(g11.4, 1x, a)

      return
      end
```