

**Stability and Control Derivative Estimation
and Engine-Out Analysis**

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Nomenclature

Symbols

A	aspect ratio
b	wing span
b_{vtail}	vertical tail span
$C_{D_{ewm}}$	drag coefficient due to windmilling of failed engine
C_L	lift coefficient
$C_{L_{htail}}$	lift curve slope of the horizontal tail
$C_{l_{vtail}}$	section lift curve slope of vertical tail
$C_{l_{vtail_{eff}}}$	effective lift curve slope of vertical tail
$C_{L_{wb}}$	lift curve slope of the wing and body
$C_{n_{avail}}$	available yawing moment coefficient at the engine-out flight condition
$C_{n_{req}}$	required yawing moment coefficient at the engine-out flight condition
C_y	variation of sideforce coefficient with yaw angle
C_l	variation of rolling moment coefficient with yaw angle
C_n	variation of yawing moment coefficient with yaw angle
D_{ewm}	drag due to windmilling of failed engine
d_{fuse}	maximum fuselage diameter
$d_{fuse_{vtail}}$	depth of the fuselage at the vertical tail quarter-chord position
d_i	engine inlet diameter
$d_{nacelle}$	nacelle diameter
l	horizontal distance between CG and vertical surface
l_e	buttlane of outboard engine
L_{ext}	external rolling moment
$k_{C_y_v}$	empirical factor for vertical tail sideslip derivative estimation
K'	empirical correction factor for large control deflections
K_b	flap span factor
K_H	factor accounting for the relative size of the horizontal and vertical tails
K_M	compressibility correction to dihedral
K_N	empirical factor for body and body + wing effects
K_{R_l}	Reynold's number factor for the fuselage
K_M	compressibility correction to sweep
K_{wb}	factor for fuselage loss in the lift curve slope
K_{wbi}	wing-body interference factor
l_{tv}	horizontal distance between CG and engine nozzle
l_{vtail}	horizontal distance between CG and aerodynamic center of vertical tail

M	Mach number
$N_{engines}$	number of engines
N_{req}	required yawing moment
N_{max}	maximum attainable yawing moment
q_{eo}	dynamic pressure at the engine-out flight condition
S_{htail}	horizontal tail area
S_o	cross-sectional area of fuselage
S_{ref}	wing reference area
S_{vtail}	vertical tail area
T	maximum available thrust at given mach and altitude
T_o	static thrust at sea level
$\frac{V_n}{V}$	ratio of mean nozzle exit velocity to freestream velocity
Y_{ext}	external sideforce
z_{tv}	vertical distance between CG and engine nozzle
z_{vtail}	vertical distance between CG and aerodynamic center of vertical tail
$C_{L_{cc}}$	change in vertical tail C_L due to circulation control
	angle of attack (rad)
	sideslip angle (positive with relative wind from right)
M	compressibility factor = $\sqrt{1 - M^2}$
a	aileron deflection (positive for right up, left down)
r	rudder deflection (positive right)
$htail$	dynamic pressure ratio at the horizontal tail
	bank angle (positive right roll)
	dihedral angle (deg)
	ratio of actual lift curve slope to 2
$c/2$	half-chord sweep angle
$c/4$	quarter-chord sweep angle
	ratio of density at a given altitude to density at sea level

Subscripts

$avail$	available
bs	body side
cc	circulation control
eff	effective
$fuse$	fuselage
$htail$	horizontal tail
req	required

<i>tv</i>	thrust vectoring
<i>vtail</i>	vertical tail
<i>wb</i>	wing-body
<i>wing</i>	wing

The FORTRAN code variable names and definitions are given in the Appendix.

1. Introduction

This report describes the estimation of stability and control derivatives using the method of Reference [1] (which is essentially DATCOM [2]), and the establishment of the engine-out constraint based on the required yawing moment coefficient. The use of thrust vectoring and circulation control to provide additional yawing moment is also described.

1.1. Control Surface Sign Conventions

The control surface sign conventions are defined such that a positive control deflection generates a positive roll or yaw moment according to the right hand rule with a conventional body axis coordinate system, as shown in Figure 1-1. A positive aileron deflection is defined with the right aileron up and the left aileron down. The aileron deflection is the average deflection of the two surfaces from the neutral position. A positive rudder deflection is defined with the trailing edge to the right, as viewed from above.

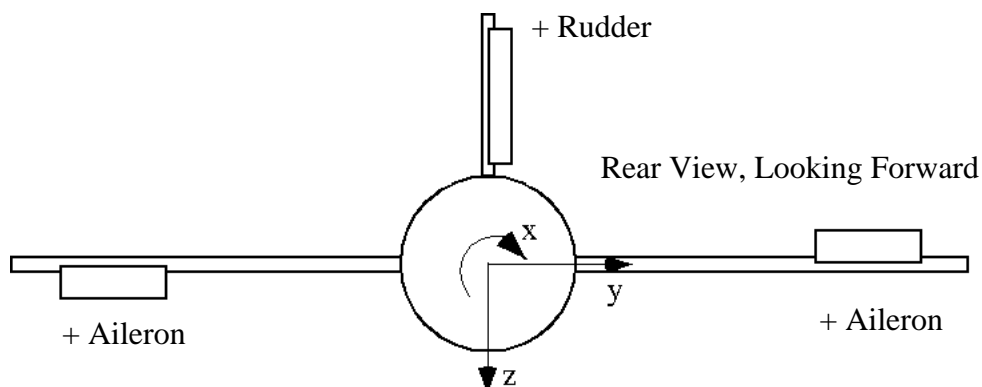


Figure 1-1: Control surface sign conventions

2. Engine-out Methodology

The engine-out constraint is established by constraining the maximum available yawing moment coefficient ($C_{n_{avail}}$) to be greater than the required yawing moment coefficient ($C_{n_{req}}$) for the engine-out flight condition:

$$C_{n_{avail}} > C_{n_{req}} \quad (2-1)$$

2.1. Required Yawing Moment Coefficient

The required yawing moment coefficient is the yawing moment coefficient required to maintain steady flight with one failed outboard engine at 1.2 times the stall speed, as specified by FAR 25.149. The remaining outboard engine must be at the maximum available thrust, and the bank angle cannot be larger than 5°.

Figure 2-1 shows the engine-out geometry for a twin-engine configuration. The yawing moment coefficient required to maintain steady flight with an inoperative engine is given by:

$$C_{n_{req}} = \frac{(T + D_{ewm})l_e}{qS_{ref}b} \quad (2-2)$$

where T is the maximum available thrust at the given Mach number and altitude, and D_{ewm} is the drag due to the windmilling of the failed engine.

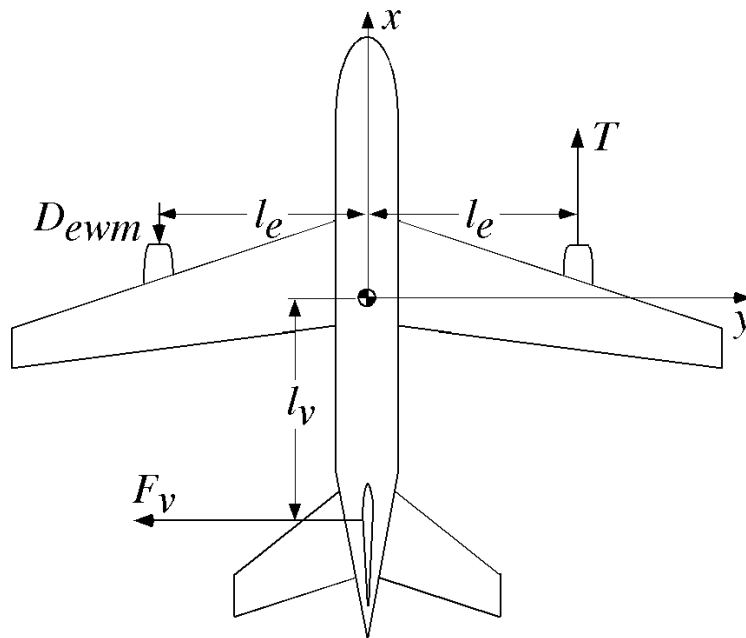


Figure 2-1: Engine-out geometry

The drag due to the windmilling of the failed engine is calculated using the method described in Appendix G-8 of Torenbeek [3].

$$D_{ewm} = qS_{ref}C_{D_{ewm}} \quad (2-3)$$

$$C_{D_{ewm}} = \frac{0.0785 d_i^2 + \frac{2}{1 + 0.16 M^2} \frac{1}{4} d_i^2 \frac{V_n}{V} \left(1 - \frac{V_n}{V}\right)}{S_{ref}} \quad (2-4)$$

where:

d_i is the engine inlet diameter

M is the Mach number

V_n is the nozzle exit velocity

$\frac{V_n}{V}$ 0.92 for high bypass ratio engines

S_{ref} is the wing reference area

Torenbeek's windmilling drag equation was validated against the flight test data of the 747. As shown in Figure 2-2, Torenbeek's equation shows relatively good agreement with the flight test data over a range of Mach numbers.

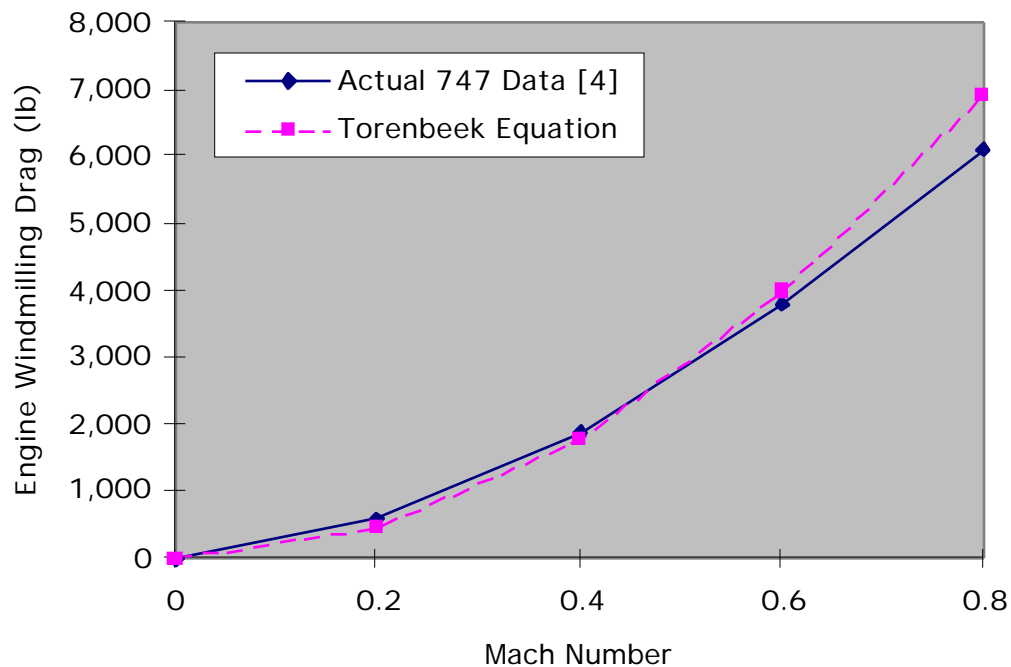


Figure 2-2: Engine windmilling drag validation

2.2. Maximum Available Yawing Moment Coefficient

The maximum available yawing moment coefficient is obtained at an equilibrium flight condition with a given bank angle () and a given maximum rudder deflection (δ_r). The bank angle is limited to a maximum of 5° by FAR 25.149, and the aircraft is allowed to have some sideslip (β).

The sideslip angle is found by summing the forces along the y-axis:

Sideforce Equation:

$$C_{y_a} a + C_{y_r} r + C_{y_\beta} \beta + C_L \sin \delta_r - \frac{T \sin \delta_T}{q S_{ref}} - C_{L_{cc}} \frac{S_{vtail}}{S_{ref}} = - \frac{Y_{ext}}{q S_{ref}} \quad (2-5)$$

In a conventional control system, the vertical tail is the dominant controller for generating a yawing moment. However, thrust vectoring and circulation control can be used to generate additional yawing moments. Since the engine-out condition is a critical constraint for a truss-braced wing with tip-mounted engines, the capability to model thrust vectoring and circulation control on the vertical tail was added to the code. The fifth term in the equation above ($\frac{T \sin \delta_T}{q S_{ref}}$) is due to the thrust being vectored at an angle δ_T to the centerline, and the sixth term ($C_{L_{cc}} \frac{S_{vtail}}{S_{ref}}$) is due to the change in C_L at the vertical tail due to circulation control. Since the external sideforce (Y_{ext}) is zero, and C_{y_a} is assumed to be zero, this equation can be simplified and solved for the sideslip angle:

$$\beta = \frac{- C_{y_r} r - C_L \sin \delta_r + \frac{T \sin \delta_T}{q S_{ref}} + C_{L_{cc}} \frac{S_{vtail}}{S_{ref}}}{C_{y_\beta}} \quad (2-6)$$

The aileron deflection required to maintain equilibrium flight is obtained by summing the rolling moments about the x-axis:

Rolling Moment Equation:

$$C_{l_a} a + C_{l_r} r + C_{l_\beta} \beta - \frac{T \sin \delta_T}{q S_{ref}} \frac{z_{tv}}{b} - C_{L_{cc}} \frac{S_{vtail}}{S_{ref}} \frac{z_{vtail}}{b} = - \frac{L_{ext}}{q S_{ref} b} \quad (2-7)$$

By setting the external rolling moment (L_{ext}) equal to zero, this equation can be solved for the aileron deflection:

$$a = \frac{- C_{l_r} r - C_{l_\beta} \beta + \frac{T \sin \delta_T}{q S_{ref}} \frac{z_{tv}}{b} + C_{L_{cc}} \frac{S_{vtail}}{S_{ref}} \frac{z_{vtail}}{b}}{C_{l_a}} \quad (2-8)$$

The rudder deflection is initially set to the given maximum allowable steady-state value, and the sideslip angle and aileron deflection for equilibrium flight are determined by Eqs. (2-6) and (2-8). The maximum allowable steady-state deflection is typically 20°-25°. This allows for an additional 5° of deflection for maneuvering. A warning statement is printed if the calculated deflection exceeds the maximum allowable deflection.

The maximum available yawing moment is found by summing the contributions due to the ailerons, rudder, and sideslip:

Yawing Moment Equation:

$$C_{n_{avail}} = C_{n_{a}} + C_{n_{r}} + C_{n_{\beta}} + \frac{T \sin \delta_T}{q S_{ref}} \frac{l_{ty}}{b} + C_{L_{cc}} \frac{S_{vtail}}{S_{ref}} \frac{l_{vtail}}{b} \quad (2-9)$$

This value of the available yawing moment coefficient is then constrained in the optimization problem to be greater than the required yawing moment coefficient, as shown in Eq. (2-1).

2.3. Why can't the vertical tail achieve its maximum lift coefficient?

The Output section shows the results of the above methodology for a 747 with no thrust vectoring and no circulation control. The maximum available yawing moment is achieved with a bank angle of 5° and a sideslip angle of 3°. This orientation would be used for a failure of the left engine. The pilot or automatic flight control system would roll the aircraft 5° in the direction of the operating engine and yaw slightly away from it. Note that in this flight condition, the vertical tail is only flying at an angle of attack of 3°, which is far below the angle of attack corresponding to the maximum lift coefficient of a typical vertical tail. One might expect that the maximum available yawing moment is obtained when the vertical tail is flying at its maximum lift coefficient, but this is not true because the equilibrium equations above must always be satisfied for steady flight. To illustrate this point, Eq. (2-5) has been solved for the bank angle with no thrust vectoring and no circulation control:

$$\delta_T = \sin^{-1} \left[- \frac{(C_{y_{r}} + C_{y_{\beta}})}{C_L} \right] \quad (2-10)$$

According to Reference [5], the angle of attack corresponding to the maximum lift coefficient for a NACA 66(215)-216 airfoil section with 15° of flap deflection is 15°. Therefore if the vertical tail in the 747 example mentioned above were flying at the maximum lift coefficient, the rudder deflection (δ_r) would be 15°, and the vertical tail angle of attack (α_{vtail}) would be at least 15° (3D effects would require an even larger angle).

If these values are plugged into Eq. (2-10) with a C_L of 1.11 and the 747 values for the stability and control derivatives (as given in Nelson [6]), the bank angle required to maintain equilibrium flight is 15.5° . Since this bank angle is much larger than the maximum allowable bank angle of 5° specified in FAR 25.149, the vertical tail cannot fly at the maximum lift coefficient and maintain equilibrium flight.

This brief analysis shows the need for circulation control or thrust vectoring. Since both of these mechanisms can generate a larger side force at the vertical tail without requiring a change in β , they can create a larger yawing moment coefficient at the same flight condition.

3. Stability and Control Derivative Estimation

The stability and control derivatives are estimated using the method of Roskam [1], which was adapted from the USAF Stability and Control DATCOM [2].

MacMillin [7] used a similar approach for the High-Speed Civil Transport. In MacMillin's work, however, the baseline stability and control derivatives were estimated using a vortex-lattice method, and the DATCOM method was only used to augment these baseline values with the effects due to changing the geometry of the vertical tail.

The Fortran source code for the stability subroutine is shown in the Appendix.

3.1. Angle of Sideslip Derivatives

3.1.1. Sideforce Coefficient

The variation of sideforce coefficient with sideslip angle has contributions from the wing, fuselage, and vertical tail. Note that all of the stability and control derivatives have units of rad^{-1} .

$$C_y = C_{y_{wing}} + C_{y_{fuse}} + C_{y_{vtail}} \quad (3-1)$$

The wing contribution is a function of the dihedral angle (in deg).

$$C_{y_{wing}} = -0.0001 \left| \beta \right| \frac{180}{\text{deg}} \quad (3-2)$$

The fuselage and nacelle contributions are estimated by:

$$C_{y_{fuse}} = -2K_{wbi} \frac{S_o}{S_{ref}} \quad (3-3)$$

where:

K_{wbi} is the wing-body interference factor, which is determined from a curve fit to Figure 7.1 in Roskam:

$$K_{wbi} = 0.85 \frac{-z_{wing}}{d_{fuse}/2} + 1 \quad \text{for } \frac{z_{wing}}{d_{fuse}/2} < 0 \quad (3-4)$$

$$K_{wbi} = 0.5 \frac{z_{wing}}{d_{fuse}/2} + 1 \quad \text{for } \frac{z_{wing}}{d_{fuse}/2} > 0 \quad (3-5)$$

and

$$S_o = \left(\frac{d_{fuse}}{2} \right)^2 + N_{engines} \left(\frac{d_{nacelle}}{2} \right)^2 \quad (3-6)$$

The contribution of a vertical tail in the plane of symmetry is found from:

$$C_{y_{vtail}} = -k_{C_{y_v}} C_{l_{vtail\,eff}} \left(1 + \frac{d}{d} \right) \frac{S_{vtail}}{S_{ref}} \quad (3-7)$$

where:

$k_{C_{y_v}}$ is determined from a curve fit to Figure 7.3 in Roskam:

$$k_{C_{y_v}} = 0.75 \quad \text{for } \frac{b_{vtail}}{d_{fuse\,vtail}} < 2 \quad (3-8)$$

$$k_{C_{y_v}} = \frac{1}{6} \frac{b_{vtail}}{d_{fuse\,vtail}} + \frac{5}{12} \quad \text{for } 2 < \frac{b_{vtail}}{d_{fuse\,vtail}} < 3.5 \quad (3-9)$$

$$k_{C_{y_v}} = 1 \quad \text{for } \frac{b_{vtail}}{d_{fuse\,vtail}} > 3.5 \quad (3-10)$$

$$C_{l_{vtail\,eff}} = \frac{2 A}{2 + \sqrt{\frac{A^2}{2} \frac{2}{M} \left(1 + \frac{\tan^2 c/2}{\frac{2}{M}} \right) + 4}} \quad (3-11)$$

$$= \frac{C_{l_{vtail}}}{2} \quad (3-12)$$

$C_{l_{vtail}}$ is assumed to have a value of 2 .

$$M = \sqrt{1 - M^2} \quad (3-13)$$

$$\left(1 + \frac{d}{d}\right)_v = 0.724 + 3.06 \frac{\left(\frac{S_{vtail}}{S_{ref}}\right)}{1 + \cos c/4} + 0.4 \frac{z_w}{d} + 0.009A \quad (3-14)$$

Note that the effective aspect ratio of the vertical tail must be used in place of A in Eqs. (3-11) and (3-14).

$$A_{vtail_{eff}} = \frac{A_{V(B)}}{A_V} A_{vtail} \left[1 + K_H \left(\frac{A_{V(HB)}}{A_{V(B)}} - 1 \right) \right] \quad (3-15)$$

where:

$\frac{A_{V(B)}}{A_V}$ is the ratio of the aspect ratio of the vertical tail in the presence of the body to that of the isolated panel, which is determined from the following curve fit to Figure 7.5 in Roskam, with the taper ratio assumed to be less than or equal to 0.6:

$$\frac{A_{V(B)}}{A_V} = 0.002 \left(\frac{b_{vtail}}{d_{fuse_{vtail}}} \right)^5 - 0.0464 \left(\frac{b_{vtail}}{d_{fuse_{vtail}}} \right)^4 + 0.404 \left(\frac{b_{vtail}}{d_{fuse_{vtail}}} \right)^3 - 1.6217 \left(\frac{b_{vtail}}{d_{fuse_{vtail}}} \right)^2 + 2.7519 \left(\frac{b_{vtail}}{d_{fuse_{vtail}}} \right) + 0.0408 \quad (3-16)$$

$\frac{A_{V(HB)}}{A_{V(B)}}$ is the ratio of the vertical tail aspect ratio in the presence of the horizontal tail and body to that of the tail in the presence of the body alone. It is assumed to have a value of 1.1, based on Figure 7.6 in Roskam. This is valid for the 747 and 777 tail geometries.

K_H is a factor accounting for the relative size of the horizontal and vertical tails, which is determined from the following curve fit to Figure 7.7 in Roskam:

$$K_H = -0.0328 \left(\frac{S_{htail}}{S_{vtail}} \right)^4 + 0.2885 \left(\frac{S_{htail}}{S_{vtail}} \right)^3 - 0.9888 \left(\frac{S_{htail}}{S_{vtail}} \right)^2 + 1.6554 \left(\frac{S_{htail}}{S_{vtail}} \right) - 0.0067 \quad (3-17)$$

3.1.2. Rolling Moment Coefficient

The variation of rolling moment coefficient with sideslip angle has contributions from the wing-body, horizontal tail, and vertical tail.

$$C_l = C_{l_{wb}} + C_{y_{htail}} + C_{y_{vtail}} \quad (3-18)$$

The contribution from the wing-body is estimated by:

$$C_{l_{wb}} = \left[C_L \left(\left(\frac{C_l}{C_L} \right)_{c/2} K_M K_f + \left(\frac{C_l}{C_L} \right)_A \right) + \left(\frac{C_l}{C_L} K_M + \frac{C_l}{C_L} \right) + \left(C_l \right)_{Z_w} \right] \frac{180}{c} \quad (3-19)$$

where:

$\left(\frac{C_l}{C_L} \right)_{c/2}$ is the wing sweep contribution, obtained from the following curve fit to Figure 7.11 in Roskam for $\alpha = 0.5$:

$$\left(\frac{C_l}{C_L} \right)_{c/2} = \frac{-0.004}{45} \frac{c/2}{180} \quad (3-20)$$

K_M is the compressibility correction to sweep, assumed to have a value of 1.0, based on Figure 7.12 in Roskam. This is valid for the 747 and 777 geometries at low Mach numbers.

K_f is the fuselage correction factor, assumed to have a value of 0.85, based on Figure 7.13 in Roskam. This is valid for the 747 and 777 geometries.

$\left(\frac{C_l}{C_L} \right)_A$ is the aspect ratio contribution, assumed to have a value of 0, based on Figure 7.14 in Roskam for $\alpha = 0.5$ and a high aspect ratio. This is valid for the 747 and 777 geometries.

$\frac{C_l}{C_L}$ is the wing dihedral effect, obtained from a curve fit to Figure 7.15 in Roskam for $\alpha = 0.5$, low sweep, and a high aspect ratio. Note that for extremely high aspect ratios, the curve fit is an extrapolation from the plot in Roskam.

K_M is the compressibility correction to dihedral, assumed to have a value of 1.0, based on Figure 7.16 in Roskam. This is valid for the 747 and 777 geometries at low Mach numbers.

$$\frac{C_l}{C_L} = -0.0005 \sqrt{A} \left(\frac{d}{b} \right)^2 \quad (3-21)$$

$$d = \sqrt{\frac{\left(\frac{d_{fuse}}{2}\right)^2}{0.7854}} \quad (3-22)$$

$$(C_l)_{z_w} = \frac{-1.2\sqrt{A} z_w 2d}{180 b b} \quad (3-23)$$

The contribution from the horizontal tail is approximately zero, since it has a small lift coefficient, small dihedral, and small area relative to the wing.

$$C_{l_{htail}} = 0 \quad (3-24)$$

The contribution from the vertical tail is estimated by:

$$C_{l_{vtail}} = C_{y_{vtail}} \frac{(z_{vtail} \cos - l_{vtail} \sin)}{b} \quad (3-25)$$

The fuselage angle of attack is the ratio of the lift coefficient to the lift curve slope minus the effective wing incidence angle. The effective wing incidence angle with 20° of flap deflection is approximately 5°.

$$= \frac{C_L}{C_L} - 5.0 \frac{1}{180} \quad (3-26)$$

The aircraft lift curve slope is calculated by:

$$C_L = C_{L_{wb}} + C_{L_{htail}} \frac{S_{htail}}{S_{ref}} \quad (3-27)$$

where:

$$C_{L_{wb}} = K_{wb} C_{L_w} \quad (3-28)$$

C_{L_w} and $C_{L_{htail}}$ are found using the following equation with the appropriate values of aspect ratio and sweep.

$$C_L = \frac{2 A}{2 + \sqrt{\frac{A^2}{2} \left(1 + \frac{\tan^2 c/2}{M}\right)} + 4} \quad (3-29)$$

$$M = \sqrt{1 - M^2} \quad (3-30)$$

The dynamic pressure ratio at the horizontal tail is assumed to be 0.95.

$$h_{tail} = 0.95 \quad (3-31)$$

3.1.3. Yawing Moment Coefficient

The variation of yawing moment coefficient with sideslip angle has contributions from the wing, fuselage, and vertical tail.

$$C_n = C_{n_{wing}} + C_{n_{fuse}} + C_{n_{vtail}} \quad (3-32)$$

The wing contribution to the yawing moment coefficient is negligible for small angles of attack.

$$C_{n_{wing}} \approx 0 \quad (3-33)$$

The fuselage contribution to the yawing moment coefficient is determined by:

$$C_{n_{fuse}} = -K_N K_{Rl} \frac{S_{bs}}{S_{ref}} \frac{l_{fuse}}{b} \frac{180}{b} \quad (3-34)$$

where:

K_N is an empirical factor for body and body + wing effects, assumed to have a value of 0.0011, based on Figure 7.19 in Roskam. This is valid for the 747 and 777 geometries.

K_{Rl} is a Reynolds number factor for the fuselage, obtained from a curve fit to Figure 7.20 in Roskam, based on the calculated fuselage Reynolds number.

The fuselage side area is approximated as 83% of the fuselage length times diameter. This is a good approximation for the 747 and 777 geometries.

$$S_{bs} = 0.83 l_{fuse} d_{fuse} \quad (3-35)$$

The contribution from the vertical tail is estimated by the following equation, where is defined in Eq. (3-26).

$$C_{n_{vtail}} = -C_{y_{vtail}} \frac{(l_{vtail} \cos \alpha + z_{vtail} \sin \alpha)}{b} \quad (3-36)$$

3.2. Lateral Control Derivatives

3.2.1. Sideforce Coefficient

The variation of sideforce coefficient with aileron deflection is assumed to be zero.

$$C_{y_a} = 0 \quad (3-37)$$

3.2.2. Rolling Moment Coefficient

The first step in the estimation of the rolling moment coefficient is to estimate the rolling moment effectiveness parameter (C_l') from Figure 11.1 in Roskam. For 747 and 777-like configurations with $\lambda = 0.5$ and $M = 0.25$, it is approximately 0.18.

The rolling effectiveness of two full-chord controls is estimated by:

$$C_l' = \frac{C_{l_{\text{actual}}}}{M} \quad (3-38)$$

where the section lift curve slope is assumed to be $2 / M$, and λ is the ratio of the actual section lift curve slope to $2 / M$.

The aileron lift effectiveness is estimated from Roskam's Figures 10.5 and 10.6 with $c_f/c = 0.20$ and $t/c = 0.08$. These assumptions result in a value of 3.5 from Figure 10.5, and a value of 1.0 from Figure 10.6. The aileron effectiveness is given by:

$$C_l = \left(\frac{C_l}{C_{l_{\text{Theory}}}} \right) C_{l_{\text{Theory}}} \quad (3-39)$$

$$= \frac{C_l}{C_l} \quad (3-40)$$

The rolling effectiveness of the partial-chord controls is estimated by:

$$C_l = C_l \quad (3-41)$$

The λ in the equation above refers to the sum of the left and right aileron deflections. Since we define the aileron deflection (δ_a) as one half of the sum of the deflections, the variation of rolling moment coefficient with aileron deflection is given by:

$$C_{l_a} = \frac{C_l}{2} \quad (3-42)$$

3.2.3. Yawing Moment Coefficient

The variation of yawing moment coefficient with aileron deflection is given by:

$$C_{n_a} = KC_L C_{l_a} \quad (3-43)$$

where K is estimated from Figure 11.3 in Roskam with $\delta = 0.5$, $A = 8$, and $\delta_i = 0.74$.

3.3. Directional Control Derivatives

3.3.1. Sideforce Coefficient

The variation of sideforce coefficient with rudder deflection is given by:

$$C_{y_r} = C_{l_{vtail}} \left(\frac{C_L}{C_l} \right) K' K_b \frac{S_{vtail}}{S_{ref}} \quad (3-44)$$

where:

$\left(\frac{C_L}{C_l} \right)$ is the ratio of the 3D flap-effectiveness parameter to the 2D flap-effectiveness parameter. It is estimated with a piecewise curve fit to Figure 10.2 in Roskam with an assumed value of $c_f/c = 0.33$.

K_b is the flap span factor, which is estimated to be 0.95 from Figure 10.3 in Roskam with $\delta = 0.85$.

K' is an empirical correction factor for large control deflections. It is estimated with a curve fit to Figure 10.7 in Roskam with $c_f/c = 0.3$.

3.3.2. Rolling Moment Coefficient

The variation of rolling moment coefficient with rudder deflection is given by:

$$C_{l_r} = C_{y_r} \left(\frac{z_{vtail} \cos \delta - l_{vtail} \sin \delta}{b} \right) \quad (3-45)$$

3.3.3. Yawing Moment Coefficient

The variation of yawing moment coefficient with rudder deflection is given by:

$$C_{n_r} = -C_{y_r} \left(\frac{l_{vtail} \cos \delta + z_{vtail} \sin \delta}{b} \right) \quad (3-46)$$

4. Validation

4.1. Boeing 747-100

The stability and control derivatives were validated with the 747-100. Table 4-1 shows a comparison of the predicted stability and control derivatives with the flight test derivatives presented in Nelson [6]. Note that the sign differences in the last three values are due to a different sign convention for the rudder deflection.

Table 4-1: Comparison of stability and control derivatives for 747-100

<u>Derivative</u>	<u>Flight Test</u>	<u>Prediction</u>	<u>Error</u>
C_y	-0.96	-0.6824	0.2776
C_l	-0.221	-0.2988	0.0778
C_n	0.150	0.0562	0.0938
C_{l_a}	0.0461	0.0501	0.0040
C_{n_a}	0.0064	0.0070	0.0006
C_{y_r}	0.175	-0.2854	0.1104
C_{l_r}	0.007	-0.0185	0.0115
C_{n_r}	-0.109	0.1496	0.0406

A correction factor was applied to each of the derivatives to increase their accuracy. Each correction factor shown in Table 4-2 is the ratio of the actual value to the predicted value for the 747-100 for the $M = 0.25$ flight condition given in NASA CR-2144 [8]. These correction factors may have to be recalibrated if the configuration is significantly different from the 747.

Table 4-2: Stability and control derivative correction factors

<u>Derivative</u>	<u>Correction Factor</u>
C_y	1.4068
C_l	0.7396
C_n	2.6690
C_{l_a}	0.9202
C_{n_a}	0.9143
C_{y_r}	0.6132
C_{l_r}	0.3784
C_{n_r}	0.7286

5. Input

The following listing is a sample input file for the Boeing 747-100. The input variables are given in the Appendix. This set of inputs was used to create the correction factors shown in the Validation section.

```

input file for stab
boeing747
1
7.0      dihedral_wing (deg)
6.2      z_wing (ft)
23.0     dia_fuse (ft)
5500.    sref (ft^2)
33.5     hspan_vtail (ft)
14.4     depth_fuse_vtail (ft)
36.4     c_vtail_root (ft)
11.5     c_vtail_tip (ft)
0.25     mach_eo
45.      sweep_vtail_1_4 (deg)
33.5     sweep_wing_1_2 (deg)
97.8     hspan_wing (ft)
36.4     hspan_htail (ft)
31.16    sweep_htail_1_2 (deg)
1.11     cl
26.      z_vtail (ft)
100.     l_vtail (ft)
225.2    length_fuse (ft)
4        new
0        nef
8.4      dia_nacelle (ft)
1467.    sh (ft^2)
2.3769e-3 rho_eo (slug/ft^3)
1116.4   a_eo (ft/s)
3.7372e-7 mu_eo (slug/(ft-s))
15.      dr_max (deg)
25.      da_max (deg)
0.       thrust_tv (lb)
0.       angle_tv (deg)
122.     l_tv (ft)
7.       z_tv (ft)
0.0     cl_circ_ctrl

```

6. Output

The following listing is the output file for the Boeing 747-100. The definitions of the variables are given in the Appendix. Note that the stability and control derivatives in this file represent the corrected values for the calibration case shown above.

stab output file
boeing747

Input

```

1 = write_flag
7.0000 = dihedral_wing (deg)
6.2000 = z_wing (ft)
23.0000 = dia_fuse (ft)
5500.0000 = sref (ft^2)
33.5000 = hspan_vtail (ft)
14.4000 = depth_fuse_vtail (ft)
36.4000 = c_vtail_root (ft)
11.5000 = c_vtail_tip (ft)
0.2500 = mach_eo
45.0000 = sweep_vtail_1_4 (deg)
33.5000 = sweep_wing_1_2 (deg)
97.8000 = hspan_wing (ft)
36.4000 = hspan_htail (ft)
31.1600 = sweep_htail_1_2 (deg)
1.1100 = cl
26.0000 = z_vtail (ft)
100.0000 = l_vtail (ft)
225.2000 = length_fuse (ft)
4 = new
0 = nef
8.4000 = dia_nacelle (ft)
1467.0000 = sh (ft^2)
0.0024 = rho_eo (slug/ft^3)
1116.4000 = a_eo (ft/s)
0.3737E-06 = mu_eo (slug/(ft-s))
15.0000 = dr_max (deg)
25.0000 = da_max (deg)
0.0000 = thrust_tv (lb)
0.0000 = angle_tv (deg)
122.0000 = l_tv (ft)
7.0000 = z_tv (ft)
0.0000 = cl_circ_ctrl

```

Output

```

-0.9601 = cy_beta (rad-1)
-0.2210 = cl_beta (rad-1)
0.1500 = cn_beta (rad-1)

0.0000 = cy_da (rad-1)
0.0461 = cl_da (rad-1)
0.0064 = cn_da (rad-1)

-0.1750 = cy_dr (rad-1)
-0.0070 = cl_dr (rad-1)
0.1090 = cn_dr (rad-1)

3.0396 = beta (deg)
5.0000 = phi (deg)
16.8350 = da (deg)
15.0000 = dr (deg)
2.3776 = ar_vtail_eff
0.0384 = cn_avail

```

References

- [1] Roskam, J., *Methods for Estimating Stability and Control Derivatives of Conventional Subsonic Airplanes*, Roskam Aviation and Engineering Corporation, Lawrence, Kansas, 1971.
- [2] Hoak, D.E. et al., *USAF Stability and Control DATCOM*, Flight Control Division, Air Force Flight Dynamics Laboratory, WPAFB, Ohio, 1978.
- [3] Torenbeek, E., *Synthesis of Subsonic Airplane Design*, Delft Univ. Press, Delft, The Netherlands, 1982.
- [4] Hanke, C.R., "The Simulation of a Large Jet Transport Aircraft, Vol. I: Mathematical Model," NASA CR-1756, March 1971.
- [5] Abbott, I.H., and von Doenhoff, A.E., *Theory of Wing Sections*, Dover, New York, 1959.
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- [7] MacMillin, P.E., Golovidov, O.B., Mason, W.H., Grossman, B., and Haftka, R.T., *Trim, Control, and Performance Effects in Variable-Complexity High-Speed Civil Transport Design*, MAD 96-07-01, July, 1996.
- [8] Heffley, R.K., and Jewell, W.F., *Aircraft Handling Qualities Data*, NASA CR-2144, December, 1972.

Appendix: Code Listing for Stability Subroutine (stab.f)

```

c////////////////////////////////////
c
c  subroutine stab
c
c    This subroutine calculates the maximum available yawing moment
c    coefficient of a given aircraft configuration at a given flight
c    condition. Note that right rudder deflection is defined as
c    positive, and right aileron up, left aileron down is defined as
c    positive. Both of these control deflections generate positive
c    moments about their respective axes. This is the convention used
c    by Roskam. The thrust vectoring angle (angle_tv) is also defined
c    as positive for a right deflection.
c
c  Inputs
c
c  outfile          output filename
c  title            title of aircraft configuration
c  write_flag       write flag (0 = no output file, 1 = output file written)
c  dihedral_wing    wing dihedral angle (deg)
c  z_wing           distance from body centerline to quarter-chord point of
c                  exposed wing root chord, positive for the quarter-chord
c                  point below the body centerline (ft)
c  dia_fuse         fuselage diameter (ft)
c  sref             wing reference area (ft^2)
c  hspan_vtail     vertical tail span (ft)
c  depth_fuse_vtail fuselage depth at the fuselage station of the
c                  quarter-chord of the vertical tail (ft)
c  c_vtail_root    root chord of vertical tail
c  c_vtail_tip     tip chord of vertical tail
c  mach_eo         mach number
c  sweep_vtail_1_4_deg vertical tail quarter-chord sweep angle (deg)
c  sweep_wing_1_2_deg average wing half-chord sweep angle (deg)
c  hspan_wing      wing half-span (ft)
c  hspan_htail    horizontal tail half-span (ft)
c  sweep_htail_1_2_deg horizontal tail half-chord sweep angle (deg)
c  cl              lift coefficient
c  z_vtail        vertical distance from CG to AC of vertical tail (ft)
c  l_vtail        horizontal distance from CG to AC of vertical tail (ft)
c  length_fuse    fuselage length (ft)
c  new            number of engines on the wing
c  nef           number of engines on the fuselage
c  dia_nacelle   nacelle diameter (ft)
c  rho_eo        density at engine-out flight condition (slug/ft^3)
c  a_eo          speed of sound at engine-out flight condition (ft/s)
c  mu_eo         viscosity at engine-out flight condition (slug/(ft-s))
c  dr_max       maximum allowable steady-state rudder deflection (deg)
c  da_max       maximum allowable steady-state aileron deflection (deg)
c  thrust_tv    maximum available thrust of the aft engine (lb)
c  angle_tv     horizontal angle between the fuselage centerline and the
c              effective thrust vector (deg, positive to the right)
c  l_tv        horizontal distance between CG and thrust vectoring
c              nozzle (ft)

```



```

c  z_tv          vertical distance between CG and thrust vectoring
c                nozzle (ft)
c  cl_circ_ctrl  change in lift coefficient due to circulation control
c                (nondimensionalized by q and the vertical tail area)
c
c  Outputs
c
c  ar_vtail_eff  effective aspect ratio of vertical tail
c  cn_avail      maximum available yawing moment coefficient
c
c  Internal Variables
c
c  alpha         angle of attack (rad)
c  alpha_d       section lift effectiveness
c  alpha_d_cl    section flap effectiveness (from Figure 10.2)
c  ar            wing aspect ratio
c  ar_vtail      actual aspect ratio of vertical tail
c  ar_htail      actual aspect ratio of horizontal tail
c  avb_av        ratio of the aspect ratio of the vertical panel in the
c                presence of the body to that of the isolated panel
c                (from Figure 7.5)
c  avhb_avb     ratio of the vertical panel aspect ratio in the
c                presence of the horizontal tail and body to that of
c                the panel in the presence of the body alone (from
c                Figure 7.6)
c  bcld_kappa    rolling moment effectiveness parameter (from Figure
c                11.1)
c  beta         sideslip angle, positive from the right (rad)
c  beta_m        square root of (1 - mach_eo)**2
c  cf_c         ratio of flap chord to wing or tail chord
c  cf_factor     flap chord factor (from Figure 10.2)
c  cl_alpha      lift-curve slope of entire aircraft (rad^-1)
c  cl_alpha_2d   2-dimensional lift-curve slope at MAC (rad^-1)
c  cl_alpha_h    lift-curve slope of horizontal tail (rad^-1)
c  cl_alpha_vtail original lift-curve slope of vertical tail (rad^-1)
c  cl_alpha_vtail_eff effective lift-curve slope of vertical tail (rad^-1)
c  cl_alpha_w    lift-curve slope of wing (rad^-1)
c  cl_alpha_wb   lift-curve slope of wing-body combination (rad^-1)
c  cl_beta       variation of rolling moment coefficient with sideslip
c                angle
c  cl_beta_cor   corrected value of cl_beta
c  cl_beta_htail horizontal tail contribution to cl_beta
c  cl_beta_vtail vertical tail contribution to cl_beta
c  cl_beta_wingbody wing-body contribution to cl_beta
c  cl_d         rolling effectiveness of partial-chord controls
c  cl_da        variation of rolling moment coefficient with aileron
c                deflection
c  cl_da_cor    corrected value of cl_da
c  cl_dr        variation of rolling moment coefficient with rudder
c                deflection
c  cl_dr_cor    corrected value of cl_dr
c  clb_cl_a     aspect ratio contribution to cl_beta_wingbody (from
c                Figure 7.14)
c  clb_cl_lambda wing sweep contribution to cl_beta_wingbody (from
c                Figure 7.11)
c  clb_gamma    dihedral effect on cl_beta (from Figure 7.15)
c  cld_prime    rolling effectiveness of two full-chord ailerons
c                (Equation 11.2)
c  cld_ratio    empirical correction for plain TE flaps (Fig. 10.6)
c  cld_theory   theoretical lift effectiveness of plain TE flaps
c                (Fig. 10.5)

```

```

c  cn_beta          variation of yawing moment coefficient with sideslip
c                    angle
c  cn_beta_cor      corrected value of cn_beta
c  cn_beta_fuse     fuselage contribution to cn_beta
c  cn_beta_vtail    vertical tail contribution to cn_beta
c  cn_beta_wing     wing contribution to cn_beta
c  cn_da            variation of yawing moment coefficient with aileron
c                    deflection
c  cn_da_cor        corrected value of cn_da
c  cn_dr            variation of yawing moment coefficient with rudder
c                    deflection
c  cn_dr_cor        corrected value of cn_dr
c  cy_beta          variation of side force coefficient with sideslip angle
c  cy_beta_cor      corrected value of cy_beta
c  cy_beta_fuse     fuselage contribution to cy_beta
c  cy_beta_vtail    vertical tail contribution to cy_beta
c  cy_beta_wing     wing contribution to cy_beta
c  cy_da            variation of side force coefficient with aileron
c                    deflection
c  cy_dr            variation of side force coefficient with rudder
c                    deflection
c  cy_dr_cor        corrected value of cy_dr
c  d                d in Equation 7.10 (estimated from Equation 7.11)
c  da              aileron deflection, positive for right aileron up, left
c                    aileron down (rad)
c  dclb_gamma       body-induced effect on wing height (from Equation 7.10)
c  dclb_zw          another body-induced effect on wing height (from
c                    Equation 7.12)
c  debug_flag       printing flag for debugging output (0 = no debugging
c                    info printed, 1 = debugging info printed)
c  dr              rudder deflection, positive for right deflection (rad)
c  eff_vtail        vertical tail effectiveness factor estimated by
c                    Equation 7.5
c  eta_h           dynamic pressure ratio at the horizontal tail
c  f_cy_beta        correction factor for cy_beta
c  f_cl_beta        correction factor for cl_beta
c  f_cn_beta        correction factor for cn_beta
c  f_cl_da          correction factor for cl_da
c  f_cn_da          correction factor for cn_da
c  f_cy_dr          correction factor for cy_dr
c  f_cl_dr          correction factor for cl_dr
c  f_cn_dr          correction factor for cn_dr
c  flap_eff_ratio   flap effectiveness ratio (from Figure 10.2)
c  i               index
c  k               empirical factor for estimating the variation of yawing
c                    moment coefficient with aileron deflection
c  k_b             span factor for plain flap (from Figure 10.3)
c  k_cy_beta_v     empirical factor from Figure 7.3
c  k_f             fuselage correction factor (from Figure 7.13)
c  k_h            factor accounting for relative size of horizontal and
c                    vertical tails (from Figure 7.7)
c  k_m_lambda      compressibility correction to wing sweep (from Figure
c                    7.12)
c  k_m_gamma       compressibility correction to dihedral effect (from
c                    Figure 7.16)
c  k_n            factor for body and body + wing effects (from Figure
c                    7.19)
c  k_prime         empirical correction for lift effectiveness of plain
c                    flaps at high flap deflections (from Figure 10.7)
c  k_r_l          Reynold's number factor for the fuselage (from Figure
c                    7.20)

```

```

c k_wbi          wing-body interference factor from Figure 7.1
c k_wb          factor for loss in lift curve due to body
c kappa         ratio of the actual lift-curve slope to 2*pi
c phi          bank angle, positive to the right (rad)
c q            dynamic pressure (lb/ft^2)
c re_fuse      fuselage Reynolds number
c sbs         body side area (ft^2)
c sh          area of horizontal tail (ft^2)
c sv          area of vertical tail (ft^2)
c sweep_htail_1_2 horizontal tail half-chord sweep angle (rad)
c sweep_vtail_1_2 vertical tail half-chord sweep angle (rad)
c sweep_vtail_1_4 vertical tail half-chord sweep angle (rad)
c sweep_wing_1_2 average wing half-chord sweep angle (rad)
c x           temporary variable for curve fits
c
c Created by:  Joel Grasmeyer
c Last Modified: 03/01/98
c
c/////////////////////////////////////////////////////////////////

      subroutine stab(outfile,title,write_flag,dihedral_wing,z_wing,
& dia_fuse,sref,hspan_vtail,depth_fuse_vtail,c_vtail_root,
& c_vtail_tip,mach_eo,sweep_vtail_1_4_deg,sweep_wing_1_2_deg,
& hspan_wing,hspan_htail,sweep_htail_1_2_deg,cl,z_vtail,l_vtail,
& length_fuse,new,nef,dia_nacelle,sh,rho_eo,a_eo,mu_eo,dr_max,
& da_max,thrust_tv,angle_tv,l_tv,z_tv,cl_circ_ctrl,ar_vtail_eff,
& cn_avail)

      implicit none

      character*72 outfile, title
      integer i, write_flag, unit_out, new, nef, debug_flag
      real pi, dihedral_wing, z_wing, dia_fuse, sref, hspan_vtail, ar,
& depth_fuse_vtail, c_vtail_root, c_vtail_tip, mach_eo, sv, sh,
& sweep_wing_1_2, hspan_wing, cy_beta, ar_vtail, k, thrust_tv,
& cy_beta_wing, cy_beta_fuse, cy_beta_vtail, ar_vtail_eff, alpha,
& cl_beta, cl_beta_htail, cl_beta_vtail, cl_beta_wingbody, sbs,
& cl, k_wbi, avb_av, avhb_avb, k_h, clb_cl_lambda, cn_da, l_tv,
& k_m_lambda, k_f, clb_cl_a, clb_gamma, k_m_gamma, dclb_gamma,
& d, dclb_zw, cl_alpha, z_vtail, l_vtail, cn_beta_fuse, da,
& cn_beta_vtail, cn_beta, cn_beta_wing, k_n, k_r_l, phi, angle_tv,
& length_fuse, re_fuse, cl_da, cy_da, bcl_d_kappa, cld_prime, cl_d,
& alpha_d, cld_theory, cld_ratio, cl_alpha_2d, cl_dr, dr, cn_dr,
& cy_dr, cf_factor, k_prime, alpha_d_cl, flap_eff_ratio, k_b,
& cf_c, beta, rho_eo, a_eo, mu_eo, cn_avail, k_cy_beta_v, da_max,
& dia_nacelle, dr_max, f_cy_beta, f_cl_beta, z_tv, cl_circ_ctrl,
& f_cn_beta, f_cl_da, f_cn_da, f_cy_dr, f_cl_dr, f_cn_dr, x,
& sweep_vtail_1_4, sweep_vtail_1_2, sweep_wing_1_2_deg, k_wb,
& sweep_vtail_1_4_deg, cy_beta_cor, cl_beta_cor, cn_beta_cor,
& cl_da_cor, cn_da_cor, cy_dr_cor, cl_dr_cor, cn_dr_cor,
& hspan_htail, sweep_htail_1_2_deg, sweep_htail_1_2, eta_h,
& ar_htail, cl_alpha_h, cl_alpha_w, cl_alpha_wb

      pi = acos(-1.)

c Initialize value of debug_flag
      debug_flag = 0

c Convert sweep angles from degrees to radians
      sweep_wing_1_2 = sweep_wing_1_2_deg*pi/180.

```

```

sweep_htail_1_2 = sweep_htail_1_2_deg*pi/180.
sweep_vtail_1_4 = sweep_vtail_1_4_deg*pi/180.

c Append extension to idrag output filename
i = 1
do while (outfile(i:i) .ne. '.')
  i = i + 1
end do
outfile(i+1:i+5) = 'stab'
outfile(i+6:) = ''

c Write input data to output file for confirmation
if (write_flag .eq. 1) then
  unit_out = 171
  open(unit_out,file=outfile)
  write(unit_out, "('stab output file')")
  write(unit_out, "(a72)") title
  write(unit_out, *)
  write(unit_out, "(a5)") 'Input'
  write(unit_out, *)
  write(unit_out,101) write_flag, '= write_flag'
  write(unit_out,100) dihedral_wing, '= dihedral_wing (deg)'
  write(unit_out,100) z_wing, '= z_wing (ft)'
  write(unit_out,100) dia_fuse, '= dia_fuse (ft)'
  write(unit_out,100) sref, '= sref (ft^2)'
  write(unit_out,100) hspan_vtail, '= hspan_vtail (ft)'
  write(unit_out,100) depth_fuse_vtail, '= depth_fuse_vtail (ft)'
  write(unit_out,100) c_vtail_root, '= c_vtail_root (ft)'
  write(unit_out,100) c_vtail_tip, '= c_vtail_tip (ft)'
  write(unit_out,100) mach_eo, '= mach_eo'
  write(unit_out,100) sweep_vtail_1_4*180./pi,
& ' = sweep_vtail_1_4 (deg)'
  write(unit_out,100) sweep_wing_1_2*180./pi,
& ' = sweep_wing_1_2 (deg)'
  write(unit_out,100) hspan_wing, '= hspan_wing (ft)'
  write(unit_out,100) hspan_htail, '= hspan_htail (ft)'
  write(unit_out,100) sweep_htail_1_2*180./pi,
& ' = sweep_htail_1_2 (deg)'
  write(unit_out,100) cl, '= cl'
  write(unit_out,100) z_vtail, '= z_vtail (ft)'
  write(unit_out,100) l_vtail, '= l_vtail (ft)'
  write(unit_out,100) length_fuse, '= length_fuse (ft)'
  write(unit_out,101) new, '= new'
  write(unit_out,101) nef, '= nef'
  write(unit_out,100) dia_nacelle, '= dia_nacelle (ft)'
  write(unit_out,100) sh, '= sh (ft^2)'
  write(unit_out,100) rho_eo, '= rho_eo (slug/ft^3)'
  write(unit_out,100) a_eo, '= a_eo (ft/s)'
  write(unit_out,103) mu_eo, '= mu_eo (slug/(ft-s))'
  write(unit_out,100) dr_max, '= dr_max (deg)'
  write(unit_out,100) da_max, '= da_max (deg)'
  write(unit_out,100) thrust_tv, '= thrust_tv (lb)'
  write(unit_out,100) angle_tv, '= angle_tv (deg)'
  write(unit_out,100) l_tv, '= l_tv (ft)'
  write(unit_out,100) z_tv, '= z_tv (ft)'
  write(unit_out,100) cl_circ_ctrl, '= cl_circ_ctrl'
end if

c Calculate stability and control derivatives via Roskam's methods

c Sideslip angle derivatives

```

```

    cy_beta_wing = -0.0001*abs(dihedral_wing)*180./pi

c Estimate k_wbi from Figure 7.1 (curve fit)
  if (z_wing/(dia_fuse/2.) .le. 0.) then
    k_wbi = 0.85*(-z_wing/(dia_fuse/2.)) + 1.
  elseif (z_wing/(dia_fuse/2.) .gt. 0.) then
    k_wbi = 0.5*z_wing/(dia_fuse/2.) + 1.
  end if

c Estimate the side force coefficient due to the fuselage and nacelles
  cy_beta_fuse = -2.*k_wbi*( pi*(dia_fuse/2.)**2 +
&                (new + nef)*pi*(dia_nacelle/2.)**2 )/sref

c Estimate k_cy_beta_v from Figure 7.3 (curve fit)
  x = hspan_vtail/depth_fuse_vtail
  if (x .le. 2.) then
    k_cy_beta_v = 0.75
  elseif (x .gt. 2. .and. x .lt. 3.5) then
    k_cy_beta_v = x/6. + 5./12.
  elseif (x .ge. 3.5) then
    k_cy_beta_v = 1.
  end if

c Estimate avb_av from Figure 7.5 (curve fit for taper ratio <= 0.6)
  x = hspan_vtail/depth_fuse_vtail
  avb_av = 0.002*x**5 - 0.0464*x**4 + 0.404*x**3 - 1.6217*x**2 +
&        2.7519*x + 0.0408

c Factor from Figure 7.6 is for zh/bv = 0.
  avhb_avb = 1.1

c Estimate k_h from Figure 7.7 (curve fit)
  sv      = hspan_vtail*(c_vtail_root + c_vtail_tip)/2.
  x       = sh/sv
  k_h     = -0.0328*x**4 + 0.2885*x**3 - 0.9888*x**2 + 1.6554*x -
&        0.0067

c Estimate the effective aspect ratio for the vertical tail
  ar_vtail      = hspan_vtail**2/sv
  ar_vtail_eff  = avb_av*ar_vtail*(1. + k_h*(avhb_avb - 1.))

c Assume the section lift-curve slope is 2.*pi
  cl_alpha_vtail = 2.*pi

c Estimate the effective lift-curve slope for the vertical tail
  kappa      = cl_alpha_vtail/(2.*pi)
  beta_m     = sqrt( 1. - mach_eo**2 )
  sweep_vtail_1_2 = atan( (c_vtail_root/4. + hspan_vtail*
&                        tan(sweep_vtail_1_4) + c_vtail_tip/4. -
&                        c_vtail_root/2.)/hspan_vtail )
  cl_alpha_vtail_eff = 2.*pi*ar_vtail_eff/( 2. +
&      sqrt( ar_vtail_eff**2*beta_m**2/kappa**2*
&      ( 1. + tan(sweep_vtail_1_2)**2/
&      beta_m**2 ) + 4. ) )

c Estimate the third term in eqn. 7.4 from eqn. 7.5
  eff_vtail      = 0.724 + 3.06*sv/sref/(1. +
&      cos(sweep_vtail_1_4)) + 0.4*z_wing/dia_fuse +
&      0.009*ar_vtail_eff
  cy_beta_vtail = -k_cy_beta_v*cl_alpha_vtail_eff*eff_vtail*sv/sref

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```

c Calculate total variation of side force coefficient with sideslip angle
  cy_beta = cy_beta_wing + cy_beta_fuse + cy_beta_vtail

c Factor from Figure 7.11 is approximated by a curve fit for lambda = 0.5
  clb_cl_lambda = -0.004/45*sweep_wing_1_2*180./pi

c Factor from Figure 7.12 is approximated for 747 and 777 configurations
c at low Mach numbers
  k_m_lambda = 1.0

c Factor from Figure 7.13 is approximated for 747 and 777 configurations
  k_f = 0.85

c Factor from Figure 7.14 is approximated for lambda = 0.5 and high AR
  clb_cl_a = 0.000

c Factor from Figure 7.15 is approximated by a linear curve fit for
c lambda equal to 0.5, low sweep, and high AR
  ar      = (2.*hspan_wing)**2/sref
  clb_gamma = -0.00012 - 0.00013/10*ar

c Factor from Figure 7.16 is approximated for 747 and 777 configurations
c at low Mach numbers
  k_m_gamma = 1.0

c Estimate body-induced effect on wing height from eqns. 7.10, 7.11, and 7.12
  d      = sqrt(pi*(dia_fuse/2.)**2/0.7854)
  dclb_gamma = -0.0005*sqrt(ar)*(d/(2.*hspan_wing))**2
  dclb_zw   = -1.2*sqrt(ar)/(180./pi)*z_wing/(2.*hspan_wing)*
&          2.*d/(2.*hspan_wing)

c Wing-body contribution to cl_beta (wing twist effect is neglected)
  cl_beta_wingbody = ( cl*(clb_cl_lambda*k_m_lambda*k_f +
& clb_cl_a) + dihedral_wing*(clb_gamma*k_m_gamma + dclb_gamma) +
& dclb_zw )*180./pi

c Since the horizontal tail has a small lift coefficient, small dihedral,
c and small area relative to the wing, it is negligible.
  cl_beta_htail = 0.

c Calculate the lift curve loss factor due to the fuselage
  x = dia_fuse/(2.*hspan_wing)
  k_wb = 1 - 0.25*x**2 + 0.025*x

c Assume the 2D lift-curve slope is 2*pi/beta_m
  cl_alpha_2d = 2*pi/beta_m
  kappa      = cl_alpha_2d/(2.*pi/beta_m)

c Calculate the lift curve slope of the wing alone and wing-body combination
  cl_alpha_w = 2.*pi*ar/( 2. + sqrt( ar**2*beta_m**2/kappa**2*
& ( 1. + tan(sweep_wing_1_2)**2/beta_m**2 ) + 4. ) )
  cl_alpha_wb = k_wb*cl_alpha_w

c Calculate the lift curve slope of the horizontal tail
  ar_htail = (2.*hspan_htail)**2/sh
  cl_alpha_h = 2.*pi*ar_htail/( 2. + sqrt( ar_htail**2*beta_m**2/
& kappa**2*( 1. + tan(sweep_htail_1_2)**2/beta_m**2 )
& + 4. ) )

c Assume the dynamic pressure ratio at the horizontal tail is 0.95
  eta_h = 0.95

```

```

c Calculate the lift curve slope of the total aircraft
  cl_alpha = cl_alpha_wb + cl_alpha_h*eta_h*sh/sref

c Calculate the angle of attack of the fuselage centerline. The wing
c incidence angle is assumed to be 5 deg.
  alpha = cl/cl_alpha - 5.*pi/180.

c Estimate the vertical tail contribution to cl_beta
  cl_beta_vtail = cy_beta_vtail*( z_vtail*cos(alpha) - l_vtail*
&                               sin(alpha) )/(2.*hspan_wing)

c Calculate total variation of rolling moment coefficient with sideslip angle
  cl_beta = cl_beta_wingbody + cl_beta_htail + cl_beta_vtail

c Wing contribution to cn_beta is negligible for small angles of attack.
  cn_beta_wing = 0.

c Estimate empirical factor for body and body + wing effects from Figure 7.19
c Constant value assumed for 747 and 777-like configurations
  k_n = 0.0011

c Calculate fuselage Reynolds number at the engine-out flight condition
  re_fuse = rho_eo*mach_eo*a_eo*length_fuse/mu_eo

c Estimate fuselage Reynolds number effect on wing-body from Figure 7.20
  k_r_l = 1. + 1.2/log(350.)*log(re_fuse/1000000.)

c Estimate fuselage contribution to cn_beta
  sbs = 0.83*dia_fuse*length_fuse
  cn_beta_fuse = -180./pi*k_n*k_r_l*sbs/sref*
&               length_fuse/(2.*hspan_wing)

c Estimate vertical tail contribution to cn_beta
  cn_beta_vtail = -cy_beta_vtail*( l_vtail*cos(alpha) +
&                               z_vtail*sin(alpha) )/(2.*hspan_wing)

c Calculate total variation of yawing moment coefficient with sideslip angle
  cn_beta = cn_beta_wing + cn_beta_fuse + cn_beta_vtail

c Assume variation of sideforce coefficient with aileron deflection is zero
  cy_da = 0.

c Estimate the rolling moment effectiveness parameter from Figure 11.1
c for lambda = 0.5, and for 747 and 777-like ailerons at mach 0.25
  bcl_d_kappa = 0.18

c Estimate the rolling effectiveness of two full-chord controls by Eqn. 11.2
  cld_prime = kappa/beta_m*bcl_d_kappa

c Estimate aileron effectiveness by assuming cf/c = 0.20 and t/c = 0.08
  cld_theory = 3.5
  cld_ratio = 1.0
  cl_d = cld_ratio*cld_theory
  alpha_d = cl_d/cl_alpha_2d

c Determine the rolling effectiveness of the partial-chord controls by
c Eqn. 11.3. Note that this is the change in cl with respect to a change
c in the sum of the left and right aileron deflections (d).
  cl_d = alpha_d*cld_prime

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c Estimate variation of rolling moment coefficient with aileron deflection
c by neglecting differential control effects. Since the aileron deflection
c (da) is defined as half of the sum of the left and right deflections, cl_d
c from the equation above must be divided by 2.
  cl_da = cl_d/2.

c The method in Roskam for estimating cn_da does not account for the
c effect of differential ailerons and the use of spoilers for roll control
c on the yaw moment. Therefore, the factor k is estimated
c based on the ratio of cn_da to cl_da from the 747 flight test data
c presented in Nelson. Note that the effect of cl is absorbed into
c the factor k.
  k = 0.0064/0.0461

c Estimate variation of yawing moment coefficient with aileron deflection
  cn_da = k*cl_da

c Estimate the flap chord factor from Figure 10.2 for cf/c = 0.33
c The flap effectiveness ratio is estimated with a piecewise curve fit
  cf_c = 0.33
  alpha_d_cl = -sqrt( 1. - (1. - cf_c)**2 )
  if (alpha_d_cl .ge. -0.5) then
    flap_eff_ratio = 1.42 + 1.8*alpha_d_cl
  elseif (alpha_d_cl .ge. -0.6) then
    flap_eff_ratio = 1.32 + 1.6*alpha_d_cl
  elseif (alpha_d_cl .ge. -0.7) then
    flap_eff_ratio = 1.08 + 1.2*alpha_d_cl
  else
    flap_eff_ratio = 0.94 + alpha_d_cl
  end if
  flap_eff_ratio = 1. + flap_eff_ratio/( ar_vtail_eff -
&      0.5*(-alpha_d_cl - 2.1) )
  cf_factor = flap_eff_ratio*alpha_d_cl

c Estimate empirical correction for lift effectiveness of plan flaps at
c from Figure 10.7 for cf/c = 0.33.
  x = dr_max
  if (x .lt. 15.) then
    k_prime = 1.
  else
    k_prime = 4e-7*x**4 - 7e-5*x**3 + 0.0047*x**2 - 0.1453*x +
&      2.3167
  end if

c Estimate span factor for plain flap from Figure 10.3 for delta eta = 0.85
  k_b = 0.95

c Estimate variation of sideforce coefficient with rudder deflection
  cy_dr = cl_alpha_vtail_eff*cf_factor*k_prime*k_b*sv/sref

c Estimate variation of rolling moment coefficient with rudder deflection
  cl_dr = cy_dr*( z_vtail*cos(alpha) - l_vtail*sin(alpha) )/
&      (2.*hspan_wing)

c Estimate variation of yawing moment coefficient with rudder deflection
  cn_dr = -cy_dr*( l_vtail*cos(alpha) + z_vtail*sin(alpha) )/
&      (2.*hspan_wing)

c Multiply empirical estimates by their respective correction factors
c The correction factors are the ratio of the actual 747 derivatives to
c the 747 derivatives predicted by the method above at the M=0.25 flight

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c condition defined in NASA CR-2144 and Nelson. The rudder deflection
c was 15 deg for this calibration.
  cy_beta_cor = 1.4068*cy_beta
  cl_beta_cor = 0.7396*cl_beta
  cn_beta_cor = 2.6690*cn_beta
  cl_da_cor   = 0.9202*cl_da
  cn_da_cor   = 0.9143*cn_da
  cy_dr_cor   = 0.6132*cy_dr
  cl_dr_cor   = 0.3784*cl_dr
  cn_dr_cor   = 0.7286*cn_dr

c Calculate the dynamic pressure
  q = 0.5*rho_eo*(mach_eo*a_eo)**2

c Set the rudder deflection to 20 deg, and the bank angle to 5 deg
  dr = dr_max*pi/180.
  phi = 5.*pi/180.

c Solve for the sideslip angle and aileron deflection
  beta = ( -cy_dr_cor*dr - cl*sin(phi) +
&         sign( thrust_tv*sin(angle_tv*pi/180.)/(q*sref),
&         angle_tv ) + cl_circ_ctrl*sv/sref )/cy_beta_cor
  da   = ( -cl_dr_cor*dr - cl_beta_cor*beta + sign( thrust_tv*
&         sin(angle_tv*pi/180.)*z_tv/(q*sref*2.*hspan_wing),
&         angle_tv ) + cl_circ_ctrl*z_vtail/(2.*hspan_wing)*
&         sv/sref )/cl_da_cor

c Check if the aileron deflection is greater than the max allowable value
  if (da .gt. da_max) then
    print*, 'Warning from stab.f: Required aileron deflection is ',
&         'greater than the maximum allowable value.'
  end if

c Calculate the maximum available yawing moment coefficient
  cn_avail = cn_da_cor*da + cn_dr_cor*dr + cn_beta_cor*beta +
&         sign( thrust_tv*sin(angle_tv*pi/180.)*l_tv/
&         (q*sref*2.*hspan_wing), angle_tv ) +
&         cl_circ_ctrl*l_vtail/(2.*hspan_wing)*sv/sref

c Write output data
  if (write_flag .eq. 1) then
    write(unit_out,*)
    write(unit_out,"(a6)") 'Output'
    write(unit_out,*)

c This section is normally commented out. It can be used to print the
c uncorrected values of the derivatives for debugging purposes.
  if (debug_flag .eq. 1) then
    write(unit_out,100) cy_beta_wing, '= cy_beta_wing (rad-1)'
    write(unit_out,100) cy_beta_fuse, '= cy_beta_fuse (rad-1)'
    write(unit_out,100) cy_beta_vtail, '= cy_beta_vtail (rad-1)'
    write(unit_out,100) cy_beta, '= cy_beta (rad-1)'
    write(unit_out,*)
    write(unit_out,100) cl_beta_wingbody,
& '= cl_beta_wingbody (rad-1)'
    write(unit_out,100) cl_beta_htail, '= cl_beta_htail (rad-1)'
    write(unit_out,100) cl_beta_vtail, '= cl_beta_vtail (rad-1)'
    write(unit_out,100) cl_beta, '= cl_beta (rad-1)'
    write(unit_out,*)
    write(unit_out,100) cn_beta_wing, '= cn_beta_wing (rad-1)'
    write(unit_out,100) cn_beta_fuse, '= cn_beta_fuse (rad-1)'

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write(unit_out,100) cn_beta_vtail, '= cn_beta_vtail (rad-1)'
write(unit_out,100) cn_beta,      '= cn_beta (rad-1)'
write(unit_out,*)
write(unit_out,100) cy_da,        '= cy_da (rad-1)'
write(unit_out,100) cl_da,        '= cl_da (rad-1)'
write(unit_out,100) cn_da,        '= cn_da (rad-1)'
write(unit_out,*)
write(unit_out,100) cy_dr,        '= cy_dr (rad-1)'
write(unit_out,100) cl_dr,        '= cl_dr (rad-1)'
write(unit_out,100) cn_dr,        '= cn_dr (rad-1)'
write(unit_out,*)
end if

c This section prints the corrected values of the derivatives
write(unit_out,100) cy_beta_cor,  '= cy_beta (rad-1)'
write(unit_out,100) cl_beta_cor,  '= cl_beta (rad-1)'
write(unit_out,100) cn_beta_cor,  '= cn_beta (rad-1)'
write(unit_out,*)
write(unit_out,100) cy_da,        '= cy_da (rad-1)'
write(unit_out,100) cl_da_cor,    '= cl_da (rad-1)'
write(unit_out,100) cn_da_cor,    '= cn_da (rad-1)'
write(unit_out,*)
write(unit_out,100) cy_dr_cor,    '= cy_dr (rad-1)'
write(unit_out,100) cl_dr_cor,    '= cl_dr (rad-1)'
write(unit_out,100) cn_dr_cor,    '= cn_dr (rad-1)'
write(unit_out,*)
write(unit_out,100) beta*180./pi,  '= beta (deg)'
write(unit_out,100) phi*180./pi,  '= phi (deg)'
write(unit_out,100) da*180./pi,   '= da (deg)'
write(unit_out,100) dr*180./pi,   '= dr (deg)'
write(unit_out,100) ar_vtail_eff, '= ar_vtail_eff'
write(unit_out,100) cn_avail,     '= cn_avail'
write(unit_out,*)
close(unit_out)
endif
100 format(f11.4, 1x, a)
101 format(7x, i4, 1x, a)
102 format(f11.0, 1x, a)
103 format(g11.4, 1x, a)

return
end

```