AOE 4134 Astromechancs Problem Sheet Four

Due September 25, 2002

Read Chapter 1 again! (You should have it memorized by now)
Read Chapter 3 all of it (first time), Read Chapter 8, 8.1 - 8.3.2

21. We have determined that the global positioning satellites (GPS) are in 12 hour circular orbits that have a radius of 4.1721 DU. It is desired to raise the satellite to a 24 hour circular orbit that has a radius of 6.6228 DU. The procedure is to use a Hohmann transfer orbit. This orbit is such that its perigee distance is the same as that of the GPS satellite orbit, and the apogee distance is the same as that of the 24 hour (Geosynchronous, GEO) orbit. Determine the following:

a) energy of the GPS orbit $(DU/TU)^2$

b) energy of the transfer orbit $(DU/TU)^2$

c) eccentricity of the transfer orbit

d) ΔV at GPS-transfer orbit perigee to go from GPS orbit to transfer orbit (DU/TU)

e) Calculate the exact change in energy from GPS to transfer orbit $(DU/TU)^2$ and compare it with the change in energy calculated using the differential approximation method, i.e. % error

f) Calculate the exact change in the semi-major axis (DU) from the GPS orbit to the transfer orbit, and compare it with the value calculated using the differential approximation, i.e. % error.

22. Problem (21) dealt with the initiation of the transfer orbit. We now want to calculate the maneuver required to transfer from the transfer orbit to the outer circular orbit.

a) Determine the ΔV required to go from the transfer orbit to the 24 hour orbit (DU/TU)

b) Determine the total ΔV for the mission, transfer from 12 to 24 hour orbit (DU/TU)

c) Compare with the ΔV that would be required to escape for the original 12 hour orbit.

d) The time of a perigee to apogee transfer is half the orbit period. a) Determine the time to transfer from the 12 hour orbit to the 24 hour orbit. (TU and minutes).

23. Consider a circular orbit of radius r_0. We would like to transfer to circular orbits that are twice the radius, 4 times the radius and 8 times the radius of the original orbit. Determine the transfer times, assuming a Hohmann transfer orbit in terms of the initial (circular orbit period, T_{P_0}). Eg. 0.5 T_{P_0}, or 5 T_{P_0}, etc.

24. A vehicle is in a circular orbit of radius r_0. A small ΔV is added in the ***radial*** direction. Find an expression for the eccentricity of the resulting orbit in terms of $\Delta V, r_0,$ and μ. Note: this is a simple expression!

25. A mail rocket is fired from the surface of the Earth at an angle of 30 degrees with a velocity of ½ V_{cir}.

a) Determine the true anomaly at launch, ν_L (deg)

b) Determine the apogee height above the Earth's surface (DU, km)

c) Determine the angular range over the Earth before it strikes the Earth again. (Deg)

d) Determine the range (km) over the Earth' surface.