AOE Astrodynamics Problem Sheet 2

Due: September 11, 2002

Read: Chapter 1 BMW (this should be the 3rd reading. It should start to make sense now)

In all problems where you use a code (MATLAB or other), include the code with your answer sheet. In all these problems, assume no atmosphere (just the inverse gravitational force)

11. In class we looked at the problem of launching from the Earth's surface at a speed of 3 km/s. We found that it can reach a different maximum altitude depending on the angle at which it is launched. The maximum altitude is achieved when it is launched straight up, and the minimum max altitude when it is launched horizontally. ***For a launch speed of 4 km/sec***.

   a) Make a table and plot of the maximum radius vs launch angle (use 5 degree increments from 0 to 90 degrees)
   b) Make a table and plot of the minimum radius vs launch angle (same increments and range)

12. The orbits calculated in (11) are elliptic with the center of the Earth located at a focus. The eccentricity of an ellipse is given by (without proof, to be shown later):

$$e = \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}}$$

For the launch conditions that you calculated in (11), make a table and plot of eccentricity vs launch angle.

13. Global Positioning satellites (used in the Global positioning system, GPS) are located in circular orbits that have a 12 hour period.

   a) Determine the radius of these orbits (km)
   b) Determine the altitude of these orbits (km)
   c) Determine the energy of these orbits $(km/s)^2$
   d) Determine (magnitude of) the angular momentum of these orbits (km^2/s)

14. A satellite is in a 90 minute low Earth circular orbit (LEO). It is desired to move it into a 24 hour (circular) geosynchronous orbit (GEO). How much energy must be added to the satellite to make this change? $(km/s)^2$

15. A probe is fired vertically (radially) from the surface of the (non-rotating) Earth at circular speed.

   a) Determine the maximum altitude that the probe will reach. (DU and km)
   b) Determine the angular moment of this orbit (DU^2/TU)
   c) Determine the semi-major axis (a) of this orbit.
   d) What is the minimum amount of additional velocity (at launch) would be required to have this probe escape the Earth's gravitational field? (ΔV = ? In DU/TU)
   e) What would be the minimum additional velocity that would need to be added at the max altitude to cause the probe to escape the Earth's gravitational field? (In DU/TU)
   f) In light of the results of parts d) and e), and assuming the ΔV is proportional to the fuel expended, would it be cheaper (fuel-wise) to add the ΔV at the launch or at the max altitude? - Justify your response.
   f) If the gravitational force were a constant, at the sea level value, determine the altitude (in DU) that the probe would reach, and calculate the % error from the actual altitude found in (a)