15. Topics In Astrodynamics

Effect of Oblate Earth

In all the previous developments, the assumption has been that the Earth, Sun and Planets were homogenous spheres of uniform density so that they could be treated as point masses with an inverse square attractive force. In reality the Earth is not a perfect sphere and it is certainly not uniform. It tends to bulge in the middle, and is generally larger in the southern hemisphere then in the northern hemisphere. Here we will consider (without proof) the effects only of the bulge or oblateness of the Earth. The general effect is that our orbital elements are no longer constant in time. The changes in the orbital elements can be classified into two categories: periodic and secular. Secular means increasing or decreasing with time. The following observations can be made:

changes in a, e, i	periodic
changes in Ω, ω	secular

The secular changes in the longitude of the ascending node and in the argument of perigee can be shown (analytically) to be:

$$\dot{\Omega} = -\frac{3\, n\, J_2\, R_e^2 \cos i}{2\, a^2 \left(1 - e^2\right)} \tag{1}$$

and

$$\dot{\omega} = -\frac{3\, n\, J_2\, R_e^2}{4\, a^2 \left(1 - e^2\right)} \left(5 \sin^2 i - 4\right) \tag{2}$$

where: $n = \sqrt{\frac{\mu}{a^3}}$ = mean angular rate

J_2 = 0.00108263 = oblate Earth gravitational constant

R_e = radius of the Earth

Critical Inclination

We can observe from the equation for the change in the argument of perigee, that it is possible to have an inclination where that rate will be zero. The inclination at which this occrs is called the critical inclination and can be determined from:

$$5 \sin^2 i - 4 = 0 \qquad \Rightarrow \qquad i = 63.4349 \text{ deg} \tag{3}$$

An example of an orbit that takes advantage of this phenomena is the so-called ***Molniya orbit***. This orbit was used for a communication satellite over Russia. Since Russia is located so far in the Northern hemisphere, a 24 hour satellite that stays over one point at such a latitude is not possible. The satellites were launched in an orbit with the properties:

$$e = 0.7,\ i = 63.4 \text{ deg, and } T_p = 12 \text{ hours}$$

In this case the perigee and hence apogee are fixed, with the apogee placed in the northern hemisphere. The eccentricity of the satellite orbit is large so that the satellite spends most of its time in the northern hemisphere, say 9 or so hours. Then the satellite could be used for communication during this time period before it went out of sight when it quickly swung by the perigee and stayed only a shor time in the southern hemisphere. As long as the perigee and apogee remained fixed, this strategy worked to provide a communication satellite for the northern most countries.

Sun Synchronous Orbits

We can also take advantage of the Earth's oblateness to provide a sun synchronous orbit. A sun synchronous orbit is an orbit who's plane (or normal to the plane) always points to the Sun. We can make this happen by taking advantage of the way that the longitude of the ascending node changes due to the oblateness of the Earth. We can illustrate the idea with a problem.

Example

Suppose that we have a circular orbit. For energy reasons (solar panels) we would like to have the plane of the orbit always face the Sun. The satellite is to be at an altitude of 400 km. What inclination is required?

The orbit plane has to progress $\dot{\Omega} = \dfrac{360}{365.25} = 0.9856 \text{ deg / day}$

$$r = a = \frac{6378.1363 + 400}{6378.1363} = 1.0627 \text{ DU} \qquad n = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{1}{1.0627^3}} = 0.9128 \text{ rad/TU}$$

We can now substitute into the nodal equation and solve for the inclination required:

$$\dot{\Omega} = -\frac{3\, n\, J_2 R_e^2 \cos i}{2\, a^2 \left(1 - e^2\right)} = -\frac{(3)\,(0.9128)\,(1.08263x10^{-3}\,(1^2)}{2\,(1.0627^2)\cos i} = 0.9856\,\frac{\pi \text{ rad}}{180 \text{ deg}}\,\frac{1 \text{ day}}{(24)\,(60) \text{ min}}\,\frac{13.44685 \text{ min}}{1 \text{ TU}}$$

$$\cos i = -0.1224 \qquad \Rightarrow \qquad i = 97.03 \text{ deg}$$

Therefore, for a 400 km circular orbit, the orbit must have an inclination of 97 degrees (retrograde) to be Sun synchronous.