13. Time Considerations- Local Sidereal Time

The time that is used by most people is that called the ***mean solar time***. It is based on the idea that if the Earth revolved around the Sun at a uniform rate, the time between two successive meridian crossings would be one mean solar day. If we divide that into 24 hours, and each hour into 60 minutes, and each minute into 60 seconds, we can define a mean solar second. Related to this idea is that of a mean sidereal day. Here the assumption is that the day is defined by the time that it would take the same distant star to pass over a specified meridian two successive times. Because the Earth revolves around the Sun, and it advances about 1/365.25 x 360 degrees each day, the mean solar day is longer than the mean sidereal day. The ratio of these two fundamental time units is :

1 solar day = 1.002737909350795 sidereal days

This conversion factor serves as a means to convert from solar to sidereal time. We can now make the following definition:

<u>LOCAL SIDEREAL TIME</u>

The angle between the Vernal Equinox (x inertial axis) and the local meridian is the ***Local Sidereal Time (LST)***. Note that the local sidereal time is an ***ANGLE***.

However it is often expressed in terms of time. The conversion factor for angles expressed as time is that, by definition, 24 hours = 360 degrees. Hence we can convert information given to us in terms of time to an angle as follows: Suppose we are given that the sidereal time at some location is

$LST = 6^h\, 42^m\, 51.5354^s = 6{:}42{:}51.5354$

The LST ANGLE is determined from the following conversion:

$$LST = \frac{\left\{6 + \dfrac{\left[42 + \dfrac{51.5354}{60}\right]}{60}\right\}}{24} \times 360$$

$$= 100.7147308 \text{ deg}$$

It is convenient to carry out all calculations in degrees or radians so has not to confuse angles with what we usually think of time. All solar time is generally referred to a reference point on the Earth. That point (or line really) is the Greenwich Meridian. The local time at that location is called the Greenwich Mean Time (GMT) or Universal Time (UT1) or Zulu time (Z). It is mean time because we are assuming a mean solar day as describe previously. To change current local time to GMT we need to add or subtract hours. In the Eastern Time Zone, we are approximately 45 degrees West of the Greenwich Meridian (to the center of the ETZ), so we add 5 hours to Eastern Standard time to get GMT. Note that we also use the 24 hour clock. So if it is 1:45:30.0 PM EST, we can convert to UT1 time by adding 5 hours:

$$GMT = EST + 5 \text{ hrs}$$

0

First we convert to a 24 hour clock,

$$1{:}45{:}30.0 \text{ PM} = 1{:}45{:}30.0 + 12^h = 13{:}45{:}30.0 \text{ EST}$$

Now convert to GMT

$$GMT = 13{:}45{:}30.0 + 5^h = 18{:}45{:}30.0 \text{ UT1} = 18^h\ 45^m\ 30.0^s \text{ UT1}$$

We are now in a position to calculate the local sidereal time. We will use the following strategy. If we assume that we know the Greenwich Sidereal Time at some epoch, we can calculate the number of elapsed days from that epoch and hence determine the additional angle that the Earth has rotated through since that time. The equation that we can use is,

$$ED = ED_0 + \frac{\left\{ hr + \left[\frac{\left(\min + \frac{\sec}{60} \right)}{60} \right] \right\}}{24}$$

where ED are the elapsed days, and ED_0 are the elapsed days from epoch to midnight prior to the day of interest. The epoch time we will select is January 1, 2001, $0^h\ 0^m\ 0^s$, for the year 2001 and January 1, 2002, 0:0:0.0 for year 2002, and likewise for successive years. The elapsed days, ED_0 is then just the day of the year -1. That is the first of January is day 1, but is elapsed day 0. Clearly midnight of January 1 is $ED_0 = 1$ so times during January 1 must be between 0 and 1, and times during January 2 must be between 1 and 2, etc. This leads to the following chart:

* Denotes leap year

Date	Day	Elapsed Day ED_0
31 January	031	030
28 February	059	058
31 March	090 (091*)	089 (090*)
30 April	120 (121*)	119 (120*)
31 May	151 (152*)	150 (151*)

30 June	181 (182*)	180 (181)
31 July	212 (213*)	211 (212*)
31 August	243 (244)	241 (242*)
30 September	273 (274*)	272 (273*)
31 October	304 (305*)	303 (304*)
30 November	334 (335*)	333 (334*)
31 December	365 (366*)	364 (365*)

The local sidereal time can then be determined in the following way. First we have to know the Greenwich Sidereal time at January 1 of the year of interest. This will be designated as θ_{gST_0}. Then the Greenwich sidereal time at some time later is given by:

$$\theta_{gST} = \theta_{gST_0} + 1.002737909350795 \cdot ED \cdot 360$$

Note that 360 can be replace by 2π if the angle change in radians is desired. The local sidereal time is determined by adding (East Longitude) or subtracting (West Longitude) the longitude of the location of interest. Hence we have:

$$\theta_{LST} = \theta_{gST} \pm \lambda \qquad \begin{cases} + & East \\ - & West \end{cases}$$

If we note that Blacksburg is at the location:

$$\begin{aligned} 37° \; 12.7 \text{ min N Latitude} &= 37.2133 \; ° \text{ N} \\ 80° \; 24.5 \text{ min W Longitude} &= 24.4083° \text{ W} \end{aligned}$$

For the time given previously, we can compute the elapsed day ED using the equation above.

$$ED = 338 + \frac{\left\{ 18 + \left[\frac{\left(45 + \frac{39}{60} \right)}{60} \right] \right\}}{24} = 338.78159722$$

The Sidereal Time given previously is the Greenwich Sidereal Time at the epoch January 1, 2001, 0:0:0.0. Hence we have,

$$\theta_{gST_0} = 100.7147308 \text{ deg}$$

and

$$\theta_{gST} = 100.7147308 + 1.00273790935 \cdot 338.781597222 \cdot 360$$
$$= 122396.0089 \text{ deg}$$
$$= 356.008896 \text{ deg}$$

Determine the local sidereal time from,

$$\theta_{LST} = \theta_{gST} =- \lambda$$
$$= 356.008896 - \left(80 + \frac{24.5}{60} \right)$$
$$= 275.6005627 \text{ deg}$$

Note that this angle can also be expressed as

$$\theta_{LST} = 18^h\ 22'\ 24.3506''$$

where $'$ indicates arc minutes and $''$ indicates arc seconds.

An alternate approach to obtaining the local sidereal angle is more accurate than the previous result and does not depend on the knowledge of the Greenwich Sidereal Time at the January 1 epoch. It requires establishing the Julian Date and using its value to establish the Greenwich sidereal angle at the current time and hence the local sidereal angle.

Julian Date

The Julian date is the number of days since noon, January 1, 4713 BC. These numbers are quite large, so sometime a modified Julian date is used. The modified Julian date is given by

MJD = JD - 2,400,000.5

and starts at midnight, rather than noon. Using the MJD can add to confusion so only the JD will be used here. We can calculate the JD for the period March 1, 1900 - February 28, 2100 using the following algorithm:

$$JD = 367(\text{yr}) - \text{int}\left\{ \frac{7\left[\text{yr} + \text{int}\left(\frac{\text{mo} + 9}{12} \right) \right]}{4} \right\} + \text{int}\left(\frac{275 \cdot \text{mo}}{9} \right) + \text{day} +$$
$$1{,}721{,}013.5 + \left\{ \frac{\left[\frac{\text{hr} + \left(\text{min} + \frac{\text{sec}}{60} \right)}{60} \right]}{24} \right\}$$

where the "int" function means to drop the decimal part of the number, for example,

$$\text{Int } 35.985 = 35,$$

and the year is the four digit year, for example 2001, mo is the month of the year, for example December is the 12 month, and day is the day of the month, for example the 5th of December would be day = 5.

Of interest in calculating the local sidereal time is the Julian day number JD_0 which is the Julian Date with the hours, minutes and seconds set equal to zero, $0^h\,0^m\,0.0$.

Continuing our example for 1:45:30.0 on December 5, 2001, we can calculate the Julian day by using the previous algorithm with the hours, minutes and seconds equal to zero.

$$JD_0 = 367(2001) - \text{int}\left\{\frac{7\left[2001 + \text{int}\left(\frac{12+9}{12}\right)\right]}{4}\right\} + \text{int}\left(\frac{275 \cdot 12}{9}\right) + 5 + 1{,}721{,}013.5$$

$$JD_0 = 734367 - 3503 + 366 + 5 + 1{,}721{,}013.5 = 2452248.5$$

Calculating the current Greenwich Sidereal Time

To calculate the current GST, we calculate the time at midnight using the result obtained above, and add to it the amount the Earth has rotated up to the current time. Hence we have to make two calculations, the first being the Greenwich sidereal time at the beginning of the day of interest, $\theta_{gsT_{00}}$, and the current GST. This last calculation is done by taking the universal time and multiplying it by the angular rate of the Earth.

$$\theta_{gST} = \theta_{gST_{00}} + \omega_e (UT1)$$

To calculate the Greenwich sidereal time at the beginning of the day of interest, the following algorithm is used:

$$\theta_{gST_{00}} = 100.4606184 + 36000.77005361\,T_{UT1} + 0.00038793\,T_{UT1}^2 - 2.6 \times 10^{-8}\,T_{UT1}^3$$

where T_{UT1} is the number of Julian centuries since the epoch J2000 (noon on January 1, 2000), and is given by:

$$T_{UT1} = \frac{JD_0 - 2451545.0}{36525}$$

For our sample problem, we have:

$$T_{UT1} = \frac{2452248.5 - 245154.0}{36525} = 0.01926078$$

and,

$$\theta_{gST_{00}} = 100.4606184 + 36000.77005361\,(0.01926078) + 0.00038793\,(0.01926078)^2$$

where we have neglected the last term $(T_{UT1})^3$ since it is negligible. Carrying out the operation, we have,

$$\theta_{gST_{00}} = 793.8635304 = 73.8635304 \text{ deg}$$

We can now calculate the Greenwich sidereal time at the time of interest from the following:

$$\begin{aligned}\theta_{gST} &= \theta_{gST_{00}} + \omega_e\,(UT1)\\ &= 73.8635304 + 0.25068447733746215\,deg/\text{min} \cdot \left(18 \cdot 60 + 45 + \frac{30.0.0}{60}\right) \text{ min}\\ &= 356.0089096 \text{ deg}\end{aligned}$$

Calculate the local sidereal time. As before, the local sidereal time is obtained by adding (subtracting-West) the longitude:

$$\begin{aligned}\theta_{LST} = \theta_{gST} \pm \lambda &= 356.0089096 - 80.4083333\\ &= 275.6006 \text{ deg}\end{aligned}$$

which is close to our previous answer. This method should be more accurate than the previous one, especially for late in the year.

One final note. These equations really calculate the mean sidereal time which is measured to the mean position of the Vernal equinox. Recall the Vernal equinox is along the intersection of the plane of the ecliptic and the equatorial plane. These planes tend to change their orientation due to various perturbation forces and hence cause the equinox position to change. The mean position is the average of the periodic changes, but does include the secular changes.