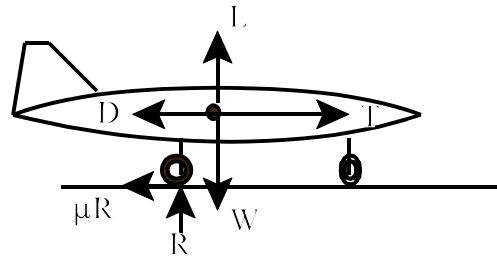


### 15. Takeoff and Landing

The takeoff distance consists of two parts, the ground run, and the distance from where the vehicle leaves the ground to until it reaches 50 ft (or 15 m). The sum of these two distances is considered the takeoff distance. (Note: sometimes a 35 ft altitude is used). The takeoff distance is generally calculated for maximum weight in a standard atmosphere. In addition, usually the worst case scenario is also calculated which is maximum weight, high altitude, and a standard “hot” day. Of immediate concern here is the calculation of the ground roll.

In order to calculate the ground roll, we need to write the equations of motion for the vehicle as it moves down the runway.

A free-body diagram of the vehicle is shown to the right. The forces that act on it are the aerodynamic forces of lift and drag ( $L$  and  $D$ ), the thrust force ( $T$ ), the ground normal force ( $R$ ) and the ground friction force ( $\mu R$ ), where  $\mu$  is the coefficient of rolling friction. We can now write the equations of motion along the runway and perpendicular to it.



Vertical:

$$L + R - W = 0 \quad \Rightarrow \quad R = W - L \quad (1)$$

Horizontal:

$$T - D - \mu R = m \frac{dV}{dt} \quad (2)$$

$$T - D - \mu (W - L) = \frac{W}{g} \frac{dV}{dt}$$

Rearranging, we have:

$$g \left( \frac{T}{W} - \mu \right) - \frac{g}{W} (D - \mu L) = \frac{dV}{dt} \quad (3)$$

$$g \left( \frac{T}{W} - \mu \right) - \frac{g}{W} \frac{1}{2} \rho S V^2 (C_{D_g} - \mu C_{L_g}) = \frac{dV}{dt}$$

where  $C_{L_g}$  is the lift coefficient during the ground run and  $C_{D_g}$  is the corresponding drag

coefficient. In general the constants in the parabolic drag coefficient are different from the ones used for up-and-away performance calculations. These differences are due to the aircraft configuration (gears, and partial flaps possibly) and due to the proximity of the ground (ground effects). How these factors affect the coefficient parameters will be discussed later.

At this stage of the development, we will assume how the thrust varies with airspeed.

**Assumption:** The thrust can vary with airspeed according to the rule:

$$T = T_0 - aV^2 \quad (4)$$

where:

$T_0$	=	Thrust at zero airspeed (Static Thrust)
$T$	=	Thrust at airspeed $V$
$a$	=	constant that can be positive, negative or zero

With this assumption, we can gather together coefficients of  $V^2$  and terms that don't depend on  $V$ , to get:

$$\begin{aligned} \frac{dV}{dt} &= g \left( \frac{T_0}{W} - \mu \right) - \frac{g}{W} \left[ \frac{1}{2} \rho S (C_{D_g} - \mu C_{L_g}) + a \right] V^2 \\ &= A - B V^2 \end{aligned} \quad (5)$$

where:

$$A = g \left( \frac{T_0}{W} - \mu \right) \quad \text{and} \quad B = \frac{g}{W} \left[ \frac{1}{2} \rho S (C_{D_g} - \mu C_{L_g}) + a \right]$$

### ***Time for Ground Run***

We can rearrange Eq. (5) to obtain the time for the ground run. This particular performance parameter is generally not of much interest, but is easily obtained, and is useful if we change thrust or lift characteristics at some time during the ground run (for example, use a jet assisted takeoff). In any case, we can solve Eq. (5) for  $dt$  to get:

$$dt = \frac{dV}{A - B V^2} \quad (6)$$

Under our assumptions,  $A$  and  $B$  are constant, so we can easily integrate Eq. (6). In most cases we can assume that both  $A$  and  $B$  are positive numbers.  $A$  certainly is always positive and  $B$  is positive unless the ground lift coefficient is large (an unlikely event). In normal conditions ( $A$  and  $B > 0$ ) the time for the ground run between two airspeeds is given by:

**Time for Ground Run Between any Two Speeds, Constant  $\alpha$ , A, B >0**

$$t_2 - t_1 = \int_{V_1}^{V_2} \frac{dV}{A - BV^2} = \frac{1}{\sqrt{AB}} \left[ \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_2 \right) - \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_1 \right) \right] \quad (7a)$$

or

**Time for Ground Run Between any Two Speeds, Constant  $\alpha$ , A, B >0**

$$t_2 - t_1 = \int_{V_1}^{V_2} \frac{dV}{A - BV^2} = \frac{1}{2\sqrt{AB}} \left\{ \ln \frac{\sqrt{A} + V_2 \sqrt{B}}{\sqrt{A} - V_2 \sqrt{B}} - \ln \frac{\sqrt{A} + V_1 \sqrt{B}}{\sqrt{A} - V_1 \sqrt{B}} \right\} \quad (7b)$$

It may be possible to have the B in the second term in our expression for acceleration to be negative Under these circumstances, we would have the situation:

**Time for Ground Run Between any Two Speeds, Constant  $\alpha$ , A > 0, B < 0**

$$t_2 - t_1 = \int_{V_1}^{V_2} \frac{dV}{A + |B| V^2} = \frac{1}{\sqrt{A|B|}} \left[ \tan^{-1} \left( \sqrt{\frac{|B|}{A}} V_2 \right) - \tan^{-1} \left( \sqrt{\frac{|B|}{A}} V_1 \right) \right] \quad (8)$$

Usually, the lift-off speed is designated to be  $1.2 V_{stall}$  :

$$V_{TO} = 1.2 V_{stall} \quad (9)$$

Then the time to takeoff from rest is given by:

Time to Takeoff

$$t_{TO} = \frac{1}{\sqrt{AB}} \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_{TO} \right) = \frac{1}{2\sqrt{AB}} \left[ \ln \frac{\sqrt{A} + V_{TO} \sqrt{B}}{\sqrt{A} - V_{TO} \sqrt{B}} \right] \quad (10)$$

### ***Distance of Ground Run***

Probably a more critical issue regarding the takeoff ground run is the distance traveled. We can determine this distance in the following manner:

$$\frac{\frac{dV}{dt}}{\frac{dS}{dt}} = \frac{dV}{dS} = \frac{A - BV^2}{V} \quad (11)$$

Rearranging, we have:

$$dS = \frac{V dV}{A - BV^2} \quad (12)$$

If we make the same assumptions as we did for the time calculations, the two parameters, A and B are constant and Eq. (12) can be integrated (regardless of the sign of B) to give:

#### **Distance for Ground Run Between two airspeeds, $V_1$ and $V_2$ .**

$$S_2 - S_1 = -\frac{1}{2B} \ln(A - BV^2) \Big|_{V_1}^{V_2} = \frac{1}{2B} \ln \frac{A - BV_1^2}{A - BV_2^2} \quad (13)$$

For the case where we start from rest and go to takeoff, Eq. (13) becomes:

#### **Takeoff Ground Run Distance**

$$S = \frac{1}{2B} \ln \frac{A}{A - BV_{TO}^2} \quad (14)$$

### ***Minimum Distance Ground Roll***

Like most other performance characteristics, we would like to find the best value and the means to achieve this best value. For takeoff distance, the best value would be the shortest distance. Hence we would like to find the flight conditions that would minimize the takeoff distance. Since the acceleration is given by  $\frac{dV}{dt} = A - BV^2$ , it is clear to maximize the acceleration (which we hope minimizes the takeoff distance) we need to maximize A and minimize B. The key ingredients of A are the thrust to weight ratio and the ground friction coefficient. Therefore we want to maximize the thrust (and minimize the weight) and to take off

from a surface that has a low rolling friction coefficient (pavement over grass for example).

If we look at the parameter B, we see that the only portions of it that we have control over are the ground aerodynamic parameters,  $C_{L_g}$ , and  $C_{D_{0L_g}}$ . We seek, therefore, to minimize the term

$$C_{d_g} - \mu C_{L_g} = C_{D_{0L_g}} + K_g C_{L_g}^2 - \mu C_{L_g} \quad (15)$$

The only terms at our disposal here is the ground lift coefficient,  $C_{L_g}$ , and the zero-lift ground drag coefficient  $C_{D_{0L_g}}$ . This latter term we would like to make as small as possible by reducing frontal area, removing appendages, and making the aircraft smooth. The only term that we really have control over is the ground lift coefficient. We can adjust this term by changing the angle and hence angle-of-attack at which the aircraft rolls along the ground. This is done by adjusting the lengths of the landing gear struts. However it is done, we can establish the best  $C_{L_g}$  by taking the derivative of Eq. (15) with respect to  $C_{L_g}$  and setting it equal to zero:

$$\frac{d}{dC_{L_g}} (C_{D_{0L_g}} + K_g C_{L_g}^2 - \mu C_{L_g}) = 2K_g C_{L_g} - \mu = 0$$

or

### Ground Lift Coefficient for Minimum Takeoff Ground Run Distance

$$C_{L_g} = \frac{\mu}{2K_g} \quad (16)$$

#### Example

Given:	$W = 56,000 \text{ lbs}$	$V_{TO} = 1.2 V_{stall}$	$\eta_p = 75\%$
	$S = 1000 \text{ ft}^2$	$C_{D_g} = 0.024 + 0.04 C_{L_g}^2$	$T_0 = 2400 \text{ BHP/engine}$
	$C_{L_{max}} = 2.4$	$\mu = 0.25$	

This is a propeller driven aircraft with a shaft (brake) horsepower of 2400 per engine. We want to calculate the minimum ground run at sea-level in a standard atmosphere.

First, calculate the takeoff speed.

$$V_{stall} = \sqrt{\frac{W}{1/2 \rho S C_{L_{max}}}} = \sqrt{\frac{56000}{1/2 (0.002377) (1000) (2.4)}} = 140.0 \text{ ft/sec}$$

$$V_{TO} = 1.2 V_{stall} = 1.2 (140) = 168 \text{ ft/sec}$$

Now determine  $a$  in the thrust equation.

$$\text{@ takeoff: } P_s \eta_p = T V = 0.75 (2400) 550 \quad \Rightarrow \quad T = \frac{(0.75) 2400 (550)}{168} = 5900 \text{ lbs}$$

Total thrust at takeoff =  $2 \times 5900 = 11,800$  lbs

Total static thrust ( $V = 0$ ) =  $2 \times 6500 = 13,000$  lbs

$$T = T_0 - a V^2 = 11,800 = 13,000 - a(168)^2 \quad \Rightarrow \quad a = 0.0422 \frac{\text{lb sec}^2}{\text{ft}^2}$$

$$T = 13000 - 0.0422 V^2$$

The shortest ground run lift coefficient is determined from:

$$C_{L_g} = \frac{\mu}{2 K_g} = \frac{0.025}{2 (0.040)} = 0.3125$$

$$C_{D_g} = 0.024 + 0.04 (0.3125)^2 = 0.0279$$

We can now determine the constants "A" and "B" in our takeoff equation:

$$A = g \left( \frac{T_0}{W} - \mu \right) = 31.174 \left( \frac{13000}{56000} - 0.025 \right) = 6.65$$

$$\begin{aligned} B &= \frac{g}{W} \left[ \frac{1}{2} \rho S (C_{D_g} - \mu C_{L_g}) + a \right] \\ &= \frac{31.174}{56000} \left[ \frac{1}{2} (0.002377) (1000) (0.0279 - (0.025) (0.3125)) + 0.0422 \right] = 3.80 \times 10^{-5} \end{aligned}$$

Then the takeoff ground roll is given by:

$$S_{TO} = \frac{1}{2B} \ln \frac{A}{A - B V_{TO}^2} = \frac{1}{2 (3.8 \times 10^{-5})} \ln \frac{6.65}{6.65 - (3.8 \times 10^{-5}) (168)^2} = 2314 \text{ ft}$$


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## Ground Effects

An aircraft acting in the presence of the ground has different aerodynamic properties than one in up and away flight. The nearness of the ground improves the efficiency of the aircraft. It does this by reducing the downwash at the wing and hence reducing the induced drag parameter. There are many different correction factors that have been developed to approximate the effect on the induced drag parameter,  $K$ . Two of these are presented next.

If we know the up and away induced drag parameter in the form:

$$K = \frac{1}{\pi AR e} \quad (17)$$

we can come up with a in-ground-effect induced drag parameter from

$$K_g = \phi K \quad (18)$$

where the two approximations of the correction factor  $\phi$  are given by:

$$\phi = 1 - \frac{2e}{\pi} \ln \left[ 1 + \left( \frac{\pi b}{8h} \right)^2 \right] \quad (19a)$$

or

$$\phi = \frac{\left( 16 \frac{h}{b} \right)^2}{1 + \left( 16 \frac{h}{b} \right)^2} \quad (19b)$$

where:  $h$  = height of wing above the ground

$b$  = wing span

$e$  = Oswald efficiency factor

The second model is found in most text books, but is slightly conservative (too large).

In addition to the ground effect on the induced drag parameter, the ground roll configuration usually has a different value of  $C_{D_{0z}}$  than is usually found in up and away flight.

There has been some empirical equations developed to estimate the change in the zero-lift drag coefficient based on the wing loading and mass of the aircraft. This empirical equation is given by:

$$\Delta C_{D_{0L}} = \frac{W}{S} K_{uc} m^{-0.215} \quad (20)$$

where:  $\frac{W}{S}$  = wing loading in N/m<sup>2</sup>

m = aircraft in kg

and

$K_{uc} = 5.81 \times 10^{-5}$  zero flap deflection

$= 3.16 \times 10^{-5}$  full flap deflection

(Sorry, I haven't converted this to U. S. Customary units!)

## **Landing Run**

The opposite of the takeoff procedure is the landing procedure. Just as in the takeoff, the landing maneuver consists of two parts:

1. The terminal glide over a 50 ft obstacle to touchdown
2. The landing ground run

Some calculations include a flare from the landing glide to the touchdown. However, for a maximum performance landing (short field landing procedure), very little flare is used, and the aircraft is flown onto the runway. Here we will neglect the flare portion of landing and assume the aircraft touches down at slightly high speed than it would after flaring.

### ***Landing Ground Run***

The equations of motion governing the landing ground run are the same as those for takeoff. However, the constants A and B can be quite different. Typically the major contributions to the differences are:

1. Thrust can be zero or even negative (reverse thrust)
2. The runway rolling friction can be much larger due to braking.

In addition, the boundary conditions on the landing run are different.

1. At the beginning of the ground roll the velocity is that at touchdown,  $V_{TD}$
2. At the end of the ground run, the velocity is  $V_2$ , usually zero

Consequently the differential equation of motion that is used to describe the landing run is the same as that for takeoff:

$$dS = \frac{V dV}{A - BV^2} \quad (21)$$

This equation can be integrated without restrictions:

$$S_2 - S_{TD} = \frac{1}{2B} \left[ \ln \left( \frac{A - V_{TD}^2}{A - BV_2^2} \right) \right] \quad (22)$$

If we come to rest,  $V_2 = 0$ , and we have:

### **Landing Ground Run Distance**

$$S = \frac{1}{2B} \ln \left( 1 - \frac{B}{A} V_{TD}^2 \right) \quad (23)$$

### **Time for Landing Ground Run**

We can calculate the time for the landing ground run in a manner similar to that for the takeoff ground run. Unfortunately the time equation that results depends on the signs of the constants A and B as we will see. The time equation can be determined from:

$$dt = \frac{dV}{A - BV^2} \quad (24)$$

The form of the resulting integration depends on the signs of the constants A and B. Thus we have four possible cases, (some of which are rarely encountered) if we look at all permutations.

#### **Case 1, A > 0, B > 0**

$$t_2 - t_1 = \frac{1}{2\sqrt{AB}} \ln \left[ \frac{\sqrt{A} + V\sqrt{B}}{\sqrt{A} - V\sqrt{B}} \right] \Bigg|_{V_{TD}}^{V_2} \quad (25)$$

#### **Case 2, A > 0, B < 0**

$$t_2 - t_1 = \frac{1}{\sqrt{-AB}} \tan^{-1} \left( V \sqrt{\frac{-B}{A}} \right) \Bigg|_{V_{TD}}^{V_2} \quad (26)$$

**Case 3,  $A < 0, B > 0$**

$$t_2 - t_1 = -\frac{1}{\sqrt{-AB}} \tan^{-1} \left( V \sqrt{\frac{B}{-A}} \right) \Big|_{V_{TO}}^{V_2} \quad (27)$$

**Case 4,  $A < 0, B < 0$**

$$t_2 - t_1 = \frac{1}{2\sqrt{AB}} \ln \left[ \frac{V\sqrt{-B} - \sqrt{-A}}{V\sqrt{-B} + \sqrt{-A}} \right] \Big|_{V_{TO}}^{V_2} \quad (28)$$

Example

An aircraft weighs 30,000 lbs, has a wing area of 750 ft<sup>2</sup>, and a  $C_{L_{max}} = 2.2$ . The runway friction coefficients are,  $\mu = 0.02$  for rolling, and  $\mu_b = 0.5$  during braking. The touchdown velocity is  $V_{TD} = 1.3 V_{stall}$ , and braking occurs when the airspeed is  $V_b = 0.8 V_{TD}$ . Additionally, at touchdown speed,  $\frac{L}{D} = 8.0$ .

We need to get the reference speeds:

$$V_{stall} = \sqrt{\frac{W}{1/2 \rho S C_{L_{max}}}} = \sqrt{\frac{30000}{1/2 (0.002377) 750 (2.2)}} = 123.6 \text{ ft/sec}$$

$$V_{TD} = 1.3 V_{stall} = 1.3 (123.6) = 160.7 \text{ ft/sec}, \quad V_B = 0.8 V_{TD} = 0.8 (160.7) = 128.56 \text{ ft/sec}$$

Before Braking:

$$T = 0, \text{ Assume ( probably not a good assumption) } C_{L_g} = C_L @ 1.3 V_{stall}$$

$$C_{L_g} = \frac{W}{1/2 \rho S V^2} = \frac{30000}{1/2 (0.002377) (750) (160.7)^2} = 1.3032$$

$$D = \frac{W}{L/D} = \frac{30000}{8} = 3750 \text{ lbs}$$

$$C_{D_g} = \frac{D}{1/2 \rho S V^2} = \frac{3750}{1/2 (0.002377) (750) (160.7)^2} = 0.1629$$

We can now compute the constants, A, B:

$$A = g \left( \frac{T_0}{W} - \mu \right) = 32.174 \left( \frac{0}{30000} - 0.02 \right) = -0.6435 \text{ ft/sec}^2$$

$$B = \frac{g}{W} \left[ \frac{1}{2} \rho S (C_{D_g} - \mu C_{L_g}) + a \right] = \frac{32.174}{30000} \left[ \frac{1}{2} (0.002377) 750 (0.1629 - 0.02 (1.3032)) \right]$$

$$B = 1.3085 \times 10^{-4} \text{ 1/ft}$$

We can calculate the distance to the braking velocity,  $V_b$

$$S_B = \frac{1}{2B} \ln \left( \frac{A - B V_{TD}^2}{A - B V_B^2} \right) = \frac{10^4}{2 (1.3085)} \ln \left[ \frac{-0.6435 - 1.3085 \times 10^{-4} (160.7)^2}{-0.6435 - 1.3085 \times 10^{-4} (128.56)^2} \right]$$

$$= 1376 \text{ ft}$$

After braking we need to recalculate the constants A and B.

$$A = g \left( \frac{T_0}{W} - \mu \right) = 32.174 (0 - 0.5) = -16.085 \text{ ft/sec}^2$$

$$B = \frac{g}{W} \left[ \frac{1}{2} \rho S (C_{D_g} - \mu C_{L_g}) \right] = \frac{32.174}{30000} \left[ \frac{1}{2} (0.002377) (750) [0.1629 - 0.5 (1.3032)] \right]$$

$$= -4.663 \times 10^{-4} \text{ 1/ft}$$

The landing roll after braking is given by:

$$S_2 = \frac{1}{2B} \ln \left( \frac{A - B V^2}{A} \right) = \frac{10^4}{2 (4.663)} \ln \left( \frac{-16.085 + 4.663 \times 10^{-4} (128.56)^2}{-16.085} \right) = 699.4 \text{ ft}$$

The total ground run is given by the sum of the two:

$$S_{total} = S_B + S_2 = 1376.0 + 699.4 = 2075 \text{ ft}$$


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### ***Balanced Field Length***

In order for a multi-engined commercial aircraft to takeoff from a runway, the runway must at least be as long as the balanced field length. The balanced field length is determined by considering two options available to the pilot if an engine fails. One is to continue the takeoff on the remaining engines to clear the 50 ft (15m) obstacle and establish a takeoff distance, and the other is to apply the brakes as soon as possible after the engine failure and to bring the aircraft to a halt in some distance. If the two distances are the same, that distance is called the balanced field length. It is determined by:

1. Assuming a speed (and hence a distance along the runway) that the engine failure occurs.
2. Continue the takeoff on the remaining engines and compute the additional distance for the vehicle to clear a 50 ft obstacle, determining the takeoff distance
3. Starting with the speed assumed in (1), assume two additional seconds go by and then the engines are shut down and the brakes are applied and the ground roll to stop calculated.
4. Compare the distance in (2) with that in (3). If the distance to stop is shorter than the distance to fly over the 50 ft obstacle, then increase the guess in step (1). If the distance to stop is shorter than that required to clear the 50 ft obstacle, then decrease the failure airspeed in step (1). Continue this procedure until the total takeoff distance and the total distance to stop are the same. This distance will be the balance field length, and the associated velocity found is called the ***critical engine failure speed***, and is usually designated as  $V_I$ .

### ***Effect of Ground Wind***

Everyone knows that aircraft takeoff into the wind. Supposedly, this action will shorten the ground run. That makes some sense since in the limit, if the ground wind were equal to the takeoff speed the aircraft would not need any distance to takeoff! In dealing with the ground wind, we must be careful about which speed is important when considering Newton's laws and acceleration, and which speed is important when determining the aerodynamic forces on the vehicle. To this end, we can define the following speeds:

- $V$  = true ground speed (of the vehicle)
- $V_{TAS}$  = true airspeed (of the vehicle)
- $V_w$  = true wind speed (ground wind) = constant

Then from the definitions, we have:

$$V_{TAS} = V \pm V_w \quad (29)$$

Newton's equation for acceleration can be written as:

$$\frac{dV}{dt} = A - B V_{TAS}^2 \quad (30)$$

The key here is to note that the acceleration is the derivative of the true ground speed, but the aerodynamic forces depend on the true airspeed, the speed of the air relative to the vehicle. We can note the following relations from Eq. (29):

$$V = V_{TAS} \mp V_w \quad (31)$$

and further,

$$\frac{dV}{dt} = \frac{dV_{TAS}}{dt} \quad (32)$$

since  $V_w = \text{const.}$

Then the acceleration equation takes the form:

$$\frac{dV_{TAS}}{dt} = A - B V_{TAS}^2 \quad (33)$$

*The time to takeoff is determined from:*

$dt = \frac{dV_{TAS}}{A - B V_{TAS}^2} \quad (34)$
--

with the following boundary conditions:

$$\text{when } t = 0, \quad V_{TAS} = \pm V_w \quad \text{and} \quad \text{when } t = t_2, \quad V_{TAS} = V_{TO} \quad (\text{or some other specified airspeed})$$

*The distance to takeoff is a little more complicated.*

Using true ground speed, we can write the inertial acceleration with respect to the ground as:

$$\frac{dV}{dt} \cdot \frac{V}{V} = \frac{V \frac{dV}{dt}}{\frac{dS}{dt}} = V \frac{dV}{dS} = A - B V_{TAS}^2$$

If we use Eqs. (31) and (32) to remove V from the equation. We get

$$(V_{TAS} \mp V_w) \frac{dV_{TAS}}{dS} = A - B V_{TAS}^2 \quad (35)$$

Solving for dS, we get the takeoff ground roll equation:

$ds = \frac{V_{TAS} dV_{TAS}}{A - B V_{TAS}^2} \mp V_w \frac{dV_{TAS}}{A - B V_{TAS}^2}$	Upper sign for headwind Lower sign for tailwind
--	--

(36)

If we integrate the above equation, we see that the first term looks like a distance term, and the second term looks like a time term times the ground wind, and that is indeed the fact. We have the following results: (presented for a headwind, upper sign).

Time for Ground Roll In the Presence of a Headwind

$t_2 - t_1 = \frac{1}{\sqrt{AB}} \left[ \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_{TAS_2} \right) - \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_{TAS_1} \right) \right] \quad (37)$
---

$S_2 - S_1 = \frac{1}{2B} \ln \left( \frac{A - B V_{TAS_1}^2}{A - B V_{TAS_2}^2} \right) - V_w (t_2 - t_1) \quad (38)$
--

If starting from rest, then  $t_1 = 0$ , and  $V_1 = 0 \Rightarrow V_{TAS1} = \pm V_w$

If proceeding to takeoff,  $t_2 =$  time to takeoff, and  $V_{TAS2} = V_{TO}$

*The time and distance equations (for headwind) become:*

$$t = \frac{1}{\sqrt{AB}} \left[ \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_{TO} \right) - \tanh^{-1} \left( \sqrt{\frac{B}{A}} V_w \right) \right] \quad (39)$$

$$S = \frac{1}{2B} \ln \left( \frac{A - B V_w^2}{A - B V_{TO}^2} \right) - V_w (t) \quad (40)$$

The equations for landing in a headwind are essentially the same, but likely have different values (and signs) for A and B. The terminal airspeed is the ground wind speed rather than zero. Hence *for the ground landing run we have:*

$$S = \frac{1}{2B} \ln \left( \frac{A - B V_{TO}^2}{A - B V_w^2} \right) - v_w (t) \quad (41)$$