3. Standard Atmosphere

The concept of a standard atmosphere is required so that we can perform comparisons of performance among different aircraft. On any given day it is likely that the atmosphere will not be standard. Consequently the results of flight tests would be different, even for the same aircraft, if the tests were performed on different days, say mid summer and mid winter. However these results can be reduced to conditions in a standard atmosphere and then compared.

The assumptions for a standard atmosphere are:

Assumptions:

1. The atmosphere is static (V = 0)

Then the differential form of Bernoulli's equation is evaluated with V = 0. The result is the aero-hydro static equation:

$$dP = -\rho\, g\, dh_G \tag{1}$$

where ρ = density
g = acceleration due to gravity
h_G = geometric (tapeline) altitude

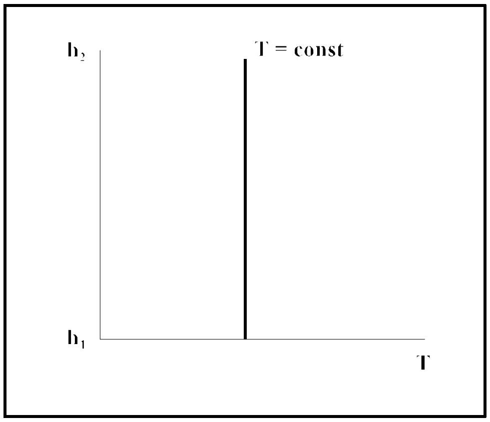
2. The atmosphere also behaves as a perfect gas and hence satisfies the perfect gas equation:

$$P = \rho R T \tag{2}$$

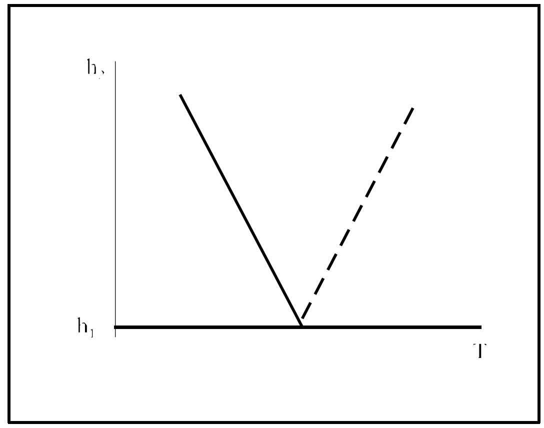
Where ρ = density
R = perfect gas constant
T = Temperature

Here we have two equations to establish three atmospheric properties, P, ρ, T , each as a function of altitude. Therefore, in order to be able to obtain a solution for the atmosphere, we need another equation!

3. It is assumed that the atmosphere is divided into standard layers that either have constant temperature, or a constant temperature gradient (change in temperature with altitude). The temperature profile would look as shown in one of the following diagrams:



Constant Temperature Layer



Constant Temperature Gradient Layer
________decreasing with altitude
_ _ _ _ _increasing with altitude

In general the temperature profile (in a given layer) can be expressed as:

$$T(h) = T_1 + K(h - h_1) \tag{3}$$

Where: T_1 = the temperature at the bottom of the layer
h_1 = the altitude at the bottom of the layer
h = altitude measured from the ground

$K = \frac{dT}{dh}$ = constant = temperature gradient in that layer $\begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$

Then the three equations that govern the description of the standard atmosphere are:

$$\begin{aligned} dP &= -\rho\, g\, dh_G \\ P &= \rho R T \\ T &= T_1 + K(h - h_1) \end{aligned} \tag{4}$$

We now need to use these equations to find P(h), and $\rho(h)$. The functions T(h) are the means by which we specify the Standard Atmosphere. We have two problems to deal with: 1) How to treat the variation in gravity as the altitude increases, and 2) How to define the temperature distribution in the standard atmosphere. We will deal with these next.

Variation of Gravity

The gravitational field of the Earth is called an inverse square gravitational field and obeys

Newton's law of gravitation: The force of gravity between two bodies is proportional to the product of the two masses and inversely proportional to the square of the distance between them: $F \propto \frac{m_1 m_2}{r^2}$. At the surface of the Earth the weight of mass 2 is, W = m_2 g_0, so that $g_0 = \frac{\mu}{R_e^2}$ where μ is the constant of proportionality times m_1, the mass of the Earth and R_e is the radius of the Earth. However, as we go up in altitude, the value of g becomes smaller. Using the relations above, we can calculate the value of g at any altitude to be:

$$g = g_0 \left(\frac{R_e}{r} \right)^2 = g_0 \left(\frac{R_e}{R_e + h_G} \right)^2$$

where r = distance from the center of the Earth and R_e is the radius of the Earth. We could substitute this value for g in our atmospheric equations and proceed, although the resulting integrations might be messy. To alleviate the mess and most likely due to historical developments, we will account for the variation in gravity in a different and ***strange*** way. We will define a "false" altitude called the ***<u>geopotential altitude</u>*** , *h*, in the following manner: We will take the hydrostatic equation, Eq. (1) and define the following:

$$dP = -\rho\, g\, dh_G \equiv -\rho\, g_0\, dh \qquad (5)$$

where h_G = geometric or tapeline altitude
h = geopotential altitude

From Eq. (5) we have:

$$g\, dh_G \equiv g_0\, dh \qquad (6)$$

From Eq (6) we can develop a relation to establish either geopotential altitude in terms of the geometric altitude, or the geometric altitude in terms of the geopotential. Can you do that?

The ***standard atmosphere is based on the geopotential altitude***. Hence the temperature profiles for the standard atmosphere are with respect to the geopotential altitude, *h*.

Temperature Distribution (Standard Atmosphere)

As indicated previously the standard atmosphere is defined by the temperature profiles in different atmospheric layers. The basis for the temperature, pressure and density distribution with altitude are these values at the Earth's surface, and are called sea-level values. Hence we have

Standard sea-level conditions:

Variable	US	SI
Pressure - P	2116.21695 lbs/ft^2	1.01325x10^5 N/m^2 (Pascals)
Density - ρ	0.002376919 slug/ft^3	1.2250 kg/m^3
Temperature	59 deg F = 518.688 deg R	15 deg C = 288.16 deg K
Gravitational Acceleration- g_0	32.174 ft/sec^2	9.80665 m/s^2
Universal Gas Constant	1716.488 ft-lb/slug deg R	287.0368 Joules/kg deg K
Radius of the Earth	2.0925644x10^7 ft	6378.1363 km

The subsequent temperature distributions that define the standard atmosphere are obtained from the 1976 standard atmosphere adopted on October 15, 1976. It is the same as previously defined atmospheres (ICAO 1952, ARDC 1956 & 1959, ICAO 1962 & 1966) up to 20 kilometers. At that point there are significant changes. In the region of the atmosphere encountered by aircraft the various “standard atmospheres” are the same. The following is the 1976 standard atmosphere, up to approximately 84 kilometers geopotential altitude. Note that the standard atmosphere is defined in SI units.

1st Layer of Standard Atmosphere (Troposphere) The region of air above the Earth’s surface where the temperature drops linearly with altitude.

Troposphere:

Range	0 - 36,089 ft	0 - 11,000 m
Temperature gradient	-0.00356616 deg R/ft	-0.0065 deg K/m

Definition: Tropopause - The boundary between the troposphere and the next layer, the stratosphere, the top of the troposphere. Hence it occurs at 36,089 ft or 11,000 m.

Stratosphere (lower): The region or layer of the atmosphere above the troposphere where the temperature is constant

Range	36,089 - 65,617 ft	11,000 - 20,000 m
Temperature gradient	0 deg R/ft	0 deg K / m
Temperature - T	389.988 deg R (-69.7 deg F)	216.66 deg K (-56.5 deg C)

Stratosphere (upper): the region or layer of the atmosphere above the lower stratosphere where the temperature increases slightly with altitude

Range	65,617 - 104,987 ft	20,000 - 32,000 m
Temperature gradient	0.0054864 deg R/ft	0.001 deg K/m

No-name 1: (Sometimes included in the upper stratosphere) The region or layer of the atmosphere above the upper stratosphere where the temperature increases more with altitude

Range	104,987 - 154,199 ft	32,000 - 47,000 m
Temperature gradient	0.0015362 deg R/ft	0.0028

No-name 2: (Sometimes included in the upper stratosphere) The region or layer of the atmosphere above the previous one where the temperature is constant with altitude

Range	154,199 - 167,323 ft	47,000 - 51,000 m
Temperature gradient	0 deg R/ft	0 deg K/m
Temperature - T	487.17 deg R (27.482 deg F)	270.65 deg K (-2.51 deg C)

Mesosphere: The region or layer of the atmosphere above the previous one (stratosphere) where the temperature gradient is negative.

Range	167,323 - 232,940 ft	51,000 - 71,000
Temperature gradient	-0.0015362 deg R/ft	-0.0028 deg K/m

No-name 3: (Sometimes included in the mesosphere) The region or layer of the atmosphere above the previous one where the temperature gradient is less negative

Range	232,940 - 278,386 ft	71,000 - 84,852 m
Temperature gradient	0.0010973 deg R/ft	-0.002 deg K/m

Note that 84,852 m geopotential altitude is approximately 86,000 m geometric or tapeline altitude. You should be aware that there are different “standard” atmospheres put out by different organizations and also revised as more information is gathered. Furthermore there are supplements for hot and cold day standards and for different latitudes. These “standards” are more likely to be used in aircraft simulations then in standardizing performance.

Standard Atmosphere Equations

The governing equations for developing the pressure and density distributions with altitude in a standard atmosphere can now be developed. First we will develop the equations for the constant temperature layers, and then for the constant temperature gradient layers.

General:

The two equations of interest are the aero-static equation, Eq. (1) and the perfect gas law, Eq. (2).

$$dP = \rho g_0 dh$$
$$P = \rho R T \tag{7}$$

Note that we use the geopotential height in these equations. By combining these equations, we can eliminate the density:

$$\frac{dP}{P} = -\frac{g_0 dh}{R T} \tag{8}$$

Equation (8) is a general relation for the atmosphere. We can now examine how the pressure changes with altitude for the different kinds of temperature layers.

Constant Temperature Layer (T = const)

Here, T = const = T_1, where T_1 is the temperature at the base of the layer. In constant temperature layers, $K = 0$ and $T = T_1$. We can substitute into Eq. (8) to obtain:

$$\frac{dP}{P} = -\frac{g_0 dh}{R T_1}$$

since the left hand side is only a function of P, and the right hand side is constant, other than *dh*, we can easily integrate this equation to obtain the pressure distribution with altitude.

$$\ln P \Big|_{P_1}^{P} = -\frac{g_0}{R T} h \Big|_{h_1}^{h}$$

$$\ln P - \ln P_1 = -\frac{g_0}{R T_1}\left(h - h_1\right)$$

Or

$$\frac{P}{P_1} = e^{-\frac{g_0}{RT_1}(h - h_1)} \tag{9}$$

From the perfect gas law we have:

$$\frac{P}{P_1} = \frac{\rho R T_1}{\rho_1 R T_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT_1}(h - h_1)} \tag{10}$$

Equations (9) and (10) tell us how pressure and density change with altitude in a constant temperature layer.

Constant Temperature Gradient Layer [$T = T_1 + K(h - h_1)$]

We can start with the general equation, Eq. (8) and replace T with the expression that includes the constant gradient:

$$\frac{dP}{P} = -\frac{g_0}{RT}dh = \frac{g_0}{R}\frac{dh}{[T_1 + K(h - h_1)]}$$

$$= -\frac{g_0}{RK}\frac{K\,dh}{[T_1 + K(h - h_1)]}$$

$$\ln P \Big|_{P_1}^{P} = -\frac{g_0}{RK}\ln[T_1 + K(h - h_1)]\Big|_{h_1}^{h}$$

$$\ln\frac{P}{P_1} = -\frac{g_0}{RK}\ln\frac{T(h)}{T(h_1)}$$

Finally we have:

$$\frac{P}{P_1} = \left(\frac{T(h)}{T_1}\right)^{-\frac{g_0}{RK}} \tag{11}$$

where $T(h) = T_1 + K(h - h_1)$.

From the perfect gas law we have:

$$\frac{P}{P_1} = \frac{\rho R T}{\rho_1 R T_1} = \left(\frac{\rho}{\rho_1} \right) \left(\frac{T}{T_1} \right) \quad \Rightarrow \quad \frac{\rho}{\rho_1} = \left(\frac{P}{P_1} \right) \left(\frac{T}{T_1} \right)^{-1}$$

Then, using Eq. (11), we can write:

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1} \right)^{-\left(\frac{g_0}{RK} + 1 \right)} \qquad (12)$$

Equations (12) and (13) tell us how the pressure and density changes with altitude in a layer of atmosphere with a constant temperature gradient.

We now have two schemes for calculating P(h) and ρ(h), and we have T(h) for constant temperature and constant temperature gradient layers of the atmosphere. We also have the values of P, ρ, and T at the surface of the Earth. We can now start with these values, and the values of the gradients presented in the tables previously, to generate the standard atmosphere.

Example:

For example, we can calculate values of the temperature, pressure, and density at any altitude in the troposphere using the constant temperature gradient equations. We can also calculate the values at the top of the troposphere (the tropopause) using the same equations, and get the starting (or base values) for the stratosphere.

Calculating values at the tropopause

It is generally better to use the SI system in all atmospheric calculations. If information is given in US customary units, then we can convert to SI units before, and back to US after, doing the calculations using SI units.

Temperature Distribution:

Using the numbers from the previous tables, we can write the temperature anywhere in the troposphere as

$$\begin{aligned} T &= T_{SL} + K(h - 0) \\ &= 2.88.16 - 0.0065\ {}^\circ K/m\ (h) \\ &= 288.16 - 0.0065\ (11{,}000) \\ &= 216.66^\circ K \end{aligned}$$

Pressure Distribution:

$$\frac{P}{P_{SL}} = \left(\frac{T}{T_{SL}} \right)^{-\frac{g_0}{RK}}$$

$$\frac{P_{tp}}{P_{SL}} = \left(\frac{216.66}{288.16} \right)^{-\frac{9.80665}{287.0368\,(-0.0065)}}$$

$$= \left(\frac{216.66}{288.16} \right)^{5.256174} = 0.22336$$

We can note that all the terms in the above equation are non-dimensional. Hence to calculate the pressure at the tropopause we can do it in either set of units by picking the sea-level value in those units:

$$P_{tp} = 2116.21695\,(0.22336) = 472.8 \text{ lbs/ft}^2$$

$$= 1.01325x10^5\,(0.22336) = 22631.95 \text{ Pa}$$

Density Distribution:

$$\frac{\rho}{\rho_{SL}} = \left(\frac{T}{T_{SL}} \right)^{-\left[\frac{g_0}{RK} + 1\right]}$$

$$= \left(\frac{216.66}{288.16} \right)^{-\left[\frac{9.80665}{287.0368\,(-0.0065)} + 1\right]}$$

$$= \left(\frac{216.66}{288.16} \right)^{4.256174}$$

$$= 0.297065$$

Again, we can compute the density in either units by using the proper sea-level value:

$$\rho_{tp} = 0.002377\,(0.297065) = 0.000706 \text{ slugs/ft}^3$$

$$= 1.2250\,(0.297065) = 0.3639 \text{ kg/m}^3$$

The values at any point in the troposphere can be calculated the same way. First calculate the temperature at the altitude of interest, and then use the pressure and density equations as above.

Example:

Calculate the atmospheric properties at 20 km. We can note that 20 km is in the stratosphere so we can use the equations that apply to constant temperature layers. However, we need to know the values at the base of the layer. Fortunately, we calculated these in the last example. T_1 = 216.66 deg K, P_1 = 22631.95 Pascals, ρ_1 = 0.3639 kg/m^3
Temperature:

$$T = T_1 = 216.66°K$$

Pressure:

$$P = P_1 e^{-\frac{g_0}{RT_1}(h - h_1)}$$

$$= (22631.95)\, e^{-\frac{9.80665}{287.0368\,(216.66)}(20{,}000-11{,}000)}$$

$$= 5474.7793 \text{ Pascals}$$

Density:

$$\rho = \rho_1 \frac{P}{P_1}$$

$$= 0.3639 \frac{5474.7793}{22631.95}$$

$$= 0.0880 \text{ kg/m}^3$$

Note that these values would serve as the base values for the upper stratosphere layer.

The standard atmosphere is tabulated in almost every aerospace book that relates to atmospheric vehicles. These tables were generated using the above relations. For the purpose of this class, you will be expected to ***use the tables***, and not the above equations.

Geometric (Tapeline) and Geopotential Altitude

We still need to resolve the geometric- geopotential altitude relations. We have shown previously that the relation between the two altitude representations is given by:

$$g\, dh_G = g_0\, dh \tag{13}$$

Substituting in for g, we have:

$$\int_0^h g_0\, dh = \int_0^{h_G} g_0 \left(\frac{R_e}{R_e + h_G} \right)^2 dh_G = -g_0 \frac{R_e^2}{R_e + h_G} \Big|_0^{h_G} = -g_0 R_e^2 \left[\frac{1}{R_e + h_G} - \frac{1}{R_e} \right]$$

or

$$h = \left(\frac{R_e}{R_e + h_G} \right) h_G \qquad \text{or} \qquad h_G = \left(\frac{R_e}{R_e - h} \right) h \tag{14}$$

In principle, given the geometric altitude, one could use Eq. (14) to convert to the geopotential altitude, and then go into the tables (or equations) and determine the temperature, pressure, and density for that altitude. We can note, however, that at about 45,000 ft, the difference between the tapeline (geometric) altitude (45,000 ft) and the geopotential altitude (44,903) is approximately 100 ft. 100 out of 45000 is about 0.22 percent. Consequently, at low altitudes we can generally neglect the difference. However there is another way that we can standardize our performance results and that is to define ***Pressure Altitude***.

Definition: Pressure Altitude - The pressure altitude is the altitude associated with a given pressure assuming a standard atmosphere.

Consequently, if we know the outside pressure, we can determine from standard atmosphere tables, the altitude associated with that pressure. We then say that we are at that pressure altitude.

Example

Given the pressure is P = 31,000 N/m^2, find the pressure altitude.

From the standard atmosphere tables:

@ h = 8000 km	P = 35580 N/m^2	(lower)
@ h = 9000 km	P = 30730 N/m^2	(upper)

Interpolate:

$$\frac{h - h_L}{h_u - h_L} = \frac{P - P_L}{P_u - P_L}$$

or

$$h = h_L + \frac{P - P_L}{P_u - P_L}(h_u - h_L)$$

$$h = 8000 + \frac{31{,}000 - 35580}{30{,}730 - 35{,}580}(9000 - 8000)$$

$$= 8944 \text{ m}$$

Hence we are at a pressure altitude of 8944 meters. (Geopotential).

Virtually all flight testing is done at a selected pressure altitude. Then temperature corrections are made to the data (beyond the scope of this work), and the results are thus reduced to standard atmosphere results. Differences between the pressure altitude and the actual altitude can be quite large on any given day!

Definition: Density Altitude - The density altitude is the altitude associated with a given density assuming a standard atmosphere. To establish the density, generally the pressure and temperature are measured, and the perfect gas law used to calculate the density.

Altimeters

The variation of pressure with altitude can be used to estimate the altitude of a vehicle. The instrument consists of a pressure source (usually a small hole in the fuselage of the aircraft positioned so that it reads static pressure (not effected by the motion of the aircraft), a corrugated box containing a vacuum, and linkage to a dial. As the pressure increases or decreases, the box contracts or expands, and these motions amplified by mechanical or electronic means to move the dial on the face of the altimeter. Hence for a given pressure, the dial will record a given altitude according to the standard atmosphere.

Unfortunately, on any given day, the atmosphere is not standard. In order to account for differences, altimeters are equipped with an adjustment knob. This knob can be used to set the sea-level atmospheric pressure in a window on the face of the altimeter called the Kollsman window. All weather reports deliver the barometric pressure corrected to sea-level. This information is given in terms of inches of mercury or millimeters of mercury. The value that is used for a standard atmosphere it is 29.92 inches of mercury or 1013 mm mercury (hg). For all flights above 10,000 ft, the altimeter is set at the standard value of 29.92 and the altitude is designated as a “flight level.” For altitudes below that, the altimeter is set to the local (sea-level) barometric pressure. If that is not available, the altimeter can be set before takeoff by turning the knob until the altitude reads the altitude of the airport. Setting the altimeter is an item on the check list! Altimeters set in this manner estimate the altitude within 50 ft or better. For flight testing, the altimeter is set to the standard 29.92 inches of mercury, and the altitude read of the altimeter is then the pressure altitude at which the test is performed.