AOE Problem Sheet 9 (ans)

The problems on this sheet deal with an aircraft with the following properties:

W = 600,000 lbs	T_max~=~180,000 lbs	S = 5128 ft^2
$C_{D_{0L}}$ = 0.017	K = 0.042	$C_{L_{max}}$ = 2.2
TSFC = 0.85 (lbs/hr)/lb	W_f = 180,000 lbs	

46. Find the range (in miles) flown under maximum range flight conditions (at least at the beginning of the flight), assuming the flight starts at 10,000 ft. for the case where:
    a) h = constant, angle-of-attack = constant
    b) V = constant, angle of attack = constant
    c) V = constant, h = constant

There are certain calculations that are common to all parts of the problem. In particular, we are interested in best range flight conditions. Best range flight conditions are obtained (for a thrust-rated aircraft) as those that maximize the ratio of $\frac{C_L^{1/2}}{C_D}$. For a parabolic drag polar, these conditions are given by:

$$C_{L_{MR}} = \sqrt{\frac{C_{D_{0L}}}{3\,K}} = \sqrt{\frac{0.017}{0.042}} = 0.3673 \qquad \text{and} \qquad C_{D_{MR}} = \frac{4}{3}\,D_{C_{0L}} = \frac{4}{3}\,0.017 = 0.0227$$

We also need to convert the thrust specific fuel consumption to basic units:

$$c_t = 0.85\,\frac{\text{lbs}}{\text{hr lb}}\,\frac{1}{3600}\,\frac{\text{hr}}{\text{sec}} = 2.3611x10^{-4}\,\frac{\text{lbs}}{\text{sec lb}}$$

a) Constant α, constant h, implies that C_L, C_D, and density are constant. For max range, we can use the values of C_L and C_D that were calculated above. At 10,000 ft, the density is ρ = 0.001756 slugs/ft^3 . Under these conditions the range equation integrates to:

$$R = \frac{2\sqrt{2}}{c_t\sqrt{\rho\,S}}\left(\frac{C_L^{1/2}}{C_D}\right)\left(\sqrt{W_1} - \sqrt{W_2}\right)$$

$$= \frac{2\sqrt{2}\,x\,10^4}{2.3611\,\sqrt{0.001756\,(5128)}}\left(\frac{0.3673^{1/2}}{0.0227}\right)\left(\sqrt{600000} - \sqrt{420000}\right)$$

$$= 13484879 \text{ ft} = 2554 \text{ miles}$$

b) Constant α, constant V, implies that C_L, and C_D are constant. We can therefore use the values calculated above for maximum range. WE need to determine the constant speed. The speed is determined from

$$V = \sqrt{\frac{W}{1/2\,\rho\,S\,C_L}} = \sqrt{\frac{600000}{1/2\,(0.001756)\,(5128)\,0.3673}} = 602.59 \text{ ft/sec}$$

With constant airspeed, we the range equation integrates to

$$R = \frac{V}{c_t}\frac{C_L}{C_D}\ln\frac{W_1}{W_2} = \frac{602.59\,x10^4}{2.3611}\left(\frac{0.3673}{0.0227}\right)\ln\frac{600000}{420000} = 14729066 \text{ ft} = 2789 \text{ mi}$$

Note that this technique is superior to that used in part (a).

c) Constant V and constant h, implies constant density and airspeed, hence the angle-of-attack must change to satatisfy the L=W requirement. As instructed, we will satisfy the best range flight condition at the beginning of the flight. Hence all the numbers calculated previously can be used.

Lets calculate the requisite constants in the range equation:

$$A = C_{D_{0L}}\,1/2\,\rho\,S\,V^2 = 0.017\,(1/2)\,(0.001756)\,(5128)\,(602.59)^2 = 27792.991$$

$$B = \frac{K}{1/2\,\rho\,S\,V^2} = \frac{0.042}{1/2\,(0.001756)\,(5128)\,(607.59)^2} = 2.56899x10^{-8}$$

Then we need the following combinations:

$$\sqrt{A\,B} = \sqrt{C_{D_{0L}}K} = \sqrt{0.017\,(0.042)} = 0.0267 \qquad \sqrt{\frac{B}{A}} = \sqrt{\frac{2.569x10^{-8}}{27792.991}} = 9.6142x10^{-7}$$

Finally, the appropriate range equation is given by:

$$R = \frac{V}{c_t}\frac{1}{\sqrt{A\,B}}\left[\tan^{-1}\left(\sqrt{\frac{B}{A}}\,W_1\right) - \tan^{-1}\left(\sqrt{\frac{B}{A}}\,W_2\right)\right]$$

$$= \frac{602.59x10^4}{2.3611}\frac{1}{0.0267}\left[\tan^{-1}\left(9.6142x10^{-7}\,(600000)\right) - \tan^{-1}\left((9.6142x10^{-7}\,(420000)\right)\right]$$

$$= 13319566 \text{ ft} = 2523 \text{ miles}$$

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47. Determine the altitude (ft) and speed (ft/sec) for each case above:

a) at the initial point (h = 10,000 ft)

b) at the final point

The governing equation to do these calculations is $C_L = \dfrac{W}{1/2\,\rho\, S\, V^2}$. Then we can impose the conditions of the flight and calculate what is unknown.

Conditions 46a, h and α constant. Then C_L and ρ are constant and the above equation tells us that W/V^2 must be a constant, or $V \propto \sqrt{W}$, Then

$$V_2 = V_1 \sqrt{\frac{W_2}{W_1}} = 602.6\sqrt{\frac{420000}{600000}} = 504.2 \text{ ft/sec}$$

For condition 46b, V and α are constant, so the above equation tells us that W/ρ is constant, or

$$\rho_2 = \frac{W_2}{W_1}\rho_1 = \frac{420000}{600000}(0.001756) = 0.001229 \qquad \Rightarrow \qquad h_2 = 21000 \text{ ft}(\rho = 0.001225)$$

In summary we have:

	h - Start	V - Start	h - finish	V - finish
h, α = const	10,000 ft	602.6 ft/sec	10,000 ft	504.2 ft/sec
V, α = const	10,000 ft	602.6 ft/sec	21,000 ft	602.6 ft/sec
h , V = const	10,000 ft	602.6 ft/sec	10,000 ft	602.6 ft/sec

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48. Find the time to achieve the three ranges above in problem 46, in hours

For the case of constant angle-of-attack, the same endurance equation holds regardless if h or V is held constant. The only assumption necessary to develop the endurance equation was that the angle-of-attack is constant. Hence for 46a, b, we have:

$$E_{1,2} = \frac{1}{c_t}\frac{C_L}{C_D}\ln\frac{W_1}{W_2} = \frac{10^4}{2.3611}\left(\frac{0.3673}{0.0227}\right)\ln\frac{600000}{420000} = 24442.9\text{sec} = 6.79 \text{ hrs}$$

Note that this is also range 46b divided by the ***constant*** velocity

For the last endurance, since range 46c is also at constant velocity, we can simply divide the range by velocity to get:

$$E_3 = \frac{R}{V} = \frac{13319566}{602.59} = 22104 \text{ secs} = 6.14 \text{ hrs}$$

49. For this aircraft, starting at 10,000 ft altitude, find

a) the conditions for maximum endurance (C_L, C_D, L/D, and V)

b) the maximum endurance for this aircraft (in hours)

c) the flight schedule used for (b)

For maximum endurance of a thrust-rated aircraft, we must fly at maximum L/D conditions or min drag conditions, For a parabolic drag polar with constant parameters, these conditions are given by:

$$C_{L_{md}} = \sqrt{\frac{C_{D_{0L}}}{K}} = \sqrt{\frac{0.017}{0.042}} = 0.6362 \qquad C_{D_{md}} = 2\,C_{D_{0L}} = 2\,(0.017) = 0.034$$

$$\left.\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{C_{D_{0L}}K}} = \frac{1}{2\,(\sqrt{0.017\,(0.042)})} = 18.712$$

$$V_{md} = \sqrt{\frac{W}{1/2\,\rho\,S\,C_{L_{md}}}} = \sqrt{\frac{600000}{1/2\,(0.0017560)\,5128\,(0.6362)}} = 457.7\ \text{ft/sec}$$

From the endurance calculated in the previous problem, it is clear that the maximum endurance would occur using flight schedule a or b in that problem. The key ingredient is constant angle-of-attack. If we use the above numbers in the endurance equation for constant angle of attack, we have:

$$E_{1,2} = \frac{1}{c_t}\frac{C_L}{C_D}\ln\frac{W_1}{W_2} = \frac{10^4}{2.3611}\left(\frac{0.6362}{0.034}\right)\ln\frac{600000}{420000} = 28266.6\text{sec} = 7.85\ \text{hrs}$$

Note that this number is about 1 hr longer than the endurance under max range conditions. Also note that the range (using schedule b) would be 2450 miles, significantly shorter than 2789 miles.

Using max endurance values in schedule c would give only 2400 miles compared to 2523 miles, and would take 7.69 hours.

Hence level flight and constant airspeed are not the best flight conditions for either range or endurance, but that is the most common flight schedule for practical reasons.

50. For the case of level flight and constant airspeed, we can not maintain the optimal conditions for maximum endurance (or range for that matter). In class we looked at the endurance for this vehicle and selected the maximum endurance V for the initial altitude given so we were not at maximum endurance conditions at the end of the flight. On the other hand, we also looked at the case where we picked the V for maximum endurance at the end of the flight so that we were not at maximum endurance conditions at the beginning of the flight. Unless there were errors in the

calculations (there could have been), the endurance calculated was close to being the same.

a) Repeat these two calculations for the case of maximum range. Pick the max range airspeed for the beginning of the flight and compare the range obtained with that obtained using the airspeed that would give us maximum range conditions at the end of the flight.

b) Investigate if we could get a better range by using an airspeed different from those in part (a), and if so, how could we determine it. Calculate the range using such a speed (if it exists).

We have already done the max range conditions at the beginning of the flight. These results were:

Maximum range conditions calculated at beginning of flight, h = 10,000 ft and constant

$$C_{L_{MR}} = 0.3673 \qquad C_{D_{MR}} = 0.0227 \qquad V_{MR} = 602.59 \text{ ft/sec}$$

The maximum range conditions at the end of the flight are the same for lift and drag coefficient, but the airspeed will be different

$$V_{MR} = \sqrt{\frac{W}{1/2\,\rho\, S\, C_{L_{MR}}}} = \sqrt{\frac{420000}{1/2\,(0.001756)\,5128\,(0.3673)}} = 503.96 \text{ ft/sec}$$

We need to calculate new constants A, and B:

$$A = C_{D_{0L}}\, 1/2\,\rho\, S\, V^2 = 0.017\,(1/2)\,0.001756\,(5128)\,503.96^2 = 1.94394x10^4 \text{ lbs}$$

$$B = \frac{K}{1/2\,\rho\, S\, V^2} = \frac{.042}{1/2\,(0.001756)\,5128\,(503.96^2)} = 3.6729x10^{-8}$$

Then we need the following combinations:

$$\sqrt{A\,B} = \sqrt{C_{D_{0L}}\,K} = \sqrt{0.017\,(0.042)} = 0.0267 \qquad \sqrt{\frac{B}{A}} = \sqrt{\frac{3.6729x10^{-8}}{19439.43}} = 1.3746x10^{-6}$$

Finally, the appropriate range equation is given by:

$$R = \frac{V}{c_t}\frac{1}{\sqrt{A\,B}}\left[\tan^{-1}\left(\sqrt{\frac{B}{A}}\,W_1\right) - \tan^{-1}\left(\sqrt{\frac{B}{A}}\,W_2\right)\right]$$

$$= \frac{503.96x10^4}{2.3611}\ \frac{1}{0.0267}\left[\tan^{-1}\left(1.37457x10^{-6}(600000)\right) - \tan^{-1}\left((1.37457x10^{-6}(420000)\right)\right]$$

$$= 13275859 \text{ ft} = 2514 \text{ miles}$$

This compares with 2523 miles calculated in problem 46c. (Could be round off errors, or could be different).

b) I don’t know of any clever way to do this part yet.