AOE 3104 Problem Sheet 7 (ans)

36. An aircraft weighs 56,000 lbs and has a 900 ft^2 wing area. Its drag polar is given by: $C_D = 0.016 + 0.04\, C_L^2$.

- a) Find the minimum thrust required for straight and level flight and the corresponding airspeeds at sea-level and at 30,000 ft
- b) Find the minimum power required and the corresponding true airspeeds for straight and level flight at sea-level and at 30,000 ft.

Minimum thrust required is equal to minimum drag. So calculate the minimum drag flight conditions:

$$C_{L_{md}} = \sqrt{\frac{C_{D_{0L}}}{K}} = \sqrt{\frac{0.016}{0.04}} = 0.6324 \qquad C_{D_{md}} = 2\,C_{D_{0L}} = 2\,(0.02) = 0.04$$

$$\left.\frac{L}{D}\right)_{\max} = \frac{C_{L_{md}}}{C_{D_{md}}} = \frac{1}{2\sqrt{C_{D_{0L}}K}} = \frac{1}{2\sqrt{0.016\,(0.04)}} = 19.764$$

a) The minimum drag can be calculated from (it the same at all altitudes):

$$D_{\min} = \frac{W}{\left(\frac{L}{D}\right)_{\max}} = \frac{56000}{19.764} = 2833.4 \text{ lbs}$$

Sea-level airspeed for minimum drag:

$$V_{md_{SL}} = \sqrt{\frac{W}{1/2\,\rho_{SL}\,S\,C_{L_{md}}}} = \sqrt{\frac{56000}{1/2\,(0.002377)\,(900)\,(0.6324)}} = 287.7 \text{ ft/sec}$$

At altitude:

$$V_{md_{30K}} = V_{md_{SL}}\sqrt{\frac{\rho_{SL}}{\rho}} = 287.7\sqrt{\frac{0.002377}{0.00089}} = 470.2 \text{ ft/sec}$$

b) Min power conditions

$$C_{L_{mp}} = \sqrt{\frac{3\,C_{D_{0L}}}{K}} = \sqrt{\frac{3\,(0.016)}{0.04}} = 1.0954 \qquad C_{D_{mp}} = 4\,C_{D_{0L}} = 4\,(0.016) = 0.064$$

compute L/D min power:

$$\left.\frac{L}{D}\right)_{mp} \quad \frac{\sqrt{3}}{4}\,\frac{1}{\sqrt{C_{D_{0L}}K}} = \frac{\sqrt{3}}{4}\,\frac{1}{\sqrt{(0.016)\,(0.04)}} = 17.1163$$

We can calculate the min-power drag (same at all altitudes) from:

$$D_{mp} = \frac{W}{(L/D)_{mp}} = \frac{56000}{17.1163} = 3271.7 \text{ lbs}$$

To get the power required, we need to calculate DV = P_{req}

$$V_{mp_{SL}} = \sqrt{\frac{W}{1/2\,\rho_{SL}\,S\,C_{L_{mp}}}} = \sqrt{\frac{56000}{1/2\,(0.002377)\,(900)\,(1.0954)}} = 218.6 \text{ ft/sec}$$

Then, at sea-level,

$$P_{req} = D\,V = 3271.7\,(218.6) = 715266\ \frac{\text{ft}-\text{lbs}}{\text{sec}} = 1300 \text{ HP}$$

At altitude we can note that the drag at minimum power is the same at all altitudes, the difference in power required is the true airspeed times the drag. But the true airspeed is just the sea-level speed divided by the square root of the density ratio, σ. Hence:

$P\sqrt{\sigma}$ is constant with altitude, so at altitude:

$$P_{req_h} = \frac{P_{req_{SL}}}{\sqrt{\sigma}} = \frac{P_{req_{SL}}}{\sqrt{\dfrac{\rho_h}{\rho_{SL}}}} = 715266\sqrt{\frac{0.002377}{0.000890}} = 1168926\ \frac{\text{ft}-\text{lbs}}{\text{sec}} = 2125 \text{ HP}$$

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37. An aircraft has the following specifications: W = 24,000 lbs, S = 600 ft^2, $C_{D_{0L}} = 0.015$, and K = 0.056

This aircraft has run out of fuel at an altitude of 30,000 ft.

a) Find the initial and final values of its airspeed for best range glide
b) Find the glide angle for best range
c) Find the rate of descent at 30,000, 15,000 ft, and sea-level
d) Estimate (find) the time to descend to sea-level

We need to set up the maximum range glide condition which corresponds to the minimum drag flight condition. The minimum drag flight condition is determined from:

$$C_{L_{md}} = \sqrt{\frac{C_{D_{0L}}}{K}} = \sqrt{\frac{0.015}{0.056}} = 0.5175 \qquad C_{D_{md}} = 2\,C_{D_{0L}} = 2\,(0.015) = 0.03$$

and

$$\left.\frac{L}{D}\right)_{\max} = \frac{C_{L_{md}}}{C_{D_{md}}} = \frac{1}{2\,\sqrt{0.015\,(0.056)}} = 17.2516$$

a) Airspeed

$$V_{md_{SL}} = \sqrt{\frac{W}{1/2\,\rho_{SL}\,S\,C_{L_{md}}}} = \sqrt{\frac{24000}{1/2\,(0.002377)\,(600)\,(0.5\,75)}} = 255.0 \text{ ft/sec}$$

$$V_{md_{30K}} = V_{md_{SL}}\sqrt{\frac{\rho_{SL}}{\rho_{30K}}} = 255.0\sqrt{\frac{0.002377}{0.000890}} = 416.8 \text{ ft/sec}$$

b) Glide angle:

$$\tan\gamma = \frac{1}{L/D} = \frac{1}{17.2516} = 0.0580 \qquad \Rightarrow \qquad \gamma = 0.0579 \text{ rad} = 3.32 \text{ deg}$$

c) Rate of descent

$$\dot{h}_{SL} = -\sqrt{\frac{W}{1/2\,\rho_{SL}\,S}}\,\frac{C_{D_{0L}}}{C_L^{3/2}} = -\sqrt{\frac{24000}{1/2\,(0.002377)\,(600)}}\,\frac{0.03}{0.5175^{3/2}} = -14.784 \text{ ft/sec} = -887.0 \text{ ft/min}$$

$$\dot{h}_{15K} = \dot{h}_{SL}\sqrt{\frac{\rho_{SL}}{\rho_{15K}}} = -14.784\sqrt{\frac{0.002377}{0.001497}} = -18.629 \text{ ft/sec} = 1117.9 \text{ ft/min}$$

$$\dot{h}_{30K} = \dot{h}_{SL}\sqrt{\frac{\rho_{SL}}{\rho_{30K}}} = -14.784\sqrt{\frac{0.002377}{0.000890}} = -24.161 \text{ ft/sec} = 1449.6 \text{ ft/min}$$

d) Time to descend

$$tof = \frac{|\,\Delta h\,|}{|\,\dot{h}_{15K}\,|} = \frac{30000}{18.629} = 1610.0 \text{ sec} = 26.84 \text{ min}$$

38. The above aircraft (prob 37) has a sea-level thrust of 6000 lbs, that is independent of airspeed. Also, the thrust varies proportional with air density. Estimate the ceiling for this aircraft.

We need to determine when the thrust available is equal to the minimum drag. From previous work, L/D $)_{max}$ = 17.2516. Then:

$$D_{min} = \frac{W}{L/D)_{max}} = \frac{24000}{17.2516} = 1391.2 \text{ lbs}$$

$$T = D_{min} = T_{sl}\frac{\rho}{\rho_{SL}} = 1391.2 = 6000\frac{\rho}{\rho_{SL}} \quad => \quad \frac{\rho}{\rho_{SL}} = \frac{11291.2}{6000} = 0.23186$$

Then at the ceiling, $\rho = \rho_{SL}\frac{\rho}{\rho_{SL}} = 0.002377\,(0.23186) = 0.000550 \text{ slugs/ft}^3$

We can now interpolate between the altitudes where this value falls.

h_L = 40,000 ft $\qquad \rho_L = 0.0005857 \text{ slugs/ft}^3$

h_u = 45,000 ft $\qquad \rho_u = 0.0004605 \text{ slugs/ft}^3$

then the ceiling is approximately:

$$h = h_L + \frac{h_u - h_L}{\rho_u - \rho_L}(\rho - \rho_L) = 40000 + \frac{45000 - 40000}{0.0004605 - 0.0005857}(0.00055 - 0.0005857) = 41426 \text{ ft}$$

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39. Find the maximum angle of climb at sea-level and at 30,000 ft.

The angle of climb is given by:

$$\sin\gamma = \frac{T - D}{W} = \frac{T}{W} - \frac{C_D}{C_L}$$ {small angle assumption L = W)

Since T/W is constant, the angle will be a maximum when C_D/C_L is a minimum, or L/D is a maximum (minimum drag condition). From problem (37) the maximum L/D = 17.2516. Hence we have:

at sea-level:

$$\sin\gamma = \frac{6000}{24000} - \frac{1}{17.2516} = 0.1920 \qquad \gamma = 0.1932 \text{ rad} = 11.071 \text{ deg}$$

at 30000 ft

$$\sin\gamma = \frac{T}{W} - \frac{C_D}{C_L} = \frac{\frac{\rho}{\rho_{SL}} T_{SL}}{W} - \frac{C_D}{C_L} = \frac{0.000890}{0.002377}\frac{6000}{24000} - \frac{1}{17.2516} = 0.0356$$

$$\gamma = 0.0356 \text{ rad} = 2.04 \text{ deg}$$

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40. Find the maximum rate of climb at sea-level and at 30,000 ft.

This is a tougher problem since the best rate of climb conditions, change with altitude. For the case where the thrust available is independent of airspeed, we can determine the conditions for maximum rate of climb from:

$$C_L = \frac{1}{2K}\left[-\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + 12\, C_{D_{0L}} K} \right]$$

at sea-level

$$C_{L_{\max R/C_{SL}}} = \frac{1}{2\,(0.056)}\left[-0.25 + \sqrt{0.25^2 + 12\,(0.015)\,(0.056)} \right] = 0.1733$$

The corresponding drag coefficient is

$$C_D = C_{D_{0L}} + k\,C_L^2 = 0.015 + 0.056\,(0.1733)^2 = 0.0167$$

Then: $$V_{SL} = \sqrt{\frac{W}{1/2\,\rho\, S\, C_{L_{SL}}}} = \sqrt{\frac{24000}{1/2\,(0.002377)\,(600)\,(0.1733)}} = 440.7 \text{ ft/sec}$$

and

$$\sin\gamma = \frac{T}{W} - \frac{C_D}{C_L} = 0.25 - \frac{0.0167}{0.1733} = 0.1536 \quad\Rightarrow\quad \gamma = 8.84 \text{ deg}$$

$$\dot{h}_{\max_{SL}} = R/C = V\sin\gamma = 440.7\,(0.1536) = 67.7 \text{ ft/sec} = 4061 \text{ ft/min}$$

I don’t know of any way to get the max rate of climb at altitude from that at sea-level in a clever way, so we just have to redo the calculations that we did above.

At 30000 ft, $T = \frac{\rho}{\rho_{SL}} T_{SL} = \frac{0.000890}{0.002377} 6000 = 2246.5 \text{ lbs}$

The lift coefficient for maximum rate of climb is then given by:

$$C_{L_{\max R/C_{30K}}} = \frac{1}{2\,(0.056)}\left[-\frac{2246.5}{24000} + \sqrt{\left(\frac{2246.5}{24000}\right)^2 + 12\,(0.015)\,(0.056)}\right] = 0.3898$$

The corresponding drag coefficient is:

$$C_D = 0.015 + 0.056\,(0.3898)^2 = 0.0235$$

The angle of climb: $\sin\gamma = \frac{2246.5}{24000} - \frac{0.0235}{0.3898} = 0.0333$ $\gamma = 0.0333 \text{ rad} = 0.91 \text{ deg}$

The airspeed is:

$$V_{\max R/C_{30K}} = \sqrt{\frac{24000}{1/2\,(0.000890)\,(600)\,(0.3898)}} = 480.2 \text{ ft/sec}$$

$$\dot{h}_{\max R/C_{30K}} = V\sin\gamma = 480.2\,(0.0333) = 15.99 \text{ ft/sec} = 959.4 \text{ ft/min}$$
