AOE 3104 Vehicle Performance Problem Sheet 4 (ans)

19. Consider an aircraft that has a wing span of 15 m, a wing area of 37.5 m^2, and a gross weight of 88000 N. In level flight, the lift equals the weight. The aircraft is flying at 200 knots. Also the Oswald efficiency factor is 0.9, and the zero-lift drag coefficient is 0.0220. Determine the following:

a) lift coefficient
b) induced drag coefficient
c) total drag coefficient
d) induced drag (N)
e) zero-lift drag (N)
f) total drag (N)
g) lift to drag ratio, (L/D)

Convert to basic units: 200kts (0.5144 m/kt)~=~102.88 m/s

Calculate basics: Dynamic pressure: $\bar{q} = 1/2\, \rho\, V^2 = 0.5\,(1.2250)\,(102.88)^2 = 6482.88\ \text{n/m}^2$

and: $\bar{q}\, S = 6482.88\,(37.5) = 243108.01\ \text{N}$

a) Lift coefficient: $C_L = \dfrac{L}{1/2\, \rho\, V^2\, S} = \dfrac{W}{\bar{q}\, S} = \dfrac{88000}{243108.01} = \underline{0.3620}$

b) $AR = \dfrac{b^2}{S} = \dfrac{15^2}{37.5} = 6.0$ $\qquad C_{D_i} = \dfrac{C_L^2}{\pi\, AR\, e} = \dfrac{0.3620^2}{\pi\,(6.0)\,0.9} = \underline{0.0077}$

c) $C_D = C_{D_{0L}} + C_{D_i} = 0.0220 + 0.0077 = \underline{0.0297}$

d) $D_i = C_{D_i}\, \bar{q}\, S = 0.0077\,(243108.01) = \underline{1871.9\ \text{N}}$

e) $D_{0L} = C_{D_{0L}}\, \bar{q}\, S = 0.0220\,(243108.01) = \underline{5348.4\ \text{N}}$

f) $D = D_{0L} + D_i = 5348.4 + 1871.9 = 7220.3\ \text{N}$

g) $\dfrac{L}{D} = \dfrac{W}{D} = \dfrac{88000}{7220.3} = \underline{12.19}$

20. Repeat problem (19) for the case where altitude is 10,000 m. From your results, discuss (for the case where true airspeed is a constant (200 knots) the effects of altitude on C_L, C_{D0L}, $C_{Di,}$ C_D, and the L/D.

@ 10,000 m ρ = 0.413 kg/m^3 Designate sea-level conditions as 1 and altitude conditions as 2.

a) $$C_{L_2} = \frac{W}{1/2\,\rho^2\,V^2\,S} = \frac{W}{1/2\,\rho_1\,V^2\,S}\,x\,\frac{\rho_1}{\rho_2} = C_{L_1}\,\frac{\rho_1}{\rho_2} = 0.3620\,x\left(\frac{1.2250}{0.413}\right) = \underline{1.0737}$$

b) $$C_{D_i} = \frac{C_{L_2}^2}{\pi\,AR\,e} = 1.\frac{0737^2}{\pi\,(6.0)\,0.9} = \underline{0.0695}$$

c) $$C_D = C_{D_{0L}} + C_{D_i} = 0.0220 + 0.695 = \underline{0.0900}$$

d) $$\bar{q}_2 = 1/2\,\rho\,V^2 = 1/2\,(0.413)\,(102.88)^2 = \bar{q}_1\,\frac{\rho_2}{\rho_1} = 6482.88\left(\frac{0.413}{1.2250}\right) = \underline{2185.66\ \text{N/m}^2}$$

$$\bar{q}\,S = 2185.66\,(37.5) = 81962.13\ \text{N}$$

e) $$D_{0L} = C_{D_{0L}}\,\bar{q}\,S = 0.0220\,(81962.13) = \underline{1803.17\ \text{N}}$$

f) $$D = D_{0L} + D_i = 1803.17 + 5696.37 = \underline{7499.5\ \text{N}}$$

g) $$\frac{L}{D} = \frac{W}{D} = \frac{88000}{7499.5} = \underline{11.734}$$

An increase in altitude at constant true airspeed causes the following changes:

1. Increase in C_L
2. Does not change $C_{D_{0L}}$
3. Increase in C_{D_i}
4. Increase in C_D
5. Decrease in L/D

Item 5 may not always be true, but the rest are always true.

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21. The aircraft in problem (19) has a wing with an airfoil that has a 2-D lift-curve slope of 5.9 /rad. Use the DATCOM formula to estimate the 3-D lift-curve slope. The wing has a leading edge sweep angle of 30 degrees, and a taper ratio of 0.5. Make the calculations for 200 knots at

a) sea-level
b) 10,000m

@ sea-level: 200 kts = 102.88 m/s , a_{SL} = 340 m/s, $M = \frac{V}{a} = \frac{102.88}{340} = 0.3024$

$$a = \frac{2\pi AR}{2 + \sqrt{\frac{AR^2(1-M^2)}{k^2}\left(1 + \frac{\tan^2\Lambda_{c/2}}{(1-M^2)}\right) + 4}}$$

$$k = \frac{a_0}{2\pi} = \frac{5.9}{2\pi} = 0.9390$$

$$\tan\Lambda_n = \tan\Lambda_m - \frac{4}{AR}\left[(n-m)\frac{1-\lambda}{1+\lambda}\right]$$ Here, n = ½, and m = 0 (leading edge)

$$\tan\Lambda_{1/2} = \tan\Lambda_0 - \frac{4}{(6.0)}\left[(1/2-0)\frac{1-0.5}{1+0.5}\right] = 0.4662 \qquad (24.997\text{ deg})$$

Substitute into DATCOM formula

$$a = \frac{2\pi(6.0)}{2 + \sqrt{\frac{(6.0)^2(1-0.3024^2)}{0.9390^2}\left(1 + \frac{0.4662}{(1-0.3024^2)}\right) + 4}} = \underline{4.157\ /\text{rad}}$$

@ 10,000 m a = 299.47 $M = \frac{V}{a} = \frac{102.88}{299.47} = 0.3435$ All other items stay the same:

$$a = \frac{2\pi(6.0)}{2 + \sqrt{\frac{(6.0)^2(1-0.3435^2)}{0.9390^2}\left(1 + \frac{0.4662}{(1-0.3435^2)}\right) + 4}} = \underline{4.192.\ /\text{rad}}$$

Note, at low speeds small changes in Mach number do not affect the lift-curve slope very much.

22. Consider a rectangular wing. Assume that it has an airfoil with a lift-curve slope of 2π. Also assume that we are at low speed so that M = 0 (neglect compressibility effects). Calculate the lift-curve slope for the wing if the aspect ratio is 6 using: A rectangular wing has a span efficiency factor of 0.83

a) Prandtl's relation
b) DATCOM formula

$a_0 = 2\pi$,   M = 0,   AR = 6

Prandtl's formula:

$$a = \frac{a_0}{1 + \frac{a_0}{\pi \, AR \, e}} = \frac{2\pi}{1 + \frac{2\pi}{\pi \, (6.0) \, 0.83}} = \underline{4.482 \ /\text{rad}}$$

In a rectangular wing, all sweep angles are the same = 0.0. Hence the general DATCOM equation becomes: (for sweep and Mach =0, also since $a_0 = 2\pi$, $k = a_0/(2\pi) = 1$

$$a = \frac{2\pi \, AR}{2 + \sqrt{AR^2 + 4}}$$

or

$$a = \frac{2\pi \, (6.0)}{2 + \sqrt{6.0^2 + 4}} = \underline{4.529 \ /\text{rad}}$$
