AOE 3104 Vehicle Performance Problem Sheet 3 (ans)

14. We wish to design a wind tunnel experiment to accurately measure the lift and drag coefficients that pertain to a Boeing 777 in actual flight at Mach 0.84 at an altitude of 35,000 ft. The wingspan of a 777 is 199.9 ft. However, in order to fit in a wind tunnel test section, the wingspan of the model is 6 ft. The pressure in the airstream of the wind tunnel is 1 atmosphere. Calculate the necessary values of the airstream velocity, temperature, and density in the test section. Assume that the viscosity varies as the square root of the temperature. Hint: for dynamic similarity we need to match Reynolds number and Mach number. Note: The answer to this problem leads to an ***absurdity***. Discuss the nature of this absurdity in relation to the real word of wind tunnel testing.

@ 35,000 ft M = 0.84 P_1 = 498.0 lbs/ft^2 T_1 = 393.95 deg R ρ = 0.000737 slugs/ft^3
(Condition 1) b = 199.9 ft

Test Conditions: P_2 = 2116.2 lbs/ft^2 b = 6 ft
(condition 2)

ρ_2, V_2, T_2 = ? Do ensure complete dynamic symmetry we want to match Re and M. Therefore we require,

$$Re = \frac{\rho_1 V_1 l_1}{\mu_1} = \frac{\rho_2 V_2 l_2}{\mu_2} \qquad M = \frac{V_1}{a_1} = \frac{V_2}{a_2}$$

In addition we know that (given) $\mu = K_1 \sqrt{T}$ and $a = \sqrt{\gamma R T} = K_2 \sqrt{T}$

Substituting these into the Re and M equations, we have:

$$\frac{\rho_1 M K_2 \sqrt{T_1}\, l_1}{K_1 \sqrt{T_1}} = \frac{\rho_2 M K_2 \sqrt{T_2}\, l_2}{K_1 \sqrt{T_2}} \qquad \frac{V_1}{K_2 \sqrt{T_1}} = \frac{V_2}{K_2 \sqrt{T_2}}$$

Cancelling out all the K_1, K_2, M, and some of $\sqrt{T_1}, \sqrt{T_2}$, we end up with the following requirements:

$$\rho_2 = \frac{l_1}{l_2}\rho_1 \qquad V_2 = \sqrt{\frac{T_2}{T_1}}\, V_1 = \sqrt{\frac{T_2}{T_1}}\, M \sqrt{\gamma R T_1}$$

We can calculate ρ_2 from the first equation, and T_2 from P = ρRT, and finally V_2 from the second equation above.

$$\rho_2 = \frac{199.9}{6}(0.000737) = 0.0246 \ \text{slugs/ft}^3$$

$$T_2 = \frac{P_2}{\rho_2 R} = \frac{2116.2}{0.0246\,(1716.5)} = 50.12 \ \text{deg}$$

$$V_2 = \sqrt{\frac{50.12}{393.95}}\,(0.84)\sqrt{1.4\,(1716.5)\,393.95} = 291.52 \ \text{ft/sec}$$

Hence to match both Re and M for this scale model, we need a very high density, and a very low temperature. If we could run at 10 atmospheres, then the temperature would come up to reality. But we still have to deal with the high density problem, 10 times that of sea-level air.

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15. Consider an NACA 2412 airfoil (data in figure attached) with a chord of 1.5 m at an angle of attack of 4 deg. For a free-stream velocity of 30 m/s at standard sea-level conditions, calculate the lift and drag per unit span. Note the viscosity coefficient at standard sea-level conditions is $1.7894\text{x}10^{-5}$ kg/(m-s).

@h = sea-level	P = 101325 N/m^2,	ρ = 1.2250 kg/m^3,	T = 288.16 deg K
c = 1.5 m	α = 4.0 deg	V = 30 m/s	

calculate the Reynolds number:

$$Re = \frac{\rho V l}{\mu} = \frac{1.2250\,(30)\,1.5}{1.7894\,x\,10^{-5}} = 3.0806\,X\,10^6$$

Use the curves with the circles (Re = 3.1 x 10^6)

@ α = 4 deg $\quad C_l$ = 0.63 $\quad C_d$ = 0.0068

Calculate dynamic pressure $\bar{q} = 1/2\,\rho\,V^2 = 1/2\,(1.2250)\,(30^2) = 551.25 \ \text{N/m}^2$

$$L = C_l\,\bar{q}\,S = 0.63\,(551.25)\,(1.5)\,(1) = 520.9 \ \text{N(/meter of length)}$$

$$D = C_d\,\bar{q}\,S = 0.0068\,(551.25)\,(1.5)\,(1) = 5.62 \ \text{N (per meter of length)}$$

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16. For the NACA 2412 airfoil, (see figure attached), show that, an α = 6 deg, C_l = 0.85 and $C_{m1/4}$ = -0.037. The aerodynamic center of this airfoil is located at h_{ac} = 0.2553. Calculate the value of the moment coefficient about the aerodynamic center.

From figure, (Reynolds Number not a big influence on these values), at α = 6 deg, $C_l = 0.85$ and $C_{m_{(1/4)}} = -0.037$. We can calculate the moment about the aerodynamic center from:

$$C_{m_{ac}} = C_{m_{(1/4)}} + C_l\left(h_{ac} - h_{(1/4)}\right)$$
$$= -0.037 + 0.85\,(0.2553 - 0.25)$$
$$= -0.037 + 0.85\,(0.0053)$$
$$= -0.032$$

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17. A model is being tested in the wind tunnel at a speed of 100 miles/hour. The flow in the test section is at standard sea-level conditions.

   a) What is the pressure at the model's stagnation point, (lbs/ft^2)
   b) If the tunnel speed is measured by a pitot-static tube connected to a U tube manometer, what is the reading of the manometer in inches of water?
   c) At one point on the model, the pressure is measured at 2058 lbs/ft^2. What is the local 0 airspeed at that point?

@ sea-level P = 2116,2 lbs/ft^2 ρ = 0.002377 $slugs/ft^3$ V = 100 mi/hr x 88 ft/sec/mph = 146.67 ft/sec

a) From the incompressible Bernoulli's equation:

$$P_0 = P + \frac{1}{2}\rho V^2 = 2116.2 + \frac{1}{2}(0.002377)(146.67^2)$$
$$= 2116.2 + 25.5660$$
$$= 2141.76 \text{ lbs/ft}^2$$

b) The pressure difference is the dynamic pressure: $\frac{1}{2}\rho V^2 = \bar{q} = 25.5660 \text{ lbs/ft}^2$

$$P_0 - P = -\rho_w g (h_0 - h)$$
$$25.5660 = (1.94)(32.174)(h - h_0)$$
$$h - h_0 = 0.4096 \text{ ft}$$
$$= 4.915 \text{ in } H_20$$

c) From the incompressible Bernoulli's equation, and noting the pressure difference is the dynamic pressure, we have:

$$V = \sqrt{\frac{2(P_0 - P)}{\rho}} = \sqrt{\frac{2(2141.76 - 2058.0)}{0.002377}} = 265.47 \text{ ft/sec}$$

18. The momentum theory tells us that the force on the fluid is the net momentum flux out of the control volume. Here we have:

$$F_{\text{engine on fluid}_x} = (\text{momentum flux out})_x - (\text{momentum flux} \in)_x$$

The momentum flux is given by: $(\text{momentum flux})_x = (\dot{m})\, V_x = (\rho\, A\, V)\, V_x$.

a)   We can write the equation in the x direction, noting that the mass flow in equals the mass flow out (assuming the x direction is in the direction of air flow through the engine)

$$F_x = (\rho A V)(V_{ex} - V_{en})_x = (0.002377\,(4)\,300)\,(1800 - 300) = 4278 \text{ lbs}$$

The force of the fluid on the engine is the thrust and acts in the negative x direction, of forward.

T = 4278 lbs

b) To include the fuel flow we will assume that the fuel starts at rest in the vehicle and leaves in the exhaust at the same speed as the exhaust. Hence we can consider the additional thrust due to the fuel:

$$\Delta F = \dot{m}\,(V_{ex} - V_{en}) = 0.15\,(1800 - 0.0) = 270 \text{ lbs}$$

Hence the total thrust is:

T = 4278 + 270 = 4548 lbs