AOE 3104 Problem Sheet 10 (ans)

Our class business jet has the following characteristics:

Gross Weight = 10,000 lbs
S = 200 ft^2
2 engines with 1000 lb / engine

b = 40 ft
$C_D = 0.02 + 0.05\,(C_L)^2$
(up and away drag polar)

C_{Lmax} = 1.8 (normal flight)
C_{Lmax} = 2.4 (with flaps)

During ground roll, the wing is 5 ft above the ground
The coefficient of friction on the paved runway is $\mu = 0.02$
The coefficient during braking is $\mu_b = 0.3$

51. Calculate the takeoff ground roll distance assuming no flaps are used for takeoff and that minimum takeoff distance conditions are used during the ground roll.

Must calculate the conditions on the ground:

$$\Delta C_{D_{0L}} = \frac{W}{S} K_{uc}\, m^{-0.215} \qquad \begin{aligned} K_{uc} &= 5.81x10^{-5} \ \text{ no flaps} \\ K_{uc} &= 3.16x10^{-5} \ \text{ full flaps} \end{aligned}$$

However, units must be SI units so:

$$\frac{W}{S} = \frac{10{,}000 \text{ lb}}{200 \text{ ft}^2} \cdot \frac{4.448 \text{ N/lb}}{(0.3048 \text{ m/ft})^2} = 2393.9 \text{ N/m}^2$$

and

$$m = \frac{10000}{32}.174 \text{ slugs} \cdot 14.5939 \text{ kg/slug} = 4535.9 \ kg$$

Then

$$\Delta C_{D_{0L}} = \frac{W}{S} K_{uc}\, m^{-0.215} = (2393.9)\,(5.81x10^{-5})\,(4535.9^{-0.215} = 0.0227$$

and

$$C_{D_{0L_g}} = C_{D_{0L}} + \Delta\, C_{D_{0L}} = 0.021 + 0.0227 = \underline{0.0427}$$

The ground effect on the induced drag parameter K.

$$AR = \frac{B^2}{S} = \frac{40^2}{200} = 8 \qquad K = \frac{1}{\pi\, AR\, e} = \frac{1}{\pi\,(8)\, e} = 0.05 \quad \Rightarrow \quad e = 0.7958$$

Then the correction factor ϕ is given by

$$\phi = 1 - \frac{2e}{\pi^2} \ln\left[1 + \left(\frac{\pi b}{8h}\right)^2\right] = 1 - \frac{2(0.7958)}{\pi^2} \ln\left[1 + \left(\frac{\Pi (40)}{8(5)}\right)^2\right] = 0.6152$$

and

$$K_g = \phi K = 0.6152 (0.05) = 0.0308$$

Therefore

$$C_{D_g} = 0.0427 + 0.0308\, C_{L_g^2}$$

This is the ground drag polar!

For minimum distance takeoff, the proper ground lift coefficient is given by:

$$C_{L_g} = \frac{\mu}{2K_g} = \frac{0.02}{2(0.0308)} = 0.3247$$

The corresponding drag coefficient is determined from the ground drag polar"

$$C_{D_g} = 0.0427 + 0.0308 (0.3247)^2 = 0.0459$$

We can now calculate the constants in the takeoff equations:

$$A = g\left(\frac{T_0}{W} - \mu\right) = 31.174\left(\frac{2000}{10000} - 0.02\right) = 5.7913 \text{ ft/sec}^2$$

$$B = \frac{g}{W}\left[1/2\, \rho\, S\left(C_{D_g} - \mu\, C_{L_g}\right) + a\right]$$

$$= \frac{31.174}{10000}\left[1/2 (0.002377) (200) \left(0.0459 - (o{,}o2)(0.3247)\right) + 0\right]$$

$$= 3.0149x10^{-5}$$

We need to calculate the reference speeds:

$$V_{stall} = \sqrt{\frac{W}{1/2\, \rho\, S\, C_{L_{\max}}}} = \sqrt{\frac{10000}{1/2 (0.002377) (200) (1.8)}} = 152.85 \text{ ft/sec}$$

$$V_{TO} = 1.2\, V_{stall} = 1.2 (152.85) = 183.42 \text{ ft/sec}$$

Finally the takeoff distance can be calculated from:

$$S = \frac{1}{2B} \ln\left(\frac{A}{A - B V^{TO^2}} \right) = \frac{10^5}{2(3.0149)} \ln\left(\frac{5.7913}{5.7913 - 3.0149x10^{-5}(183.42)^2} \right) = 3193 \text{ ft}$$

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52. Calculate the landing ground roll distance for this aircraft assuming a full flap landing and that zero lift occurs during the ground roll, and that brakes are applied at touch-down (short field landing in which you retract the flaps and hit the brakes as soon as possible after touch-down).

Here we will assume that we land with flaps so we need to calculate new reference airspeeds:

$$V_{stall} = \sqrt{\frac{W}{1/2\, \rho\, S\, C_{L_{\max}}}} = \sqrt{\frac{10000}{1/2\,(0.002377)\,(200)\,(2.4}} = 132.4 \text{ ft/sec}$$

$$V_{TD} = 1.3\, V_{stall} = 1.3\,(132.4) = 172.1 \text{ ft/sec}$$

Ground run, no flaps. From previous problem, we have $C_{D_{0L_g}} = 0.0427$

Then with no lift,

$$C_{D_g} = C_{D_{0L_g}} + \phi K C_{L_g^2} = C_{D_{0L_g}} = 0.0427$$

We can calculate the new constants in the landing equation:

$$A = g\left(\frac{T_0}{W} - \mu \right) = 32.174\,(0 - 0.3) = -9.6522 \text{ ft/sec}^2$$

$$B = \frac{g}{W}\left[1/2\, \rho\, S\,(C_{D_g} - \mu\, C_{L_g}) + a \right]$$

$$= \frac{32.174}{10000}\left[1/2\,(0.002377)\,(200)\left(0.0427 - 0\right) + 0 \right]$$

$$= 3.2670x10^{-5}$$

We can now calculate the landing ground run:

$$S = \frac{1}{2B} \ln\left(1 - \frac{B}{A} V^2\right) = \frac{10^5}{2\,(3.2670)} \ln\left(1 - \frac{3.2670x10^{-5}}{-9.6522}(172.1)^2 \right) = 1461.8 \text{ ft}$$

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53. It is determined that the runway is too short so that a Jet Assisted TakeOff (JATO) is to be used with a rocket giving an additional 2000 lbs thrust for 5 seconds.

a) Should the JATO be fired during the first 5 seconds of the takeoff roll or during the last 5 seconds of the takeoff roll (and why?).

b) What is the takeoff distance using the JATO in the most efficient way?

a) Power = TV = d(energy)/dt. Hence the maximum rate of change of the energy would occur when the velocity is the highest. Consequently the most energy can be added during the last 5 seconds when the speed is the highest.

b) Assume no flaps as in first problem (p51). Then the constants will be

$C_{L_g} = 0.3247, \qquad C_{D_g} = 0.0459 \qquad V_{TO} = 183.4 \text{ ft/sec}$

The takeoff equation constants are:

$$A = g\left(\frac{T_0}{W} - \mu\right) = 32.174\left(\frac{2000+2000}{10000} - 0.02\right) - 12.2261 \text{ ft/sec}^2$$

$$B = 3.0149x10^{-5} \text{ /ft} \quad \text{(As before)}$$

We need to calculate when to apply the JATO thrust. So we will work backward from the takeoff speed and compute the speed when we must apply JATO. We know that we have 5 seconds to takeoff after applying the JATO. So we will use the time equation:

$$t_{TO} - t_{JATO} = \frac{1}{\sqrt{A\,B}}\left[\tanh^{-1}\left(\sqrt{\frac{B}{A}}\,V_{TO}\right) - \tanh^{-1}\left(\sqrt{\frac{B}{A}}\,V_{JATO}\right)\right]$$

or

$$5 = \frac{1}{\sqrt{12.2261\,(3.0149x10^{-5}}}\left[\tanh^{-1}\left(\sqrt{\frac{3.01591x10^{-5}}{12.2261}}\,(183.4168)\right) - \tanh^{-1}\left(\sqrt{\frac{B}{A}}\,V_{JATO}\right)\right]$$

or

$$\tanh^{-1}\left(\sqrt{\frac{B}{A}}\,V_{JATO}\right) = 0.2003 = \tanh^{-1}\left(\sqrt{\frac{3.0149x10^{-5}}{12.2261}}\,V_{JATO}\right)$$

or

$$V_{JATO} = 125.9 \text{ ft/sec}$$

We can now calculate the takeoff ground roll by calculating the ground roll to the JATO velocity, and then the ground roll from that velocity to takeoff. The constants used for the first part of the run are the same as were used in the previous takeoff problem.

First run, no JATO:

$$A = g\left(\frac{T_0}{W} - \mu\right) = 32.174\left(\frac{2000}{1000} - 0.02\right) = 5.7913$$

$$B = 3.0149x10^{-5}$$

$$S_1 = \frac{1}{2B}\ln\frac{A}{A - B\,V_{JATO}^2} = \frac{10^5}{2\,(3.0149}\ln\frac{5.7913}{5.7913 - 3.0149x10^{-5}\,(125.9)^2} = 1427.6 \text{ ft}$$

After JATO:

$$S_2 = \frac{1}{2B}\ln\frac{A - B\,V_{JATO}^2}{A - B\,V_{TO}^2} = \frac{10^5}{2\,(3.0149)}\ln\left(\frac{12.2261 - 3.0149x10^{-2}\,(125.9)^2}{12.2261 - 3.0149x10^{-5}\,(183.4)^2}\right) = 775.3 \text{ ft}$$

The total takeoff ground run is the sum of the two:

$S = S_1 + S_2 = 1427.6 + 775.3 = 2203$ ft (JATO, no flaps) (Saves about 1000 ft!)

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54. Calculate the distance for takeoff with a 40 ft/sec head-wind (not JATO)

For the normal takeoff conditions, the takeoff equation constants are:

$A = 5.7913$ $\quad\quad B = 3.0149x10^{-5}$

We need to determine the time. However note that we do not go from wind equals zero, but with the initial wind equal to the ground wind speed. So the time equation looks like:

$$t = \frac{1}{\sqrt{A\,B}}\left[\tanh^{-1}\left(\sqrt{\frac{B}{A}}\,V_{TO}\right) - \tanh^{-1}\left(\sqrt{\frac{B}{A}}\,V_w\right)\right]$$

$$t = \frac{1}{\sqrt{5.7913\,(3.0149x10^{-1})}}\left[\tanh^{-1}\left(\sqrt{\frac{3.0149x10^{-5}}{5.7913}}\,(183.4)\right) - \tanh^{-1}\left(\sqrt{\frac{3.0149x10^{-5}}{5.7913}}\,(40)\right)\right]$$

t = 26.82 sec

The takeoff with wind is then:

$$S = \frac{1}{2B} \ln\left(\frac{A - B V_w^2}{A - B V_{TO}^2} \right) - V_w t$$

$$= \frac{10^5}{2\,(3.0149)} \ln\left[\frac{5.7913 - 3.0149x10^{-5}\,(40)^2}{5.7913 - 3.0149x10^{-5}\,(183.4)^2} \right] - 40\,(26.8)$$

$$= 3054.4 - 1072.6 = 1981 \text{ ft}$$

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55. How would the takeoff distance equation be modified if the runway were on a slope? Determine the takoff distance for the case where the runway is sloped upward 5 degrees in the direction of takeoff. What is the % increase over the normal takeoff distance?

Starting from the beginning with the equation of motion, we have:

$$m \frac{dV}{dt} = T - D - m g \sin\theta - \mu R \qquad \text{Also:} \qquad \begin{aligned} L + R - W\cos\theta &= 0 \\ R &= L + W\cos\theta \end{aligned}$$

Then, rearranging and assuming $T = T_0 - a\,V^2$, we have

$$\frac{dV}{dt} = g\left(\frac{T_0}{W} - \mu\cos\theta - \sin\theta \right) - \frac{g}{W}\left[1/2\,\rho\,S\,(C_{D_g} - \mu\,C_{L_g}) + a \right] V^2$$
$$= A - B\,V^2$$

where:

$$A = g\left(\frac{T_0}{W} - \mu\cos\theta - \sin\theta \right) \qquad B = \frac{g}{W}\left[1/2\,\rho\,S\,(C_{D_g} - \mu\,C_{L_g}) + a \right]$$

Then only A is different due to the slope.

For a 5 degree slope:

$$A = 31.174\left(\frac{2000}{10000} - 0{,}02\cos 5 - \sin 5 \right) = 2.9896$$

$$S = \frac{1}{2B} \ln \frac{A}{A - B V_{TO}^2} = \frac{10^5}{2\,(3.0149)} \ln\left[\frac{2.9896}{2.9896 - 3.0149x10^{-5}\,(183.4)^2} \right] = 6871 \text{ ft}$$

Compared to 3193 ft! Note that if the runway sloped downhill, then the takeoff distance would be 2081 ft. Note the big contributor here is the component of weight along the runway, the sin θ term.