8. Level, Non-Accelerated Flight

For non-accelerated flight, the tangential acceleration, $\dot{V} = 0$, and normal acceleration, $V\dot{\gamma} = 0$. As a result, the governing equations become:

$$
\begin{aligned}
T\cos\alpha_T - D - mg\sin\gamma &= 0 \\
T\sin\alpha_T + L - mg\cos\gamma &= 0 \\
\dot{h} &= V\sin\gamma \\
\dot{x} &= V\cos\gamma
\end{aligned}
\qquad (1)
$$

If we add the additional assumptions of level flight, $\gamma = 0$, and that the thrust is aligned with the velocity vector, $\alpha_T = 0$, then Eq. (1) reduces to the simple form:

$$
\begin{aligned}
T - D &= 0 \\
L - W &= 0 \\
\dot{h} &= 0 \\
\dot{x} &= V
\end{aligned}
\qquad (2)
$$

The last two equations tell us that the altitude is a constant, and the velocity is the range rate. For now, however, we are interested in the first two equations that can be rewritten as:

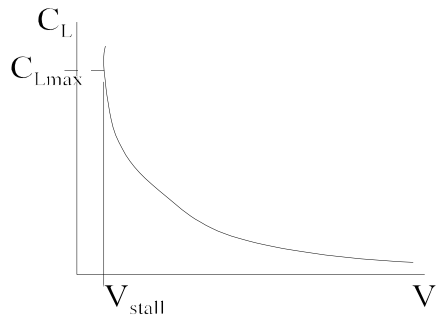
$$
\boxed{\begin{aligned}
T &= D \\
L &= W
\end{aligned}}
\qquad (3)
$$

The key thing to remember about these equations is that the ***weight, W, is a given*** and it is ***equal to the Lift***. Consequently, lift is not at our disposal, it must equal the given weight! Thus for a given aircraft and any given altitude, we can determine the required lift coefficient (and hence angle-of-attack) for any given airspeed.

$$
C_L = \frac{L}{1/2\,\rho\,V^2\,S} = \frac{W}{1/2\,\rho\,V^2\,S} = \left(\frac{W}{1/2\,\rho\,S}\right)\frac{1}{V^2} \qquad (4)
$$

Therefore at a given weight and altitude, the lift coefficient varies as 1 over V^2. A sketch of lift coefficient vs. speed looks as follows:

It is clear from the figure that as the vehicle slows down, in order to remain in level flight, the lift coefficient (and hence angle-of-attack) must increase. [You can think of it in terms of the momentum flux. The momentum flux (the mass flow rate time the velocity out in the z direction) creates the lift force. Since the mass flow rate is directional proportional to the velocity, then the slower we go, the more the air must be deflected downward. We do this by increasing the angle-of-attack!]. Unfortunately there is an upper limit to the lift coefficient called $C_{L_{\max}}$, and it occurs at the angle-of-attack called the stall angle-angle-of-attack, α_{stall}. Furthermore, the speed at which this maximum lift coefficient (or α_{stall}) is called the stall speed, V_{stall}. (Many other important speeds, especially those required for take-off and landing, are based on the stall speed).



From the definition of the lift coefficient, we can solve for the speed for any lift coefficient:

$$V = \sqrt{\frac{W}{1/2\,\rho\, S\, C_L}} \qquad (5)$$

Consequently we can also determine the ***aerodynamically limited minimum airspeed*** for level flight:

Aerodynamically limited airspeed

$$V_{stall} = V_{\min_{aero}} = \sqrt{\frac{W}{1/2\,\rho\, S\, C_{L_{\max}}}} \qquad (6)$$

Note: $C_{L_{\max}}$ is dependent only on angle-of-attack. An aircraft that is accelerating, for example, doing a pull up, can generate a large angle-of-attack and hence cause a stall. Consequently an aircraft can stall at ***any speed***.

One of the reasons that we develop equations to model the aerodynamics characteristics of an aircraft is that they can tell us how various parameters of interest affect other parameters. From Eq. (6) we can see that for a given weight aircraft, the stall speed will increase with altitude (density gets smaller, and is in the denominator). Furthermore, at a given altitude, the stall speed increases with the square root of the weight, For example, if we write Eq.(6) for two different weights and corresponding stall speeds, we can see that:

$$V_{stall_2} = \sqrt{\frac{W_2}{W_1}}\, V_{stall_1} \tag{7}$$

At landing, the weight is less than that at take-off because of the fuel used during flight. Consequently, the stall speed at landing is less than the stall speed at take-off.

How does the pilot figure all this out? Note that we can write the expression for stall speed in terms of the equivalent airspeed:

$$V_{stall_{eq}} = \sqrt{\frac{W}{1/2\, \rho_{SL}\, S\, C_{L_{\max}}}} \tag{8}$$

We can observe that the stall airspeed in terms of the equivalent airspeed is a ***constant***! We can also recall that the airspeed indicators in all aircraft read airspeeds related to the equivalent airspeed. In fact at low speeds the observed readings in all airspeed indicators (compressible and incompressible) read very close to the equivalent airspeed. Consequently from the pilot's point of view, the stall speed is the same on the airspeed indicator regardless of the altitude! Hence the pilot only has to remember one airspeed for stall. In fact s/he doesn't even have to do that. The airspeed indicator is marked with a white arc. The bottom of the white arc is the level-flight stall speed (equivalent) for the maximum (permissible) gross weight of the aircraft. Any weight less than that will have a slower stall speed.

Most aircraft have high-lift devices that increase the value of $C_{L_{\max}}$ over the nominal value that would occur if they were not used. The most common of these devices are trailing edge flaps. Other devices include: leading edge flaps, leading edge slots, slotted flaps, double and triple slotted flaps, and combinations of all the above. The purpose is to increase the value of $C_{L_{\max}}$. If we can do that, the landing and take-off speeds can be reduced as indicated by Eq. (8).

Thrust Required (level flight)

The thrust required for level flight can be determined in many different ways, depending upon what information is available. Under our usual assumptions, (thrust aligned with velocity), the level flight equations are given by Eq. (3), repeated here:

$$\begin{aligned} T &= D \\ L &= W \end{aligned} \tag{9}$$

If we divide one equation by the other, and rearrange terms, we have:

$$T = \frac{D}{L} W = \frac{W}{\left(\frac{L}{D}\right)} \tag{10}$$

Since $L = 1/2\, \rho\, V^2 S$ and $D = C_D\, 1/2\, \rho\, V^2 S$ we can right Eq. (10) in the equivalent form:

$$T = \frac{W}{\left(\frac{L}{D}\right)} = \frac{W}{\left(\frac{C_L}{C_D}\right)} \tag{11}$$

Recall that the weight *W* is given and is not for us to choose. Consequently we can see that the minimum thrust required occurs when *L/D* is a maximum. Again, since $L = W$, the maximum value of *L/D* occurs when drag is a minimum. Hence the minimum thrust required occurs at the minimum drag flight condition which is the same as the maximum *L/D* flight condition. The so-called lift-to-drag ratio, *L/D*, is often referred to as the aircraft aerodynamic efficiency, the higher the value of *L/D*, the more efficient. Typical values range from 1 - 60 with hypersonic reentry vehicles in the 1 to 2 range, fighter planes in the range of 5 to 7, transports in the range 8 - 14, and gliders from 25 - 60+.

Example

A Boeing 747 weighs 750,000 lbs. If it is flying at a condition where L/D = 10, what is the thrust required?

$$T = \frac{W}{(L/D)} = \frac{750{,}000}{10} = 75{,}000 \text{ lbs}$$

It should be clear that the thrust required is just the drag of the aircraft since $T = D$. Consequently we can calculate the thrust required in another way by computing the drag on the vehicle. Hence we note: ***Thrust required = Drag***.

We can calculate the drag using the drag coefficient, the geometry of the aircraft, and the density. We will need to use the following relations:

$$C_L = \frac{L}{1/2\, \rho\, V^2 S} \quad \text{and } D = C_D\, 1/2\, \rho\, V^2 S. \tag{12}$$

In addition we will assume a parabolic drag polar:

$$C_D = C_{D_{0L}} + K C_L^2 \tag{13}$$

If we assume level flight, then the altitude will be constant along with the density associated with that altitude. We can now calculate the drag of the vehicle for a given altitude at any airspeed.

Drag Calculation (Thrust required)

$$
\begin{aligned}
D &= C_D \, 1/2 \, \rho \, V^2 S \\
&= \left(C_{D_{0L}} + K C_L^2 \right) 1/2 \, \rho \, V^2 S \\
&= C_{D_{0L}} \, 1/2 \, \rho \, V^2 S + K \left(\frac{W}{1/2 \, \rho \, V^2 S} \right)^2 1/2 \, \rho \, V^2 S \\
&= C_{D_{0L}} \, 1/2 \, \rho \, V^2 S + \frac{K W^2}{1/2 \, \rho \, V^2 S} \\
D &= \left(C_{D_{0L}} \, 1/2 \, \rho \, S \right) V^2 + \left(\frac{K W^2}{1/2 \, \rho \, S} \right) \frac{1}{V^2}
\end{aligned}
\qquad (14)
$$

We can write this equation in two different forms, one primarily used when calculating drag for low performance vehicles (read that low speed), and one primarily used when calculating drag for high performance (read that high speed) vehicles.

Low Performance Vehicles

For low performance vehicles that operate in the low subsonic flight regime it is convenient to assume that the drag polar parameters, $C_{D_{0L}}$ and K are constant, independent of airspeed (Mach number). Under these conditions, the drag equation appears as:

$$D = A V^2 + \frac{B}{V^2} \qquad (15)$$

where:

$$A = C_{D_{0L}} \, 1/2 \, \rho \, S \qquad \text{and} \qquad B = \frac{K W^2}{1/2 \, \rho \, S}$$

and are constant at a given altitude.

High Performance Vehicles

For high performance vehicles that operate at all speeds including high subsonic and supersonic speeds, we usually think in terms of Mach number rather than airspeed. In addition, the parabolic drag polar parameters, $C_{D_{0L}}$, and K are functions of Mach number. For this case we can use the following relation for the dynamic pressure: (recall the speed of sound $a^2 = \frac{\gamma P}{\rho}$)

$$\bar{q} = 1/2\, \rho\, V^2 = 1/2\, \rho\, a^2 \frac{V^2}{a^2} = 1/2\, \rho \left(\frac{\gamma P}{\rho} \right) M^2 = 1/2\, \gamma\, P\, M^2 \qquad (16)$$

If we substitute this in for the dynamic pressure in Eq. (14) we have the high performance vehicle drag equation:

$$D = \left[C_{D_{0L}}(M)\, 1/2\, \gamma\, P\, S \right] M^2 + \left[\frac{K(M)\, W^2}{1/2\, \gamma\, P\, S} \right] \frac{1}{M^2} \qquad (17)$$

where the explicit dependence of $C_{D_{0L}}$ and K on Mach number is indicated. The terms in the square bracket are not constant, and can change with Mach number (airspeed). $\gamma = 1.4$, and P is the pressure at the altitude of interest.

General Comments About Drag

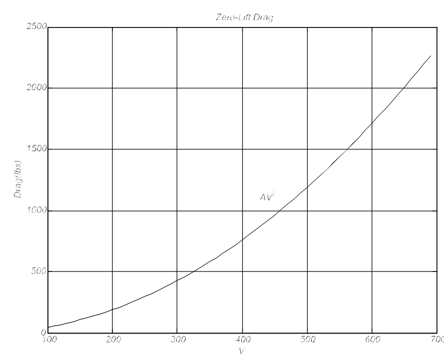
There are certain features that both Eq. (15) and Eq. (17) have in common. At high speeds or Mach numbers, the first term on the right hand side is large, and at low speeds and Mach numbers the second term on the right hand side is large. Hence the drag is large at both high speeds and at low speeds (for an aircraft in level flight). In fact in the extremes, each of these terms goes to infinity, the first term at M or $V = \infty$, and the second term at M *or* $V = 0$. Consequently, somewhere between these extremes, there must be a minimum that occurs at the minimum drag speed or Mach number.

The first term on the right hand side of each equation is called the zero-lift drag and includes parasite drag and for high speed flight ($C_{D_{0L}}$ depends on M), wave drag. The second term on the right is called the drag-due-to-lift or induced drag. The first term is what we experience in automobiles that have no lift, and the second term is the price we pay to support our vehicle on the airstream. Together they make for a very strange drag curve. It tells us that under certain conditions (at speeds below the minimum drag speed) that if we slow down, the drag will increase, and if we speed up, the drag will decrease! If we are traveling at speed higher than the minimum drag speed, if we speed up the drag goes up and vice-versa - things seem normal.

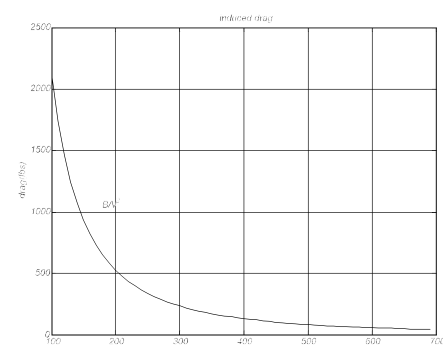
We can look at the general characteristics of the drag curve by looking at the less complicated equation, Eq. (15). The ***general shape*** of the curve is the same, assuming incompressible or low performance aircraft, as it is for high performance aircraft. Hence we will look in detail at the simpler case and note how it might change if compressible effects were to be included.

Thrust Required or Drag Curves

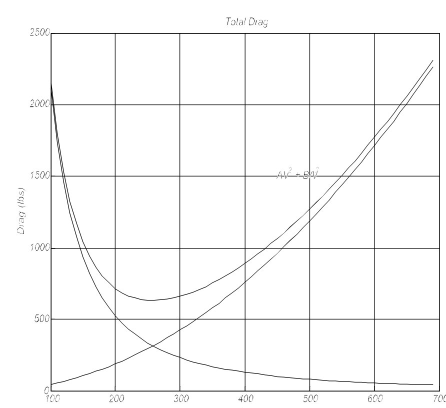
For incompressible (low speed) flight we can approximate the drag of the vehicle using Eq. (15). We see that the drag consists of two terms, the parasite, profile, or zero-lift drag that is is given by $D_{0L} = A\,V^2$ and the induced drag given by $D_i = \frac{B}{V^2}$. The total drag consists of the sum of these two terms. The first term can be plotted and it looks like the figure to the right. This type of drag is the one that most people are familiar with and increases with the square of the airspeed. This drag is the type that is encountered with all vehicles (such as automobiles).



The second term is called the induced drag or drag-due-to-lift. It behaves in an unusual way as indicated in the figure. Here we note that the drag ***decreases*** as the airspeed increases. This behavior is counter intuitive! This portion of the total drag is the cost that we have to pay for using the air to hold up the aircraft. The faster we go, the easier it is for the air to hold up the vehicles since it has to deflect the air less to create the same lift (= weight).



The total drag curve is the sum of the components of drag . The important item to note here is that the drag curve has a minimum. The airspeed at which this occurs is called the minimum drag airspeed, and the associated lift coefficient, the lift coefficient for minimum drag. From our previous work we also know that the minimum drag condition is the same as the



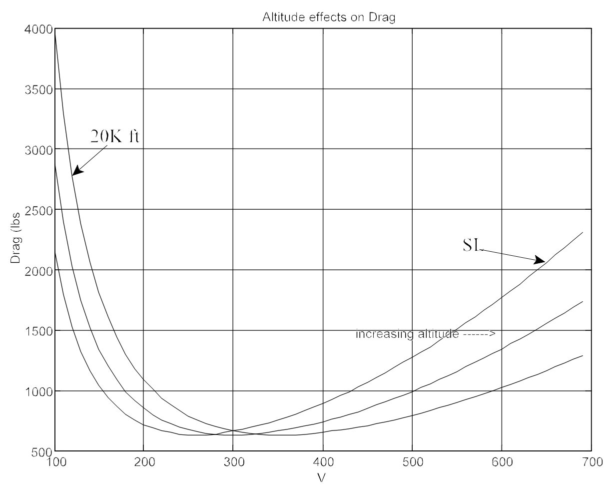
condition for maximum L/D. Note that the ***max L/D does not occur at*** $C_{L_{max}}$ ***!!!*** Furthermore, since $L/D_{max} = C_L/C_D\big)_{max}$ the value of maximum L/D is independent of altitude.

By examining the drag equation, we can determine how altitude affects the drag for a given weight aircraft, or how the change in weight would affect the drag at a given altitude.

$$D = C_{D_{0L}} \frac{1}{2} \rho S V^2 + \frac{K W^2}{\frac{1}{2} \rho S V^2} \tag{18}$$

If we assume a fixed weight, the affect of altitude can be seen by noting how the density enters into each term. As the altitude is increased, the density decreases. Consequently the first term (proportional to V^2) decreases, and the second term (proportional to $1/V^2$) gets larger. The effect then is to shift the drag curve to the right (since the minimum drag is the same at all altitudes!).

We can note from the figure that the minimum drag is the same at all altitudes, but the minimum drag speed increases with altitude. Exactly how the minimum drag speed changes with altitude is easily determined if we consider another approach to looking at drag curves.



Recall that the definition of equivalent airspeed is given by:

$$\frac{1}{2} \rho V^2 = \frac{1}{2} \rho_{SL} V_{eq}^2$$

Then if we substitute this expression into the expression for calculating drag we have:

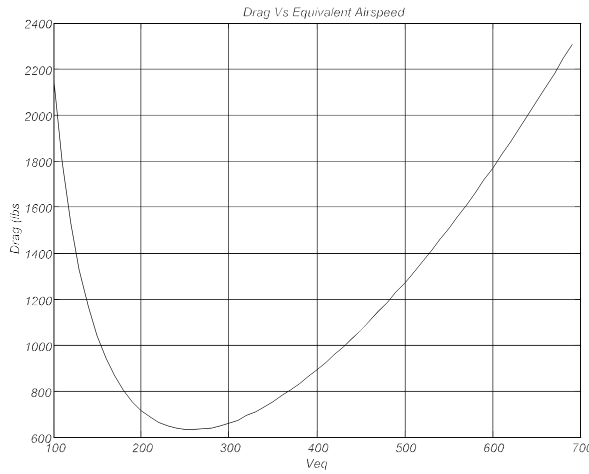
$$D = C_{D_{0L}} \frac{1}{2} \rho S V^2 + \frac{K W^2}{\frac{1}{2} \rho S V^2} = C_{D_{0L}} \frac{1}{2} \rho_{SL} S V_{eq}^2 + \frac{K W^2}{\frac{1}{2} \rho_{SL} S V_{eq}^2}$$

or

$$D = A_{SL} V_{eq}^2 + \frac{B_{SL}}{V_{eq}^2} \tag{19}$$

Consequently [for low performance (speed) aircraft] we can make a single plot of Drag vs equivalent airspeed that will allow us to calculate drag and airspeed at any altitude!

The figure at the right is the drag vs equivalent airspeed. (It is also the sea-level drag curve!). From it we can compute the drag at any value of the equivalent airspeed. To find the true airspeed that will give the same drag at any altitude we just calculate the true airspeed at altitude from our now familiar equation:



$$V = \sqrt{\frac{\rho_{SL}}{\rho}}\, V_{eq} = \frac{V_{eq}}{\sqrt{\sigma}} \qquad (20)$$

where $\sigma = \dfrac{\rho}{\rho_{SL}}$.

Example

Our class executive jet has a zero lift drag coefficient of 0.02, and an induced drag parameter of 0.05. Find the drag at sea-level if the speed is 400 ft/sec. Assume that we are below the drag rise Mach number since our Mach number is M = 0.358 << 0.7. Hence we can use the low performance(speed) drag equation assuming the drag parameter values are constant.

$$A_{SL} = C_{D_{0L}} \frac{1}{2} \rho_{SL} S V^2 = 0.02\,(0.002377)\,(200) = 0.004754 \;\frac{\text{lb sec}^2}{\text{ft}^2}$$

$$B_{SL} = \frac{K W^2}{1/2\, \rho_{SL} S} = \frac{0.05\,(10{,}000^2)}{(1/2)\, 0.002377\,(200)} = 21034918 \;\frac{\text{lb ft}^2}{\text{sec}^2}$$

The drag at sea-level at 400 ft/sec is then calculated from:

$$D = 0.004754\, V_{eq}^2 + \frac{2.10349 \;x\; 10^7}{V_{eq}^2} = 0.004754\,(400^2) + \frac{2.10349 \;x\; 10^7}{400^2}$$

$$= 761.12 + 131.46 = 892.58 \;\text{lbs}$$

Suppose now that we wanted to know the drag on a vehicle moving at 653.70 ft/sec at 30,000 ft. (Mach 0.657). We will continue to assume that it is below the drag rise Mach number (we kind of have to do that because we have no additional information!) We could just put that number in the drag equation with the proper density (ρ_{30K} = 0.000890 slugs/ft^3). Or, we can convert to equivalent airspeed and use our sea-level curve. Taking this approach we have

$$V_{eq} = \sqrt{\frac{\rho}{\rho_{SL}}}\,V = \sqrt{\frac{0.000890}{0.002377}}\,653.70 = 400 \text{ ft/sec}$$

So the drag would be 892.6 lbs.

We can now answer a question raised previously, if we know the minimum drag speed at sea-level, what would be the minimum drag speed at some altitude. The minimum drag speed at altitude would be given by:

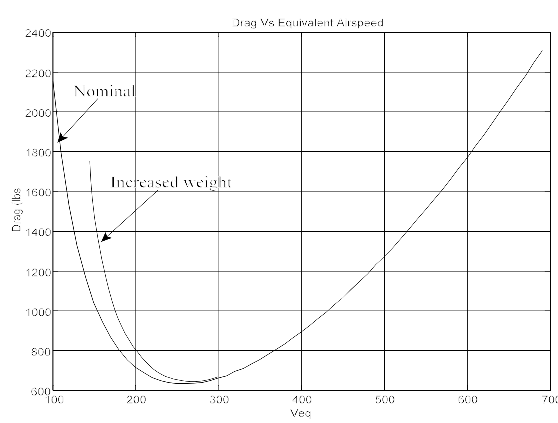
$$V_{md_h} = \sqrt{\frac{\rho_{SL}}{\rho_h}}\,V_{md_{SL}} \qquad (21)$$

Although plotting a single curve of drag vs equivalent airspeed is nice, and works for the low performance drag calculations, it does not work when we plot drag vs Mach number for high performance aircraft.

Effect of Weight on Drag at a Given Altitude

If we hold the altitude constant, we can see the effect of changing the weight of the aircraft by looking at the general drag equation (Eq. (18)). Here we can note that the weight term only enters into the induced drag portion of the equation. Consequently, at a given altitude, an increase in weight would increase the drag at low flight speeds and have little effect at high flight speeds. The value of the minimum drag would increase (since L/D_{max} wouldn't change, but L =W would!).

The figure on the right is a sketch of what the effect of added weight would be on the drag curve. There is virtually no effect on the right branch of the curve with all the effects occurring at low speeds.



High Performance Drag Considerations

The equation for high performance aircraft is generally given in terms of Mach number as

$$D = \left[C_{D_{0L}}(M)\, 1/2\, \gamma\, P\, S \right] M^2 + \left[\frac{K(M)\, W^2}{1/2\, \gamma\, P\, S} \right] \frac{1}{M^2} \qquad (22)$$

We seek a method to eliminate altitude effects from this equation in the same manner that we did for the low performance drag equation by introducing equivalent airspeed. That procedure won't work here so we will try another. By multiplying each term by P_{SL}/P_{SL}, we get:

$$D = \left[C_{D_{0L}}(M)\, 1/2\, \gamma\, P\, \frac{P_{SL}}{P_{SL}}\, S \right] M^2 + \left[\frac{K(M)\, W^2}{1/2\, \gamma\, P\, \frac{P_{SL}}{P_{SL}}\, S} \right] \frac{1}{M^2}$$

Now we can define the quantity:

<u>**Definition**</u>: $\delta \equiv \frac{P}{P_{SL}}$, the ratio of pressure at altitude to pressure at sea-level.

With this definition, we can write an equation that contains sea-level pressure and is partially independent of altitude, in the following way:

$$\frac{D}{\delta} = \left[C_{D_{0L}}(M)\, 1/2\, \gamma\, P_{SL}\, S \right] M^2 + \left[\frac{K(M) \left(\frac{W}{\delta} \right)^2}{1/2\, \gamma\, P_{SL}\, S} \right] \frac{1}{M^2} \qquad (23)$$

Here we normalize the drag force and the weight with the altitude to sea-level pressure ratio, δ, then we can plot a single curve, D/δ vs M and can determine the drag at any other altitude from it. This approach is used during flight testing. The flight test is performed at increasing altitudes so as to keep W/δ = const as the fuel is burned off.

If we return to Eq. (22), we can parameterize the drag equation in another way. First we need to define the non-dimensional wing loading:

Definition: Non-dimensional wing loading, $\omega = \frac{W}{1/2\, \gamma\, P\, S}$.

If we substitute this definition into the drag equation (22), we can arrive at the following parameterization of the drag equation:

$$\frac{D}{W}(\omega, M) = \frac{C_{D_{0L}}(M)\, M^2}{\omega} + \frac{K(M)\, \omega}{M^2} \tag{24}$$

Here we can associate the parameter ω with altitude, and the parameter M with speed. This form will not be pursued here since it doesn't gain us anything over the original equation.