1.(a)

```matlab
function r=aas(m,g)
%Class solution
r=1./m.*(2/(g+1)*(1+(g-1)/2*m.^2)).^((g+1)/2/(g-1));
```

Plot for Helium:

(b)

```matlab
clear all
g=1.13;
x=[0 0.4336 0.6532 0.9685 1.3321 1.5996 1.9797 2.314 2.6386 2.9665 3.3046 3.6573 4.0282 4.4203 4.8362 5.2787 5.7504 6.2541 6.7926 7.3691 7.9867 8.6489 9.3592 10.1218];
mdesign=[1 1.744 1.7443 1.747 1.7591 1.7758 1.8075 1.8391 1.8707 1.9023 1.9339 1.9656 1.9972 2.0289 2.0606 2.0924 2.1242 2.1562 2.1882 2.2203 2.2526 2.2849 2.3174 2.35];
figure(1)
plot(x,a);
xlabel('x');ylabel('A');
m=m_aas(a,g,1);
figure(2);
plot(x,mdesign,'b',x,m,'r');
xlabel('x');ylabel('Mach number');
legend('Design','Area-Mach no. relation');
```
We see that the 1D area Mach-number relation is exactly correct at the nozzle exit. However, there is some error over most of the nozzle presumably associated with the 1D assumption. This may be exacerbated by the fact that we are comparing the results of the A-M relation with the Mach number at the nozzle wall, rather than the Mach number averaged over the nozzle cross section.
2. (a) \( T = 273 - 44 = 229\,\text{K} \quad M = \frac{270}{\sqrt{1.4 \times 287 \times 229}} = 0.89 \)

(b) (i) \( T = 217\,\text{K} \quad M = 2.1 \quad \text{Max.} \quad \frac{T}{10} = 0.0313 \quad \therefore T_0 = \frac{217}{0.0313} = 408\,\text{K} \)

(ii) \( T_0 = \text{const.} \)

(c) \( M = 1.8 \quad \text{Max.} \quad \frac{P_0}{P_1} = 3.613, \quad \frac{P_1}{P_2} = 0.2142 \)

so \( P_1 = P_0 = 82/0.2142 = 383\,\text{kPa} \)

\( P_s = P_2 = 82 \times 3.613 = 296\,\text{kPa} \)

(d) \( P_0 = 281\,\text{kPa}, \quad P_8 = 206\,\text{kPa} \quad \text{since} \ P_s > 100\,\text{kPa} \quad \text{this must be supersonic flight} \quad \therefore \quad \frac{P_2}{P_0} = \frac{206}{281} = 0.7331 \quad \text{Max.} \quad \Rightarrow M_2 = 0.68 \)

\( \Rightarrow M_1 = 1.56 \)
3. (a) Boundary Cases

For \( M_e = 1.8 \),
\[
\frac{A_e}{A*} = 1.439 \quad \Rightarrow \quad \frac{p_m}{p_c} = 0.1740
\]

For shock at exit \( M_1 = 1.8 \), \( p/p_c = 0.1740 \),
\[
\Rightarrow \quad p_2/p_1 = 3.613
\]
so \( \frac{p_0/p_c}{\text{shock exit}} = 3.613 \times 0.1740 = 0.6289 \)

For subsonic choked, \( \frac{A_e}{A*} = 1.439 \),
\[
\Rightarrow \quad \frac{p_m}{p_c} = 0.8682
\]

so for (i) shock is in nozzle for \( p_0/p_c = 0.7 \)

\[
M_e = \sqrt{\frac{-1}{\gamma - 1} + \frac{1}{(\gamma - 1)^2} + \frac{2}{\gamma - 1} \left( \frac{\gamma M_e^2}{\gamma + 1} \right) \left( \frac{p_c A_1}{p_m A_e} \right)^2}
\]

\[
= 0.5575
\]

(ii) \( p_0/p_c = 0.4 \) — over-expanded \( M_e = 1.8 \)

(iii) \( p_0/p_c = 0.2 \) — over (expanded) \( M_e = 1.8 \)

(iv) \( p_0/p_c = 0.1 \) — under expanded \( M_e = 1.8 \)

(b) \( M_e = 0.5575 \),
\[
\Rightarrow \quad \frac{p_0}{p_02} = 0.8097
\]

so \( \frac{p_2}{p_1} = 0.7 \), \( 0.8097 \),
\[
\Rightarrow \quad M_1 = 1.679
\]

\[
\frac{A_1}{A*} = 1.318^2
\]
(a) \[
\frac{A_e}{A_t} = \frac{1.574}{0.0366} = 43
\]

\[P_e : 710 \text{ kPa}\]

\[T_e = 900 \text{ K}\]

\[M_e = 5.71\]

\[\frac{P_e}{P_0} = 8.6 \times 10^{-4}, \quad \frac{T_e}{T_0} = 0.1331\]

\[\frac{A_t}{A_t} = 0.0366 \text{ m}^2\]

\[A_e = 1.574 \text{ m}^2\]

\[P_0 = 0\]

\[P_e = 8.6 \times 10^{-4} \times 710 = 0.61 \text{ kPa}\]

\[T_e = 0.1331 \times 900 = 120 \text{ K}\]

(b) For steady flow, conservation of momentum requires:

\[\oint_{S_C} \rho \mathbf{U} \cdot \mathbf{n} \, ds = -\oint_{S_C} p \mathbf{n} \cdot ds\]

So, consider a CV enclosing the casing:

The integral \(\oint_{S_C} \rho \mathbf{U} \cdot \mathbf{n} \, ds\) is zero on all parts of the control surface except the nozzle exit itself.

\[\oint_{S_C} \rho \mathbf{U} \cdot \mathbf{n} \, ds = \rho e U e A_e \text{ in the positive x direction.}\]

\[= \frac{P_e}{R} \times M_e^2 \sqrt{\gamma T_e A_e} = 43.8 \text{ kN}\]
The integral \( \int p \, dS \) is equal to

\[ -p_0 a_e \text{ area (in positive x)} \] for the nozzle exit plane.

For the rest of the control surface this adds up to the thrust generated by the engine.

So thrust = \( 43.8 \, \text{kN} + 0.61 \times 1000 \times 1.574 \)

\[ = 44.8 \, \text{kN} \]

(This is remarkably close to the rated thrust of 42.9 kN)