Unsteady 1D Flow
Examples

• Moving aircraft, lightning, flow in a gun barrel, implosion, explosion

Shock wave hitting a bubble of low density gas

Spherical shock waves from guns of the USS Iowa

July 16, 1945 TRINITY nuclear explosion
Sudden Motion of Piston in a Tube

Expansion wave

Compression wave
Speed of the Waves

**Expansion wave**

**Compression wave**
$x - t$ Diagrams

Expansion wave

Compression wave
The Shock Tube

Time 0

Driver Gas

Diaphragm

Driven Gas

Time 1

Expansion Wave

Contact Surface

Shock

Time 2

Expansion Wave

Contact Surface

Reflected Shock
Shock Tube $x$-$t$ Diagram
The Pressure Collapse Problem

FUNDAMENTAL PHYSICAL SITUATION REPRESENTED BY THE SHOCK TUBE
Moving Normal Shocks

IDENTICAL TO A STATIONARY SHOCK – ONLY DIFFERENCE IS FRAME OF REFERENCE

Fixed Frame of Reference

Disturbed gas
Mass motion $u_p$

Undisturbed gas
Stagnant

$W$

Frame of Reference Moving With Shock
Relation for the Mass Motion Velocity

\[ u_p = f\left(\frac{p_2}{p_1}\right) \]

Frame of Reference Moving With Shock

\[ \begin{align*}
\text{②} & \quad u'_2 = W - u_p \\
\text{①} & \quad u'_1 = W \\
p_2 & \quad p_1
\end{align*} \]

\[ M_1' = \frac{W}{\sqrt{\gamma R T_1'}} \]

\[ u_p = \frac{a_1 \left(\frac{p_2}{p_1} - 1\right)}{\gamma_1 \left(\frac{p_2}{p_1} - 1\right) \sqrt{1 + \frac{\gamma_1 + 1}{2 \gamma_1} \left(\frac{p_2}{p_1} - 1\right)}} \]
Example:
Driven Flow in a Shock Tube

Schlieren shows density ratio of 3.0 across shock. Find shock Mach number, mass motion velocity and p and T after shock.
Variations

\( p_2 = 568.7 \text{kPa}, \quad T_2 = 567.6 \text{K} \)

\( M_1' = 2.24, \quad M_2' = 0.542, \quad u_p = 519 \text{m/s}, \quad W = 778 \text{m/s} \)

- Time until shock strikes probe? driver gas strikes probe?

- Mach number of probe just after shock passes

- \( p \) and \( T \) at nose of probe just after shock passes
Variations

\[ p_2 = 568.7 \text{kPa}, \quad T_2 = 567.6 \text{K} \]
\[ M_1' = 2.24, \quad M_2' = 0.542, \quad u_p = 519 \text{m/s}, \quad W = 778 \text{m/s} \]

- Given \( p_2/p_1 \) or \( T_2/T_1 \) instead of \( \rho_2/\rho_1 \)?

- Given \( u_p \) instead of \( \rho_2/\rho_1 \)?

- What happens when shock reaches end of tube?
What happens when shock reaches end of tube?

What happens when shock reaches end of tube?

\[ W \]

\[ 568.7 \text{kPa} \quad 567.6 \text{K} \]
\[ u_p = 519 \text{m/s} \]

\[ W_R \]
\[ 568.7 \text{kPa} \quad 567.6 \text{K} \]
\[ u_p = 519 \text{m/s} \]

Undisturbed
Reflected Shock Relation

Beginning with the normal shock relation for velocity ratio across a shock we can show that....

\[ \frac{M_R}{M_R^2 - 1} = \frac{M_S}{M_S^2 - 1} \left( 1 + 2 \frac{\gamma - 1}{(\gamma + 1)^2} (M_S^2 - 1) \left( \gamma + \frac{1}{M_S^2} \right) \right) \]

\[ M_S = \frac{W}{\sqrt{\gamma R T_1}} \]

\[ M_R = \frac{u_p}{\sqrt{\gamma R T_2}} + \frac{W_R}{\sqrt{\gamma R T_2}} \]
Moving Expansion Waves

Undisturbed

Moving with wavelet

\[ u = 0 \]

\[ a_4 \quad a - u \quad a_3 - u_p \quad u_p \]

\[ \rho \quad a \quad \rho + d\rho \quad a + du \]
Moving Expansion Waves

\[ \frac{2da}{\gamma - 1} + du = 0 \]
Relation for the Mass Motion Velocity

\[ u_p = f \left( \frac{p_3}{p_4} \right) \]

2a
\[ \frac{u_p}{a_4} = \frac{2}{\gamma - 1} \left( 1 - \frac{a_3}{a_4} \right) \]

\[ \frac{2a}{\gamma - 1} + u = \text{const} \]

Undisturbed

\( u = 0 \)

\[ a_4 \]

\[ a - u \]

\[ a_3 - u_p \]

\[ u_p \]
Example

Given: \( T_4 = 300K \)
\( p_3 / p_4 = 0.048 \)

Find: \( u_p \) and speed of leading and trailing edges of wave

\[
\frac{u_p}{a_4} = \frac{2}{\gamma - 1} \left[ 1 - \left( \frac{p_3}{p_4} \right)^{\frac{\gamma - 1}{2\gamma}} \right]
\]
Pressure Collapse Relation

A.K.A. SHOCK TUBE RELATION

\[ u_p = \frac{2a_4}{\gamma_4 - 1} \left[ 1 - \left( \frac{p_3}{p_4} \right)^{\frac{\gamma_4 - 1}{2\gamma_4}} \right] \]

\[ p_4 \text{(high)} \]
\[ T_4 \]
\[ \gamma_4 \]

\[ p_1 \text{(low)} \]
\[ T_1 \]
\[ \gamma_1 \]

\[ u_p = \frac{a_1 \left( \frac{p_2}{p_1} - 1 \right)}{\gamma_1 \left( \frac{p_2}{p_1} - 1 \right)} \sqrt{1 + \frac{\gamma_1 + 1}{2\gamma_1} \left( \frac{p_2}{p_1} - 1 \right)} \]

\[ \frac{p_4}{p_1} = \frac{p_2}{p_1} \left[ 1 - \frac{(\gamma_4 - 1)(a_1 / a_4)(p_2 / p_1 - 1)}{\sqrt{2\gamma_1(2\gamma_1 + (\gamma_1 + 1)(p_2 / p_1 - 1))}} \right]^{-\frac{2\gamma_4}{\gamma_4 - 1}} \]
$\frac{p_4}{p_1} = \frac{p_2}{p_1} \left[ 1 - \frac{(\gamma_4 - 1)(a_1 / a_4)(p_2 / p_1 - 1)}{\sqrt{2\gamma_1(2\gamma_1 + (\gamma_1 + 1)(p_2 / p_1 - 1))}} \right]^{2\gamma_4 / \gamma_4 - 1}$
<table>
<thead>
<tr>
<th>4</th>
<th>1</th>
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<tbody>
<tr>
<td>$p_4$ (high)</td>
<td>$p_1$ (low)</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$T_1$</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>$\gamma_1$</td>
</tr>
</tbody>
</table>

$$
\frac{p_4}{p_1} = \frac{p_2}{p_1} \left[ 1 - \frac{(\gamma_4 - 1)(a_1 / a_4)(p_2 / p_1 - 1)}{\sqrt{2\gamma_1 (2\gamma_1 + (\gamma_1 + 1)(p_2 / p_1 - 1))}} \right]^{-\frac{2\gamma_4}{\gamma_4 - 1}}
$$