Normal Shock Waves
Shock Waves

• In general – Very strong, extremely thin, waves propagating supersonically, producing almost instantaneous
  • deceleration,
  • compression,
  • increase in pressure, density and temperature, of the flow

• Normal Shocks –
  – perpendicular to the flow
  – always decelerate flow to subsonic speed
Analyzing Shocks

- Shock is so thin that we only need relations for the changes in properties across them.
- Shocks are adiabatic but not isentropic.
- In 1D flow only normal shocks are possible.
- Need...

\[
\begin{align*}
\frac{u_2}{u_1} &= F_1(M_1) \\
\frac{\rho_2}{\rho_1} &= F_2(M_1) \\
\frac{T_2}{T_1} &= F_3(M_1) \\
\frac{p_2}{p_1} &= F_4(M_1) \\
M_2 &= F_5(M_1) \\
s_2 - s_1 &= F_6(M_1) \\
\frac{\rho_{02}}{\rho_{01}} &= F_7(M_1) \\
\frac{p_{02}}{p_{01}} &= F_8(M_1) \\
\frac{A_2^*}{A_1^*} &= F_9(M_1)
\end{align*}
\]
Starting Equations

- Mass
- Momentum
- Energy
\[ \frac{u_2}{u_1} = F_1(M_1) \]

From Energy

- Energy...

\[ C_p T_2 = C_p T_1 + \frac{1}{2} u_1^2 - \frac{1}{2} u_2^2 \]
\[ \frac{u_2}{u_1} = F_1(M_1) \]

*From Mass & Momentum*
\[ \frac{a_2^2}{a_1^2} = 1 + \frac{\gamma - 1}{2} M_1^2 - \frac{\gamma - 1}{2} \frac{M_1^2}{M_1} \frac{u_2^2}{u_1^2} = \frac{u_2}{u_1} + \gamma M_1^2 \frac{u_2}{u_1} - \gamma M_1^2 \frac{u_2^2}{u_1^2} \]

\[ u_2/u_1 = F_1(M_1) \]

Solution
Velocity Ratio

![Graph showing the velocity ratio as a function of $M_1$. The graph indicates a decrease in $u_2/u_1$ with increasing $M_1$. There is a clear asymptotic behavior as $M_1$ approaches a certain value.]
Density, Temp. and Pressure Ratios

\[ \frac{\rho_2}{\rho_1} = F_2(M_1) \]

\[ \frac{T_2}{T_1} = F_3(M_1) \]

\[ \frac{a_2^2}{a_1^2} = 1 + \frac{\gamma - 1}{2} M_1^2 - \frac{\gamma - 1}{2} \frac{M_1^2 u_2^2}{u_1^2} \]

\[ \frac{p_2}{p_1} = F_4(M_1) \]
Density, Temp. and Pressure Ratios

\[
\frac{\rho_2}{\rho_1}, \quad \frac{T_2}{T_1}, \quad \frac{p_2}{p_1}
\]

\begin{figure}
\begin{center}
\includegraphics[width=\textwidth]{density_temp_pressure_ratios.png}
\end{center}
\end{figure}
\( M_2 \) and Entropy Gain

\[
M_2 = \frac{s_2 - s_1}{C_v} = \log_e \left[ \frac{T_2}{T_1} \left( \frac{\rho_1}{\rho_2} \right)^{\gamma^{-1}} \right]
\]
For Air

\[ M_2 \]

Diagram showing the relationship between \( M_1 \) and \( M_2 \) for Air.
Entropy Gain

For Air

\[
\frac{(s_2 - s_1)}{C_v} \propto C_v \quad (s_2 - s_1)
\]
Stagnation Pressure and Density Ratio

Imagine the entropy equation applied between hypothetical isentropic stagnation points on either side of the wave...

\[
\frac{S_2 - S_1}{C_v} = F_6(M_1) = \log_e \left[ \frac{T_{02}}{T_{01}} \left( \frac{\rho_{01}}{\rho_{02}} \right)^{\gamma^{-1}} \right]
\]
Stagnation Pressure and Density Ratio

For Air

\[ \frac{p_2}{p_1}, \frac{\rho_2}{\rho_1} \]

\[ M_1 \]
Rankine Hugoniot Relations

\[
\frac{u_2}{u_1} = F_1(M_1) \quad \frac{\rho_2}{\rho_1} = F_2(M_1) \quad \frac{T_2}{T_1} = F_3(M_1)
\]

\[
\frac{p_2}{p_1} = F_4(M_1) \quad M_2 = F_5(M_1) \quad s_2 - s_1 = F_6(M_1)
\]

\[
\frac{\rho_{02}}{\rho_{01}} = F_7(M_1) \quad \frac{p_{02}}{p_{01}} = F_8(M_1) \quad \frac{A_2^*}{A_1^*} = F_9(M_1)
\]

TABULATED along with \( p_1/p_{02} \) in NACA 1135

Don’t need tables for \( u_2/u_1 \) or \( \rho_{02}/\rho_{01} \) since

\[
\frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} \quad \text{and} \quad \frac{\rho_{02}}{\rho_{01}} = \frac{p_{02}}{p_{01}}
\]

Will meet later