Topic 2

Thermodynamics
Why Thermodynamics?

• To tell us the relationship between the pressure, temperature and density of a gas as it moves.
• Altogether there are 6 variables we need to describe the state of a gas...
  • $p$ pressure (Pa)
  • $\rho$ density (kg/m$^3$), or specific volume $v=1/\rho$
  • $T$ temperature (K)
  • $e$ internal energy per unit mass ($\mathrm{J/kg}$)
  • $h$ enthalpy ($\mathrm{J/kg}$), $h=e+p/\rho$
  • $s$ entropy ($\mathrm{J/kg\cdot K}$)
• To relate these variables we use
  1. *The Kinetic Theory of Gases*,
  2. *The 1st Law of Thermodynamics*,
  3. *Specific Heats*
  4. *The 2nd Law of Thermodynamics*
1. Kinetic Theory of Gases

- Assumes
  - Gas is a collection of molecules in random motion
  - Molecules bounce of each other like hard spheres
- Gives two sets of relations...

Equation of State

- Gas constant
  \[ R = 287 \text{ J/kg/K} \]
  for air

- Gases that obey these relations are called “Thermally Perfect”
Thermally Perfect

• We will always assume gases are thermally perfect in this class, but this isn’t always true...

Ceases to be valid at high densities. Why?
2. 1\textsuperscript{st} Law of Thermodynamics

“Energy is Conserved”

\[ \Delta E = \Delta Q + \Delta W \]

- Change in energy
- Amount of heat added
- Amount of work done by pressure and viscous forces

Work =
In terms of enthalpy...

\[ dq = de + pdv \]
3. Specific Heats

- Specific heat – amount of heat needed to raise the temperature of 1kg by 1 Kelvin
  \[ C = \frac{\Delta q}{\Delta T} = \frac{\partial q}{\partial T} \]

- Two kinds:
  - Specific heat at constant volume \( C_v = (\frac{\partial q}{\partial T})_v \)
  - Specific heat at constant pressure \( C_p = (\frac{\partial q}{\partial T})_p \)

Pratt and Whitney JT58 Test (Powered SR71)
\[ dq = de + pdv \]

\[ dq = dh - vdp \]

With the energy equation...
Calorically Perfect Gas

A gas that is thermally perfect and for which the...

- specific heats are constant with temperature, so...

- We will assume calorically perfect gases in this class, but...
4. 2\textsuperscript{nd} Law of Thermodynamics

Shock in a Converging Diverging Nozzle

Bourgoing & Benay (2005), ONERA, France

Schlieren visualization
Sensitive to in-plane index of ref. gradient
Processes and Entropy

• Adiabatic Process - no heat added or removed.
• Reversible Process – no dissipative phenomena occur.
• Isentropic Process – adiabatic and reversible

2\textsuperscript{nd} Law of Thermo – dissipative processes may only increase entropy, but adding or removing heat may increase or decrease entropy by an amount $\Delta Q/T$

• So...
The Entropy Equation...

For a reversible process...

\[ Tds = dq \]

**1\textsuperscript{st} Law**

\[ dq = de + pdv \]

Thermally perfect gas

\[ de = C_v dT \]

\[ p = RT / v \]
$R = C_p - C_v$

$\gamma = \frac{C_p}{C_v}$

...The Entropy Equation
Isentropic Relations

\[ \frac{s_2 - s_1}{C_v} = \log_e \left[ \frac{T_2}{T_1} \left( \frac{\rho_1}{\rho_2} \right)^{\gamma^{-1}} \right] \]

- If \( s_2 - s_1 = 0 \) then...
Summary

- **KTG**
  
  - All gases
  - Thermally perfect
  - Calorically perfect

  \[ p = \rho RT \]

  \[ e = e(T) \]

  \[ h = h(T) \]

- **1st Law**

  \[ dq = de + pdv \]

  \[ dq = dh - \nu dp \]

- **Specific heats**

  \[ C_v = (\partial e / \partial T)_v \]

  \[ C_p = (\partial h / \partial T)_p \]

- **2nd Law**

- **2nd Law**

  Isentropic

  \[ s_2 - s_1 = \log_e \left( \frac{T_2}{T_1} \left( \frac{\rho_1}{\rho_2} \right)^{\gamma - 1} \right) \]

  \[ \frac{\rho_2}{\rho_1} = \left( \frac{T_2}{T_1} \right)^{1 - \gamma} \]

  \[ \frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\gamma - 1} \]