AOE 3054 - Uncertainty

(or, How To Estimate Error)

- Relevant to *everything* from now on.
- See lab manual, “Basic Concepts in Experiments” and “Experiment 4”

Accuracy...

- Accuracy (same as error):
- Precision
- Repeatability
- Uncertainty

Thoughts

Boeing AH-64A Apache

Uncertainty

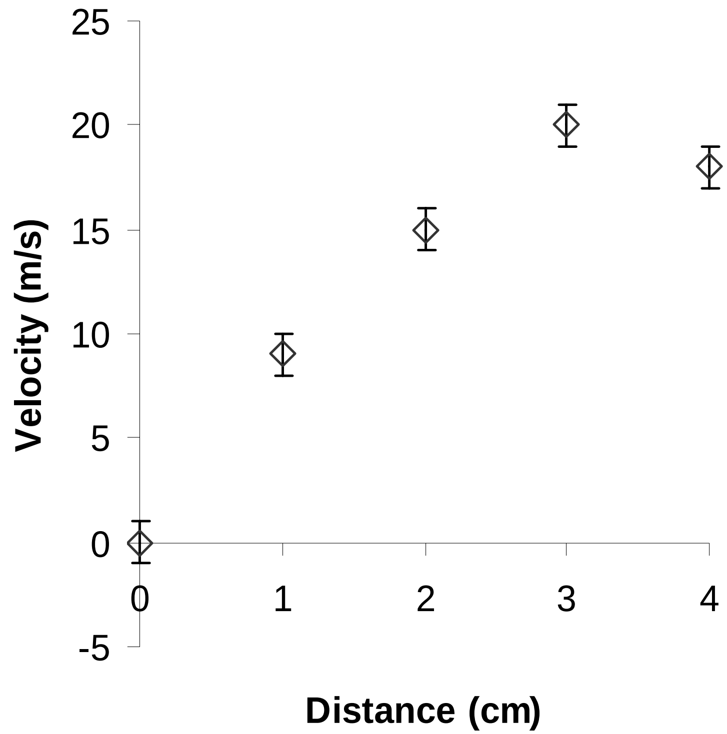
- Concept of an uncertainty interval
- Determining uncertainty
  - *in a primary measurement*
  - *in a result*
- Examples
- Using a computer to estimate uncertainty

Uncertainty Interval

- Definition:

The 95% condition is sometimes referred to as "*20:1 odds*". Depending on how critical the information being handled is, higher odds are sometimes specified, e.g. 100:1 or 99%

- Representation:
  -
  -
  -
  -
  - *Error bars...*



Determining Uncertainty
in a <u>Primary Measurement</u>

- Primary measurement:

E.g. Distance from a dial gage, height from a manometer (***<u>not</u>*** velocity from a Pitot probe)

- Make an <u>informed</u> guess based on :

  -

  -

  -

  -

  - *other factors*

    validity of measurement scheme
    operating outside the design/calibration range
    environmental conditions
    blunders

  - *experience*

Determining Uncertainty

in a Primary Measurement

Example:

Uncertainty estimate = ??

Determining Uncertainty
in a Primary Measurement

• What you consider an “uncertainty” may not always be an error in the traditional sense. It depends on what you’re hoping to determine from the primary measurement.

E.g. Suppose you aim to determine the mean value of a fluctuating quantity, like the position of a wing spar vibrating in flight, by taking a series of samples…

Determining Uncertainty

in a Derived Result

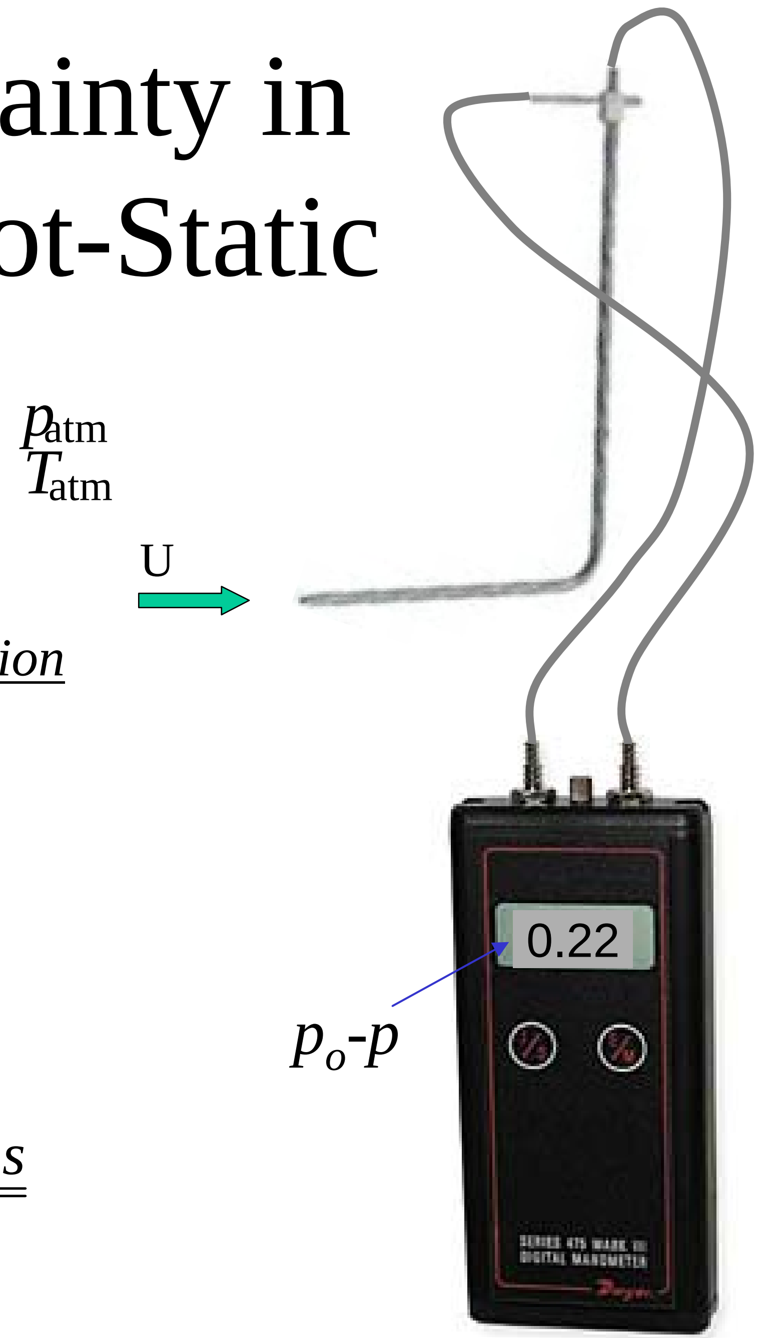
• Consider an experimental result *R* that is determined from one or more primary measurements *a*, *b*, *c*… by means of a function *f( )*, i.e. *any* derived result.

$$R = f(a, b, c, \ldots)$$

• The uncertainty in *R* due to each of the primary measurements may be determined by multiplying the uncertainty in that measurement by the sensitivity of *R* to that measurement. Since each of the primary measurements will be independent, the total uncertainty in *R* is thus,

Example 1: Uncertainty in Velocity from a Pitot-Static

Determine the uncertainty in velocity measurements made with the Pitot-static probe and digital manometer used in experiment 3



Readings

$p_o - p$ = 0.22kPa = 220Pa

p_{atm} = 941.1mBar = 94110Pa

T_{atm}= 17.0^{o}C = 290.0K

Equation

Result

$$U = \sqrt{\frac{2 \times 220}{(94110 / 287 / 290.0)}} = \underline{\underline{19.72 m/s}}$$

Step 1: Primary Uncertainties

Manometer ?

Barometer ?

Thermometer ?

$p\text{-}p_o$	$\delta(p\text{-}p_o) =$
p_{atm}	$\delta(p_{atm}) =$
T_{atm}	$\delta(T_{atm}) =$

Step 2: Partial Derivatives

$$U = \sqrt{\frac{2(p_o - p)}{(p_{atm} / R / T_{atm})}}$$

Step 3: Combine Uncertainties

$\delta(p-p_o) = 20\text{Pa}$
$\delta(p_{atm}) = 50\text{Pa}$
$\delta(T_{atm}) = 0.5^{o}\text{C}$

$$\frac{\partial U}{\partial(p_o - p)} = \underline{0.0448}$$
$$\frac{\partial U}{\partial T_{atm}} = \underline{0.0340}$$
$$\frac{\partial U}{\partial p_{atm}} = \underline{-0.000105}$$

$$\delta(U) =$$

$$d = \sqrt{[0.0448 \times 20]^2 + [0.0340 \times 0.5]^2 + [-0.000105 \times 50]^2}$$

$$= \sqrt{0.803 + 0.00029 + 0.000027} = \underline{0.80 m/s}$$

Step 4. Interpretation

$$\delta(U) = \sqrt{\left[\frac{\partial U}{\partial(p_o - p)}\delta(p_o - p)\right]^2 + \left[\frac{\partial U}{\partial T_{atm}}\delta(T_{atm})\right]^2 + \left[\frac{\partial U}{\partial p_{atm}}\delta(p_{atm})\right]^2}$$

$$= \sqrt{0.803 + 0.00029 + 0.000027} = \underline{0.80 m/s}$$

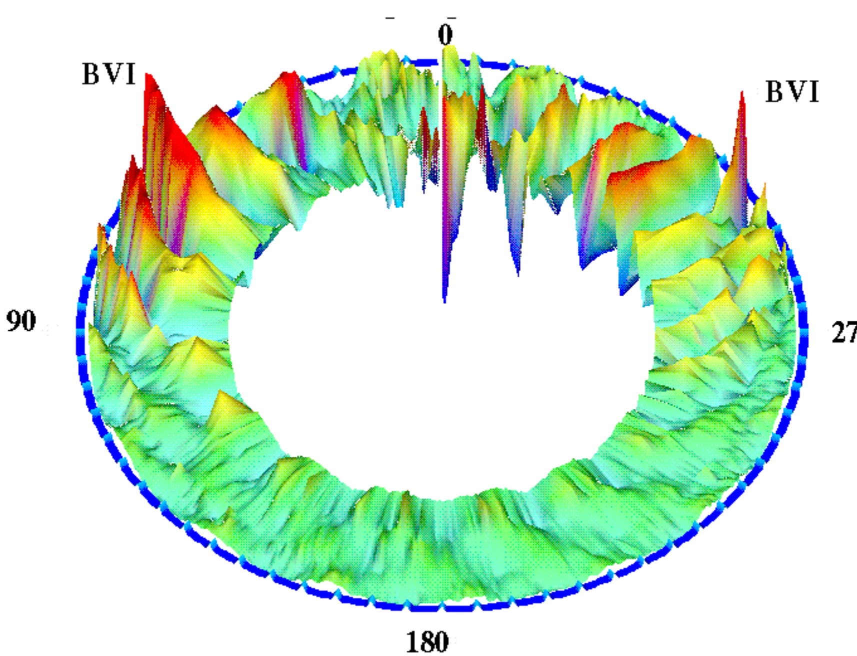
•

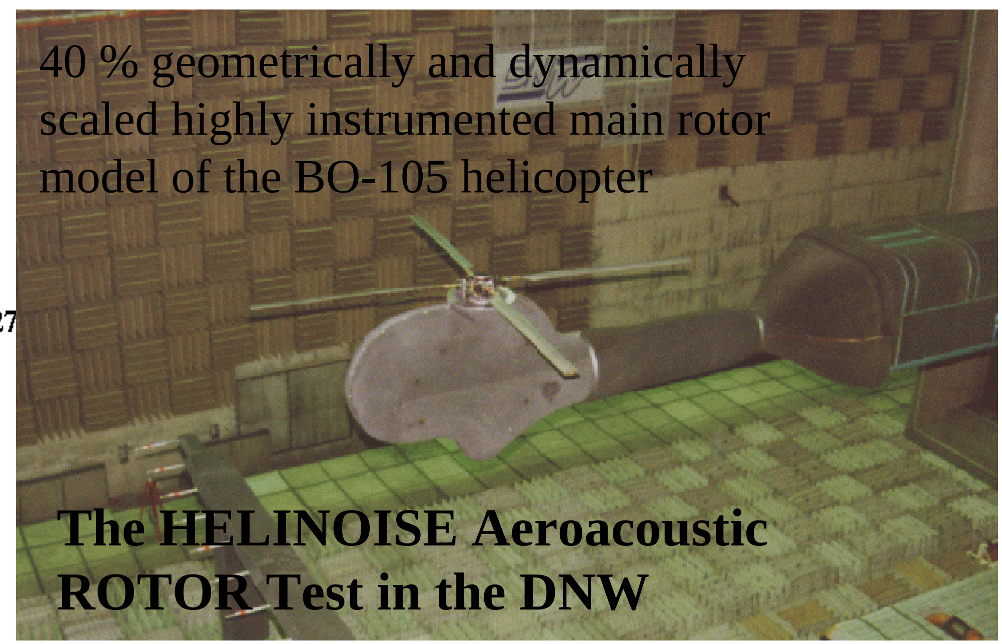
• Since it is (hopefully) a small fraction of the measurement, the accuracy of the uncertainty estimate itself is not that important. (e.g. it wouldn't change any conclusions here if the uncertainty were 0.6m/s. This is why we can tolerate informed guesswork in determining the uncertainty in the primary measurements.

Example 2: Uncertainty in a Statistical Result

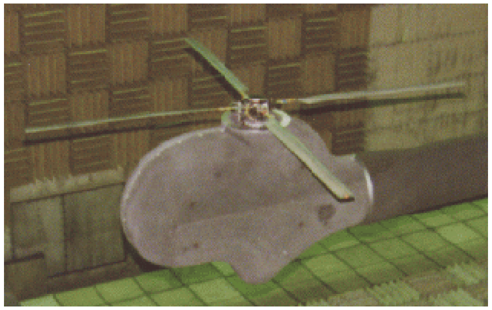
Problem: *The deflection 'y' of a helicopter rotor blade is measured, once per revolution, over N rotor revolutions. The data is used to estimate the mean and standard deviation σ of the deflection. Assuming the error in the individual deflection measurements is negligible, determine the uncertainty in the mean value.*

Mean $$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$





Step 1: Primary Uncertainties



$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{y_1 + y_2 + \ldots y_N}{N}$$

Step 2: Partial Derivatives

Step 3: Combine Uncertainties

$$\frac{\partial \bar{y}}{\partial y_i} = \frac{1}{N}$$

$$\delta(y_i) = 2\sigma_y$$

Step 4: Interpretation

- The uncertainty in the mean value goes as the square root of the number of samples. Most statistical results behave in this way.

Determining Uncertainty

using a computer

Suppose we write a computer program to determine our result *R* from the primary measurements *a*, *b*, *c*… We have:

or, as before:

The uncertainty in *R* may therefore be calculated using the same expression:

The only problem is determining the partial derivatives.

Determining Uncertainty

using a computer

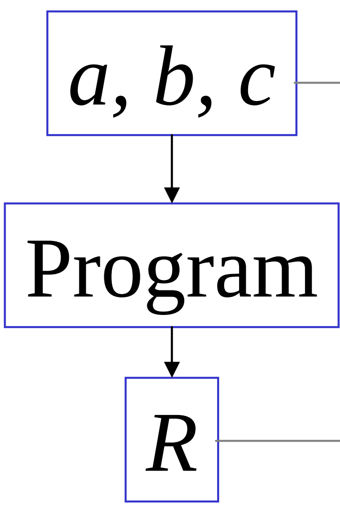
This can be done by perturbing, in turn, each of the inputs to our program by its uncertainty, and recording the change in output that results. (The modified program that does this is usually called a *jitter* program).

The δ's musn't be too large a fraction of the measured quantity – up to 20% is probably OK. (If δ's are bigger, then just use a smaller fraction and multiply the result accordingly)

Example

Computing uncertainty in measured Reynolds number

$$R = f(a, b, c, \ldots)$$



F19 =F14*F15*F5/F16

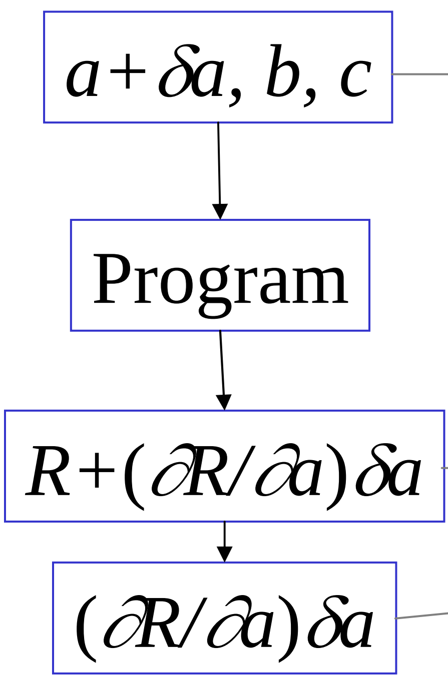
	A	B	F	G
1		**Worksheet for Uncertainty in Reynolds Number**		
2				
3				Primary
4			Quantity	Uncertainty
5		Model length (m)	0.1	
6				
7		**Primary measurements**		
8	a	Reference Pitot-static pressure kPa	0.203	0.001
9	b	Barometer reading (mBar)	944.7	0.4
10	c	Thermometer reading (deg. C)	22.5	0.5
11				
12		**Intermediate results**		
13		Absolute temperature K	295.5	
14		Air density (kg/m^3)	1.11392136	
15		Freestream velocity (m/s)	19.0913109	
16		Dynamic Viscosity (kg/m/s)	1.8244E-05	
17				
18		**Final Result**		
19		Reynolds number	116566.877	
20				

Example

Copy calculation into 3 spare columns

	A	B	C	D	E	F	G	H	I	J
1		**Worksheet for Uncertainty in Reynolds Number**								
2										
3							Primary			
4						Quantity	Uncertainty			
5		Model length (m)				0.1				
6										
7		**Primary measurements**								
8	a	Reference Pitot-static pressure kPa				0.203	0.001			
9	b	Barometer reading (mBar)				944.7	0.4			
10	c	Thermometer reading (deg. C)				22.5	0.5			
11										
12		**Intermediate results**								
13		Absolute temperature K				295.5				
14		Air density (kg/m^3)				1.113921				
15		Freestream velocity (m/s)				19.09131				
16		Dynamic Viscosity (kg/m/s)				1.82E-05				
17										
18		**Final Result**								
19		Reynolds number				116566.9				

Example



	A	B	C	D	E	F	G	H	I	J
1		**Worksheet for Uncertainty in Reynolds Number**								
2										
3							Primary			
4						Quantity	Uncertainty	a+da,b,c		
5		Model length (m)				0.1		0.1	0.1	0.1
6										
7		**Primary measurements**								
8	a	Reference Pitot-static pressure kPa				0.203	0.001	0.204	0.203	0.203
9	b	Barometer reading (mBar)				944.7	0.4	944.7	944.7	944.7
10	c	Thermometer reading (deg. C)				22.5	0.5	22.5	22.5	22.5
11										
12		**Intermediate results**								
13		Absolute temperature K				295.5		295.5	295.5	295.5
14		Air density (kg/m^3)				1.113921		1.11392	1.11392	1.11392
15		Freestream velocity (m/s)				19.09131		19.1383	19.0913	19.0913
16		Dynamic Viscosity (kg/m/s)				1.82E-05		1.8E-05	1.8E-05	1.8E-05
17										
18		**Final Result**								
19		Reynolds number				116566.9		116854	116567	116567
20							Change	286.758		
21										
22										

Example

F21 =SQRT(H20^2+I20^2+J20^2)

	A	B	C	D	E	F	G	H	I	J
1		**Worksheet for Uncertainty in Reynolds Number**								
2										
3							Primary			
4						Quantity	Uncertainty	a+da,b,c	a,b+db,c	a,b,c+dc
5		Model length (m)				0.1		0.1	0.1	0.1
6										
7		**Primary measurements**								
8	a	Reference Pitot-static pressure kPa				0.203	0.001	0.204	0.203	0.203
9	b	Barometer reading (mBar)				944.7	0.4	944.7	945.1	944.7
10	c	Thermometer reading (deg. C)				22.5	0.5	22.5	22.5	23
11										
12		**Intermediate results**								
13		Absolute temperature K				295.5		295.5	295.5	296
14		Air density (kg/m^3)				1.113921		1.11392	1.11439	1.11204
15		Freestream velocity (m/s)				19.09131		19.1383	19.0873	19.1075
16		Dynamic Viscosity (kg/m/s)				1.82E-05		1.8E-05	1.8E-05	1.8E-05
17										
18		**Final Result**								
19		Reynolds number				116566.9		116854	116592	116317
20							Change	286.758	24.6755	-250.368
21		Uncertainty in final result				381.4752				
22										

$(\partial R/\partial a)\delta a$

$(\partial R/\partial b)\delta b$

$(\partial R/\partial c)\delta c$

$$\delta(R) = \sqrt{\left[\frac{\partial R}{\partial a}\delta(a)\right]^2 + \left[\frac{\partial R}{\partial b}\delta(b)\right]^2 + \left[\frac{\partial R}{\partial c}\delta(c)\right]^2}.$$