Spectral Analysis

- This week in lab
  - Your next experiment
  - Homework is to prepare
- Next classes: 3/26 and 3/28
  - Aero Testing, Fracture Toughness Testing
  - Read the Experiments 5 and 7 sections of the course manual

Spectral Analysis

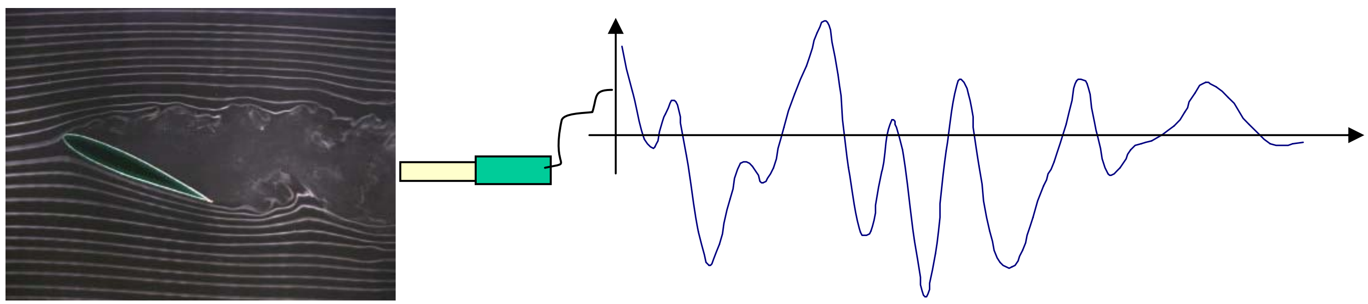
- What is spectral analysis?

- How do we estimate the spectrum of a signal?

- Getting the Spectrum right

What is Spectral Analysis?

Consider a fluctuating signal output by a transducer of some kind, e.g. unsteady lift force on a wing (strain gage balance), velocity from a hot-wire anemometer in a turbulent flow, position of a vibrating structure (proximeter)



We often need to answer the question "*What frequencies are in the signal?*" (e.g. may want to know size of eddies, motion of wing, effects of structure motion on connected objects).

This question is more artificial than it sounds. A more accurate restatement of the question would be -

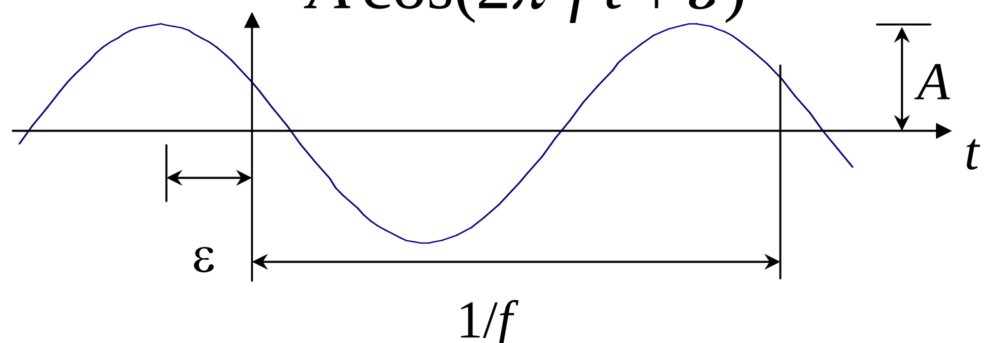
"*If we were to fit the shape of the signal to a sum of sinusoids of different frequencies, what would be the distribution of amplitudes and phases as a function of the frequency?*"

What is Spectral Analysis?

Also called Fourier Analysis

Definition: Decomposition of a signal into a series of component sinusoids, e.g.

$$A\cos(2\pi f t + \varepsilon)$$



Each sinusoid will have a different **frequency** *f*, **amplitude** *A* and **phase** ε. With such a decomposition we can plot

May be a plot of the amplitude itself or the RMS amplitude (0.7071*A*)

Amplitude *vs.* Frequency	*Amplitude Spectrum*
Phase *vs.* Frequency	*Phase Spectrum*
½Amplitude^2 *vs.* Frequency	*Power Spectrum*

May be in degrees or radians

decibels (dB)

- Logarithmic scale
- Amplitude in dB is $20\log_{10} A_n$
- RMS amplitude in dB is $20 \log_{10}(0.7071 A_n)$
- Power spectrum in dB is $10 \log_{10}(\frac{1}{2} A_n^2)$
- Used in may applications, always has the form…

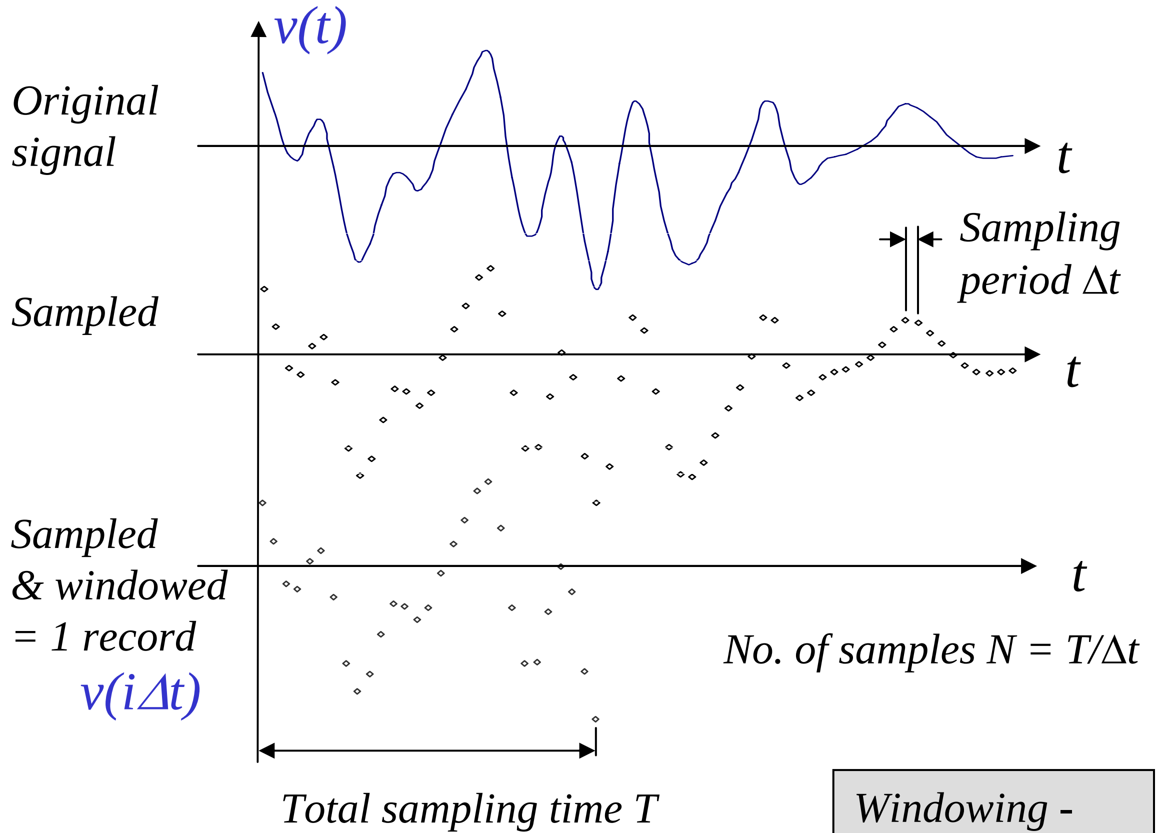
$$10\log_{10}(\text{const.} \times \text{quantity}^2)$$

How to estimate the spectrum

1. Measure the signal

Most signals, and the phenomena they represent, are continuous and go on for a very long time (if not forever). To measure the signal, however, we must take discrete samples of it at a finite rate, and we can only do that for a limited amount of time.

This process places 2 limits on the frequency information we can unambiguously extract from the measured signal.



Windowing - process by which a time-limited section of signal is extracted

Highest frequency = **Nyquist** = $1/(2\Delta t)$ (limited by sampling rate)

Lowest frequency = Frequency that lasts entire sampling time

= $1/T = 1/(N\Delta t)$ (limited by sampling time)

= *Frequency resolution*

How to estimate the spectrum

2. Calculate the spectrum

$A\cos(2\pi f t + \varepsilon)$

Original signal (as Σ sines) $v(t) = \frac{A_o}{2} + \sum_{n=1}^{N/2} A_n \cos\left(\frac{2\pi nt}{N\Delta t} + \varepsilon_n\right)$

Where:

Amplitude $A_n = \sqrt{a_n^2 + b_n^2}$

Phase $\varepsilon_n = -\arctan(b_n / a_n)$

Frequency $f = n/(N\Delta t)$

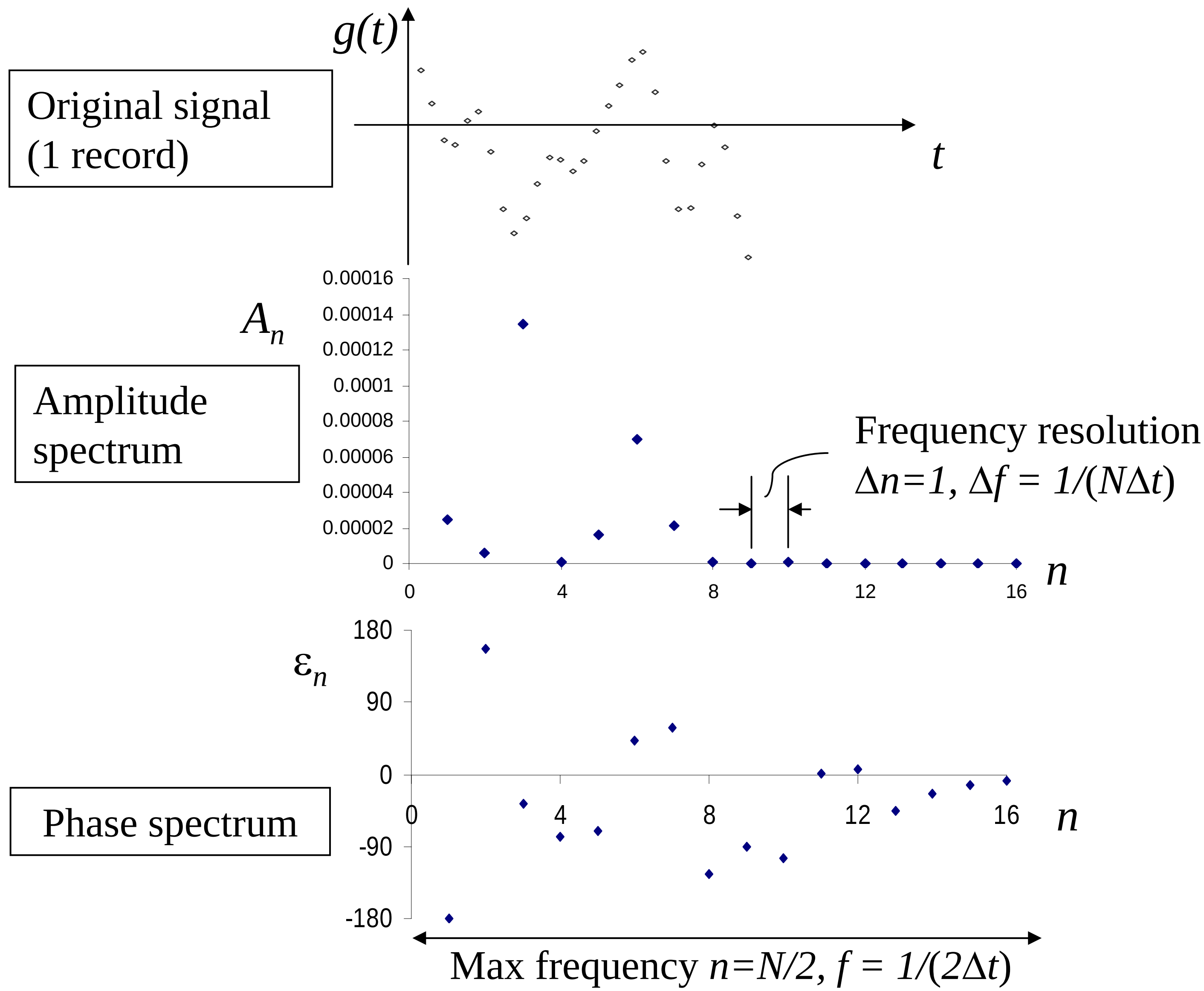
N = no. of samples
Δt = sampling period

$$a_n = \frac{2}{N}\sum_{i=1}^{N} v(i\Delta t)\cos\left(\frac{2\pi ni}{N}\right) \quad b_n = \frac{2}{N}\sum_{i=1}^{N} v(i\Delta t)\sin\left(\frac{2\pi ni}{N}\right)$$

This set of relations is known as the DISCRETE FOURIER TRANSFORM. Numerical calculations of spectra are usually performed using a short-cut algorithm (that produces identical results) known as the FAST FOURIER TRANSFORM (FFT).

How to estimate the spectrum

Results

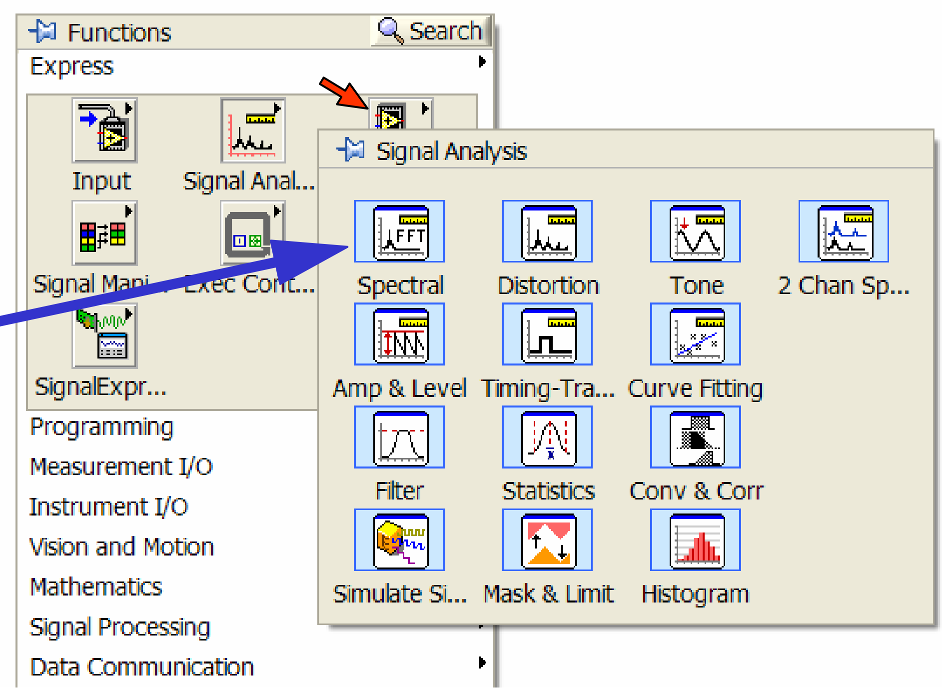


How to estimate the spectrum

3. Using standard software

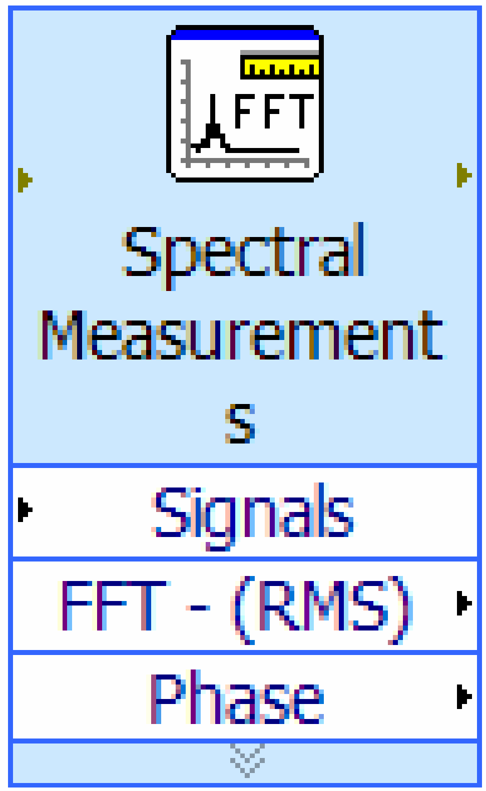
- In *Matlab*, if *v* with indices 1 to *N* is an array of samples containing the sampled time signal, then *c=fft(v)* gives a complex array *c* where $a_n+ib_n= 2i*c_{n+1}/N$. From this all needed quantities can be calculated.

- In *LabView* there are many functions that deal with computing FFTs, spectra and related quantities. The simplest to use is 'Spectral Measurements'



Spectral Measurements VI

Signal in (Voltage samples from A/D)



Output form depends on internal settings of VI

Amplitudes out

Phases out

Internal Settings

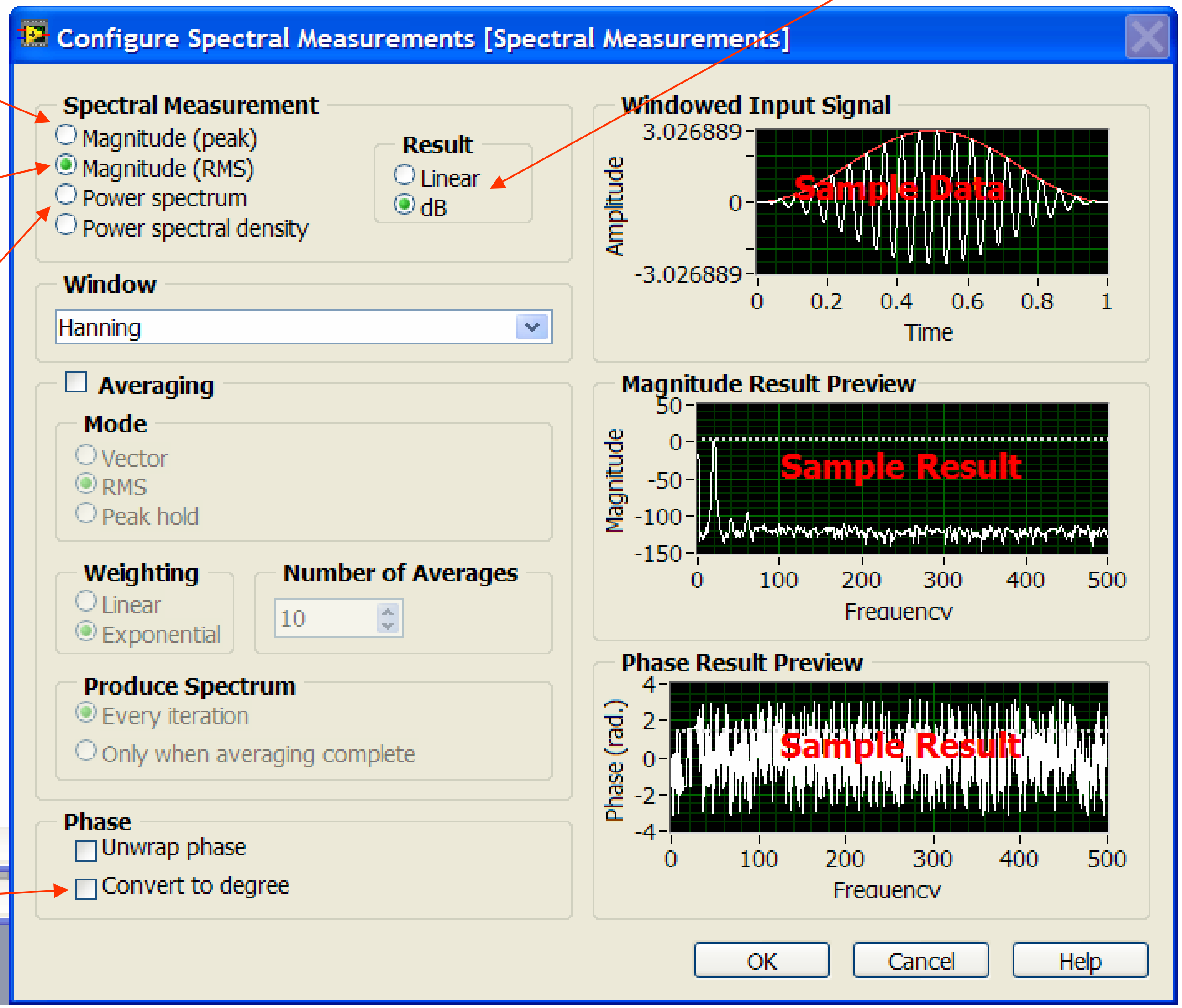
Output is actual cosine amplitudes A_n

Output is $0.7071A_n$

Output is power spectrum $\frac{1}{2}A_n^2$ (no phase output)

Phase output in degrees (otherwise radians)

Amplitude output in decibels (or not)



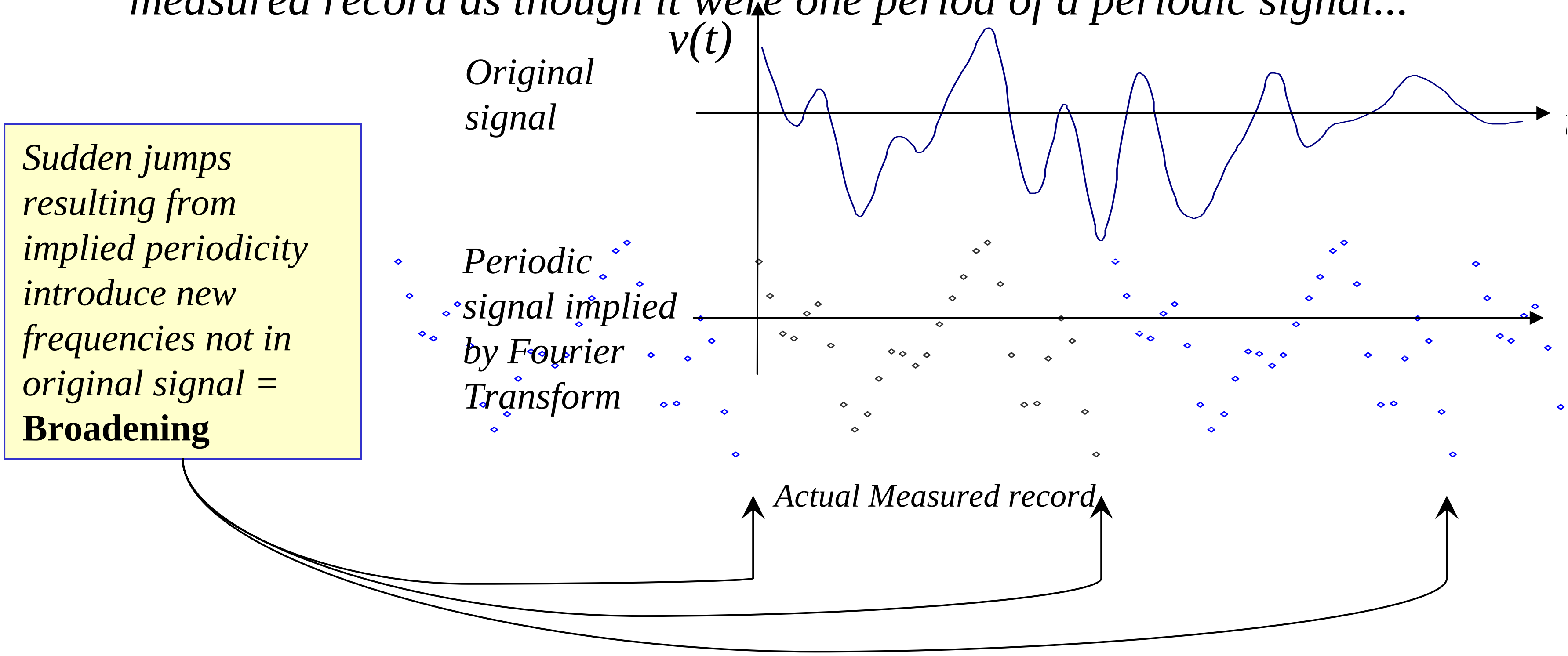
Getting the Spectrum Right

1. **Finite frequency resolution.** *Frequency resolution is limited by the total sampling time for the record.*
   - Solution*: sample for a longer time (i.e. increase the record length N) then* $\Delta f = 1/(N\Delta t)$ *will be smaller*

2. **Aliasing.** *Original signal may contain frequencies higher than the Nyquist. When we sample the signal these appear as though they are lower frequencies and thus corrupt the spectra at these frequencies.*
   - Solutions*: increase sampling rate to twice highest frequency, or, filter out all frequencies higher than the Nyquist before sampling ('anti-aliasing filter').*

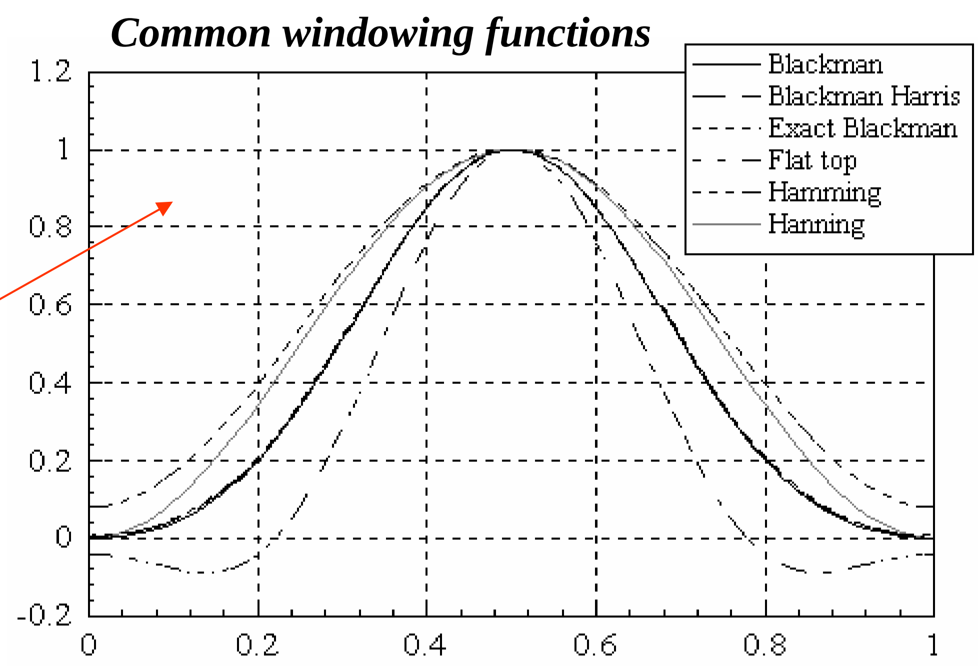
Getting the Spectrum Right

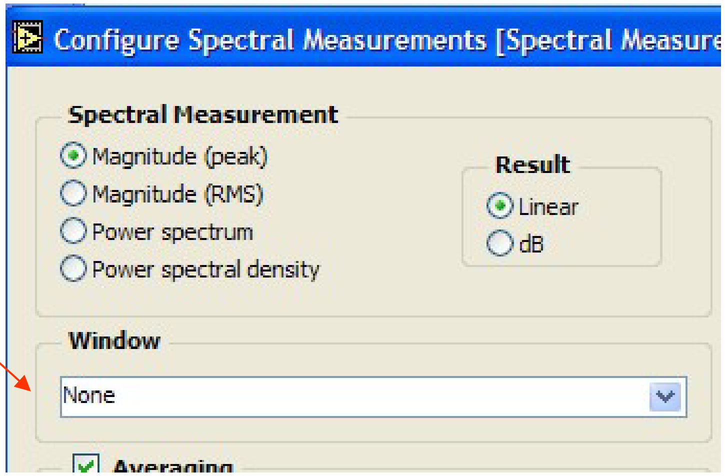
3. **Broadening.** *Sharp features in the spectrum may be smoothed (and thus inaccurately represented) in the measured spectrum.*
   – Explanation: *There is a contradiction in what we have done. We have decomposed one record (lasting only for time T) into a sum of infinitely long sinusoids. This can only be done by implicitly treating the measured record as though it were one period of a periodic signal...*



Getting the Spectrum Right

Broadening.
Solution: *Multiply record by a smooth function that is zero, or nearly so at the start and end points of the record. There are different types of these 'windowing functions' one can choose from. These functions do not substantially change the spectrum, but they tend to minimize the broadening.*





Getting the Spectrum Right

4. **Randomness in the signal**.
   a) Due to noise corrupting the signal (e.g contamination of response signal from structure due to electrical noise or floor vibrations)
   b) Due to randomness in the physical quantity being measured (turbulence in the velocity signal from a cylinder wake)
   – Introduces uncertainty into the measured spectrum
   – Solution: Average the spectrum. How you average depends on which of the above situations is present

Averaging

- To eliminate the effects of unwanted noise in the signal.
  - Take many measurements of the same signal and calculate their spectra
  - Average the a_n and b_n coefficients (= *vector averaging* in Spectral Analysis VI)
  - If you want the resulting phase spectrum to be meaningful each measurement of the signal must be synchronized with the same features of the signal (i.e. it must be triggered).
- To average the effects of random fluctuations in the measured quantity
  - Take many measurements of the same signal and calculate their spectra
  - Average the power spectral values ($\frac{1}{2}A_n^2$)
  - No phase spectrum can be usefully calculated (random signal = random phase)