

Measuring the Dynamic Response of a Structure

- This week in lab
 - Experiment 6 – application of the analogue instrumentation you met in the 1st Instrumentation Lab period.
 - Works just like a regular experiment
 - Read manual
 - Meet with your team in advance
 - Visit the lab
 - Do a logbook preparation
 - Logbook is graded
 - Note your logbook preparation should be submitted to Dustin Grissom (dgrissom@vt.edu) not your regular TA.
- Next class 3/12 (no class next week)
 - Making measurements with computers
 - Read the ‘Digital Measurements’ section of the manual

Dynamic Response of a Structure

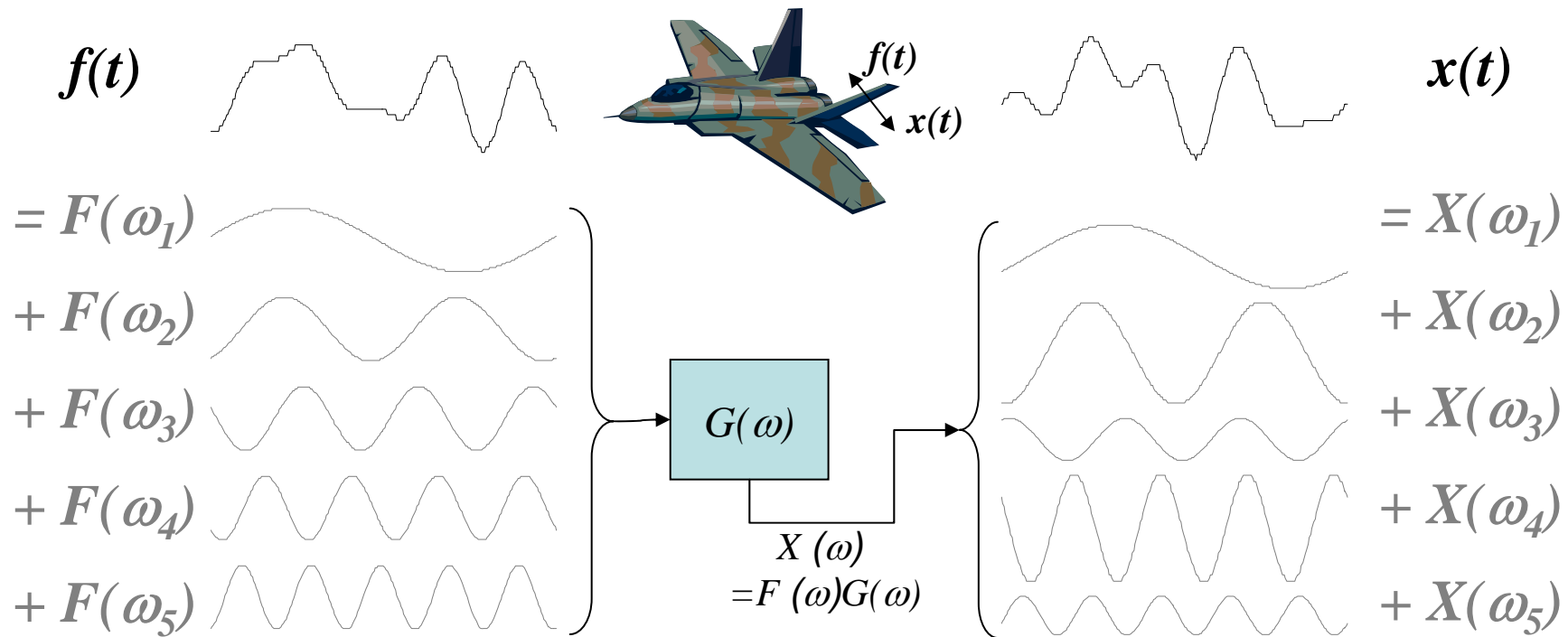
- Characterizing how a structure will respond to a time-varying set of loads is a fundamental requirement of engineering in aero, ocean and space applications



Linearity

For small deformations structures tend to respond linearly to dynamic loads.

This means that the response of the structure is the sum of its response to the individual frequencies in the load



Response function $G(\omega)$: Change in amplitude and phase at each frequency. Knowing $G(\omega)$ tells us how the structure will respond to any load fluctuation, since any load fluctuation can be decomposed into a sum of sinusoids.

Dynamic Structural Testing

We wish to define this response function. So in a lot of dynamic structural testing:

- We apply artificial unsteady loads of known magnitude
- Measure displacement amplitudes and phases
- Infer response function, or parameters of the response function.

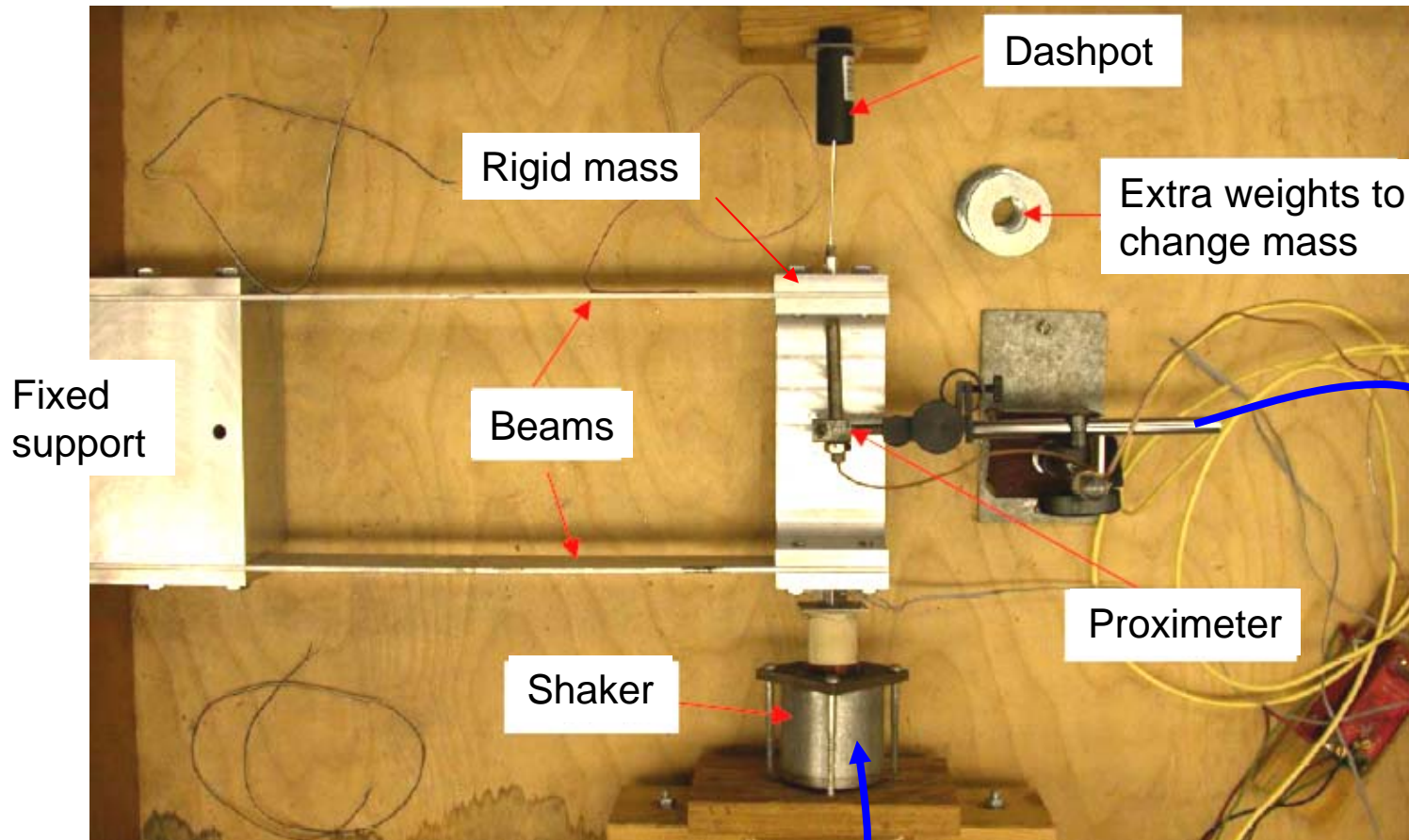
This requires:

- A means to apply the load at varying frequency
- A means to measure the displacement fluctuations
- A theoretical model of the structure that defines the parameters



Dryden Flight Research Center Photo Collection
[/www.dfrc.nasa.gov/gallery/photo/index.html](http://www.dfrc.nasa.gov/gallery/photo/index.html)
203-46 Date: August 22, 2002 Photo by: Tom Tschida
Five Aeroelastic Wing F/A-18 test aircraft are covered with accelerometers
ground vibration tests at NASA Dryden Flight Research Center.

Experiment 6



Fixed support

Rigid mass

Dashpot

Extra weights to change mass

Beams

Proximeter

Shaker

Force excitation system

Response measurement system

Application of Force (Excitation)

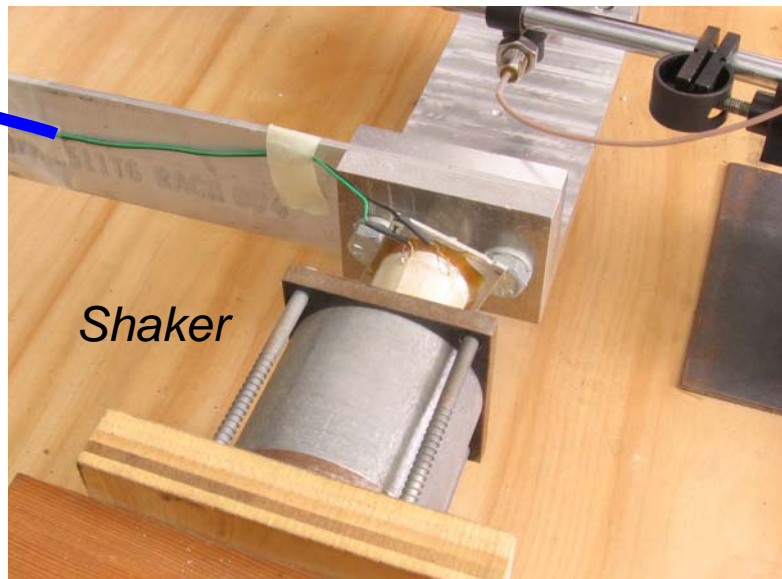


Function Generator

Power amplifier

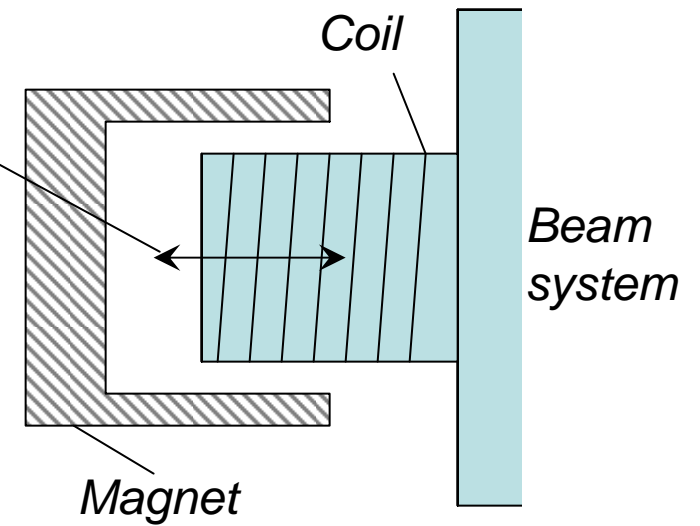


Voltage $v_1(t)$

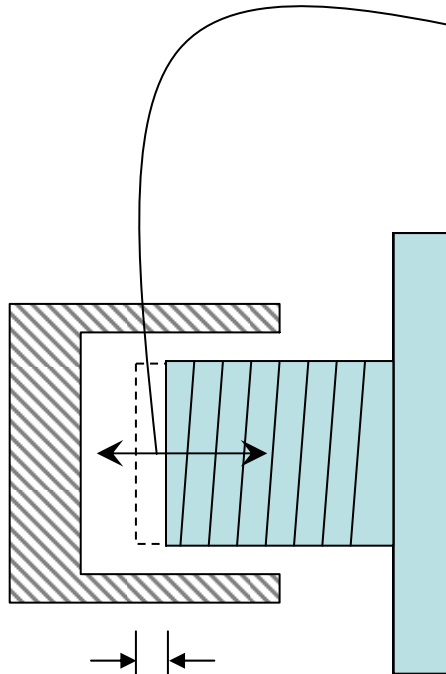


Shaker

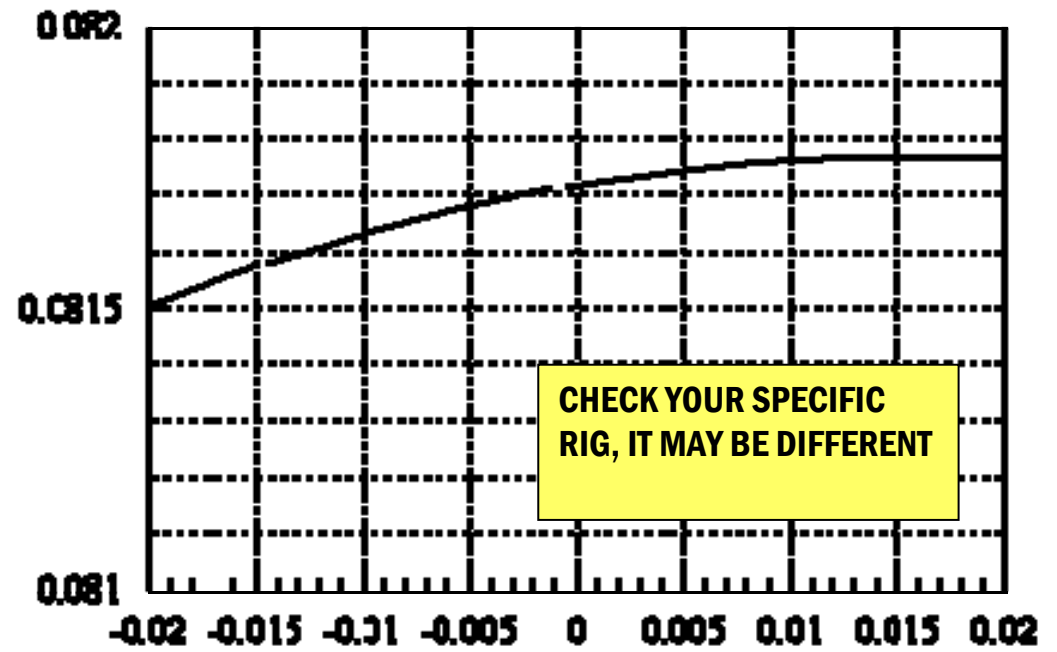
Force $f(t)$ applied without contact



Shaker calibration



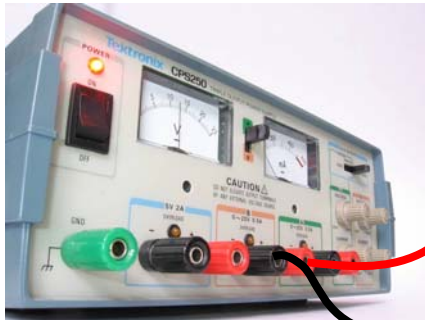
Force (lbs) f
per volt v_1



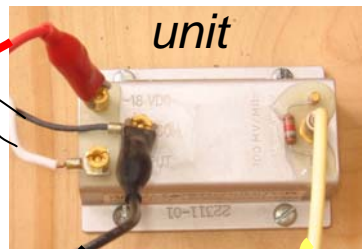
Coil displacement (inches)

Measurement of Beam Position (Response)

Power supply



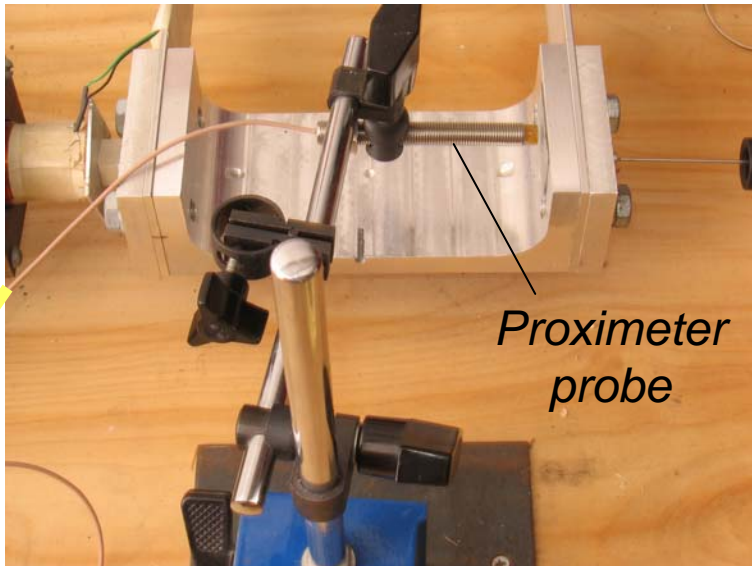
Proximeter unit



Oscilloscope

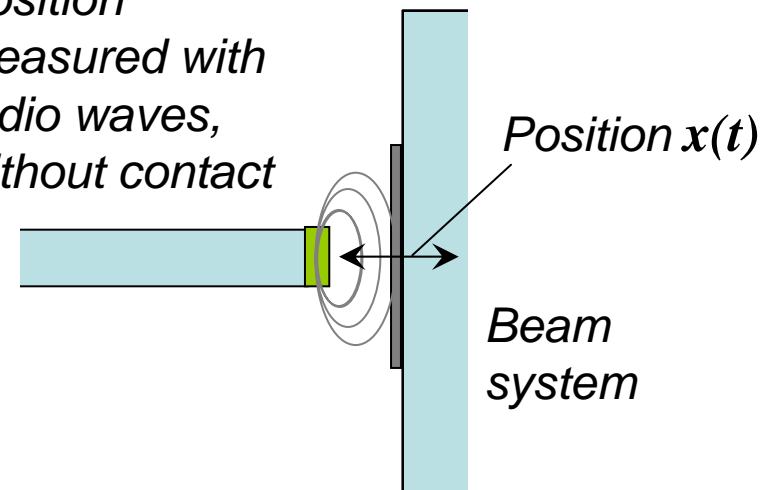


Voltage $v_2(t)$

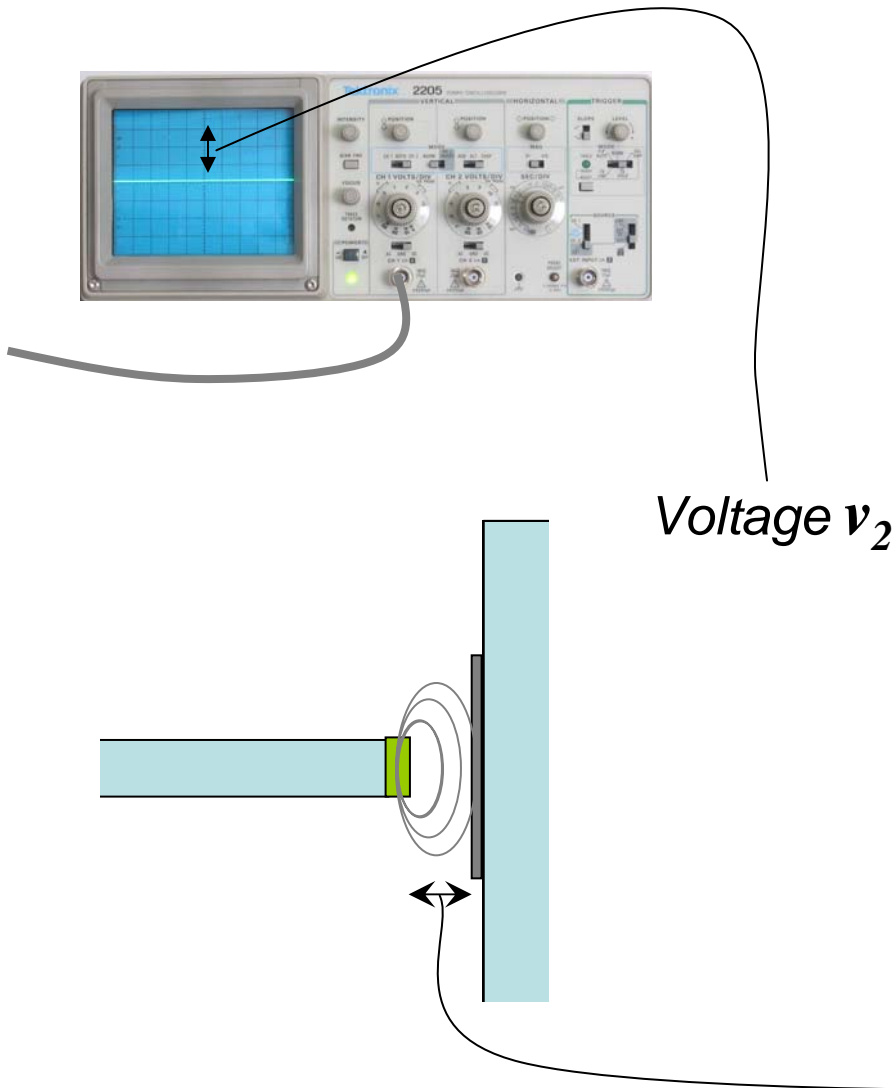


Proximeter probe

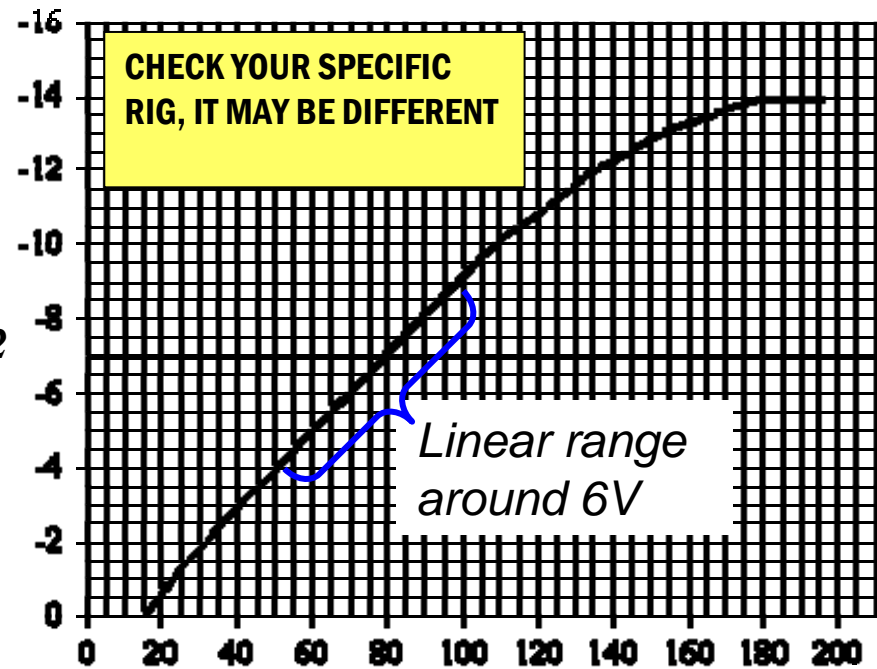
Position
measured with
radio waves,
without contact



Proximeter Calibration



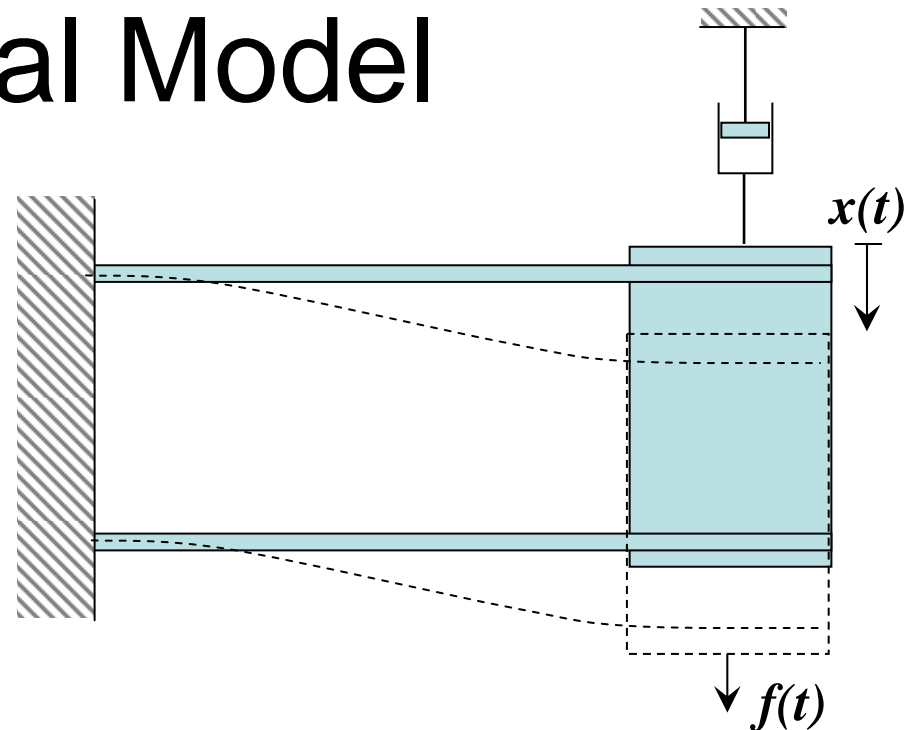
Slope in linear range ≈ 106 Volts/inch



*Position x in $1/1000''$
(from probe tip to target)*

Theoretical Model

- Mass can only move in one direction x , without rotation.
- Effective mass m includes rigid mass + a small fraction due to the beams
- Spring stiffness k derives from stiffness EI of beams and geometry
- Viscous damping b almost entirely from dashpot



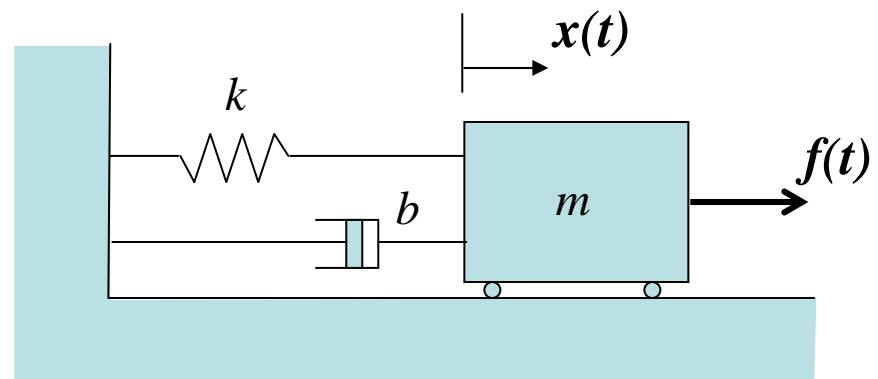
$$m\ddot{x} + b\dot{x} + kx = f(t)$$

Resisting force due to inertia of mass

Resisting force due to friction (damping).

Restoring force from springiness of beams.

Force applied



There are therefore 3 parameters (m, b, k) we need to measure to completely define the response function of this system

Solving the Governing Equation

$$m\ddot{x} + b\dot{x} + kx = f(t)$$

- Since governing equation is linear we can add solutions together to create new solutions
- Since we can add sine waves together to make any waveform, the sine wave solution can be thought of as entirely general
- So we assume a force and displacement of the form

Force amplitude

$$f(t) = f_m \cos(\omega t + \psi_f)$$

Phase of force

$$x(t) = x_m \cos(\omega t + \psi_x)$$

Displacement amplitude

Phase of displacement

Using Complex Exponentials

The math is much easier if we write these in terms of complex exponentials, e.g. :

$$x(t) = x_m \cos(\omega t + \psi_x) + \underbrace{jx_m \sin(\omega t + \psi_x)}_{\text{Fake imaginary part}} = x_m e^{j(\omega t + \psi_x)} = \underbrace{X(\omega)}_{\text{Complex number.}} e^{j\omega t}$$

Magnitude = x_m
Argument = ψ_x

And, likewise: $f(t) = f_m e^{j(\omega t + \psi_f)} = F(\omega) e^{j\omega t}$

So, $m\ddot{x} + b\dot{x} + kx = f(t)$

$$-m\omega^2 X e^{j\omega t} + jb\omega X e^{j\omega t} + kX e^{j\omega t} = F e^{j\omega t}$$

$$G = \frac{X}{F} = \frac{1}{-m\omega^2 + jb\omega + k}$$

$$-m\omega^2 X + jb\omega X + kX = F$$

Alternative Approach Using Laplace Transforms

$$m\ddot{x} + b\dot{x} + kx = f(t)$$

Take Laplace Transform : $(ms^2 + bs + k)X(s) = F(s)$

Re-arrange : $G(s) = \frac{X}{F} = \frac{1}{(ms^2 + bs + k)}$

Let $s \rightarrow j\omega$:

$$G = \frac{X}{F} = \frac{1}{-m\omega^2 + jb\omega + k}$$

Response Function Solution

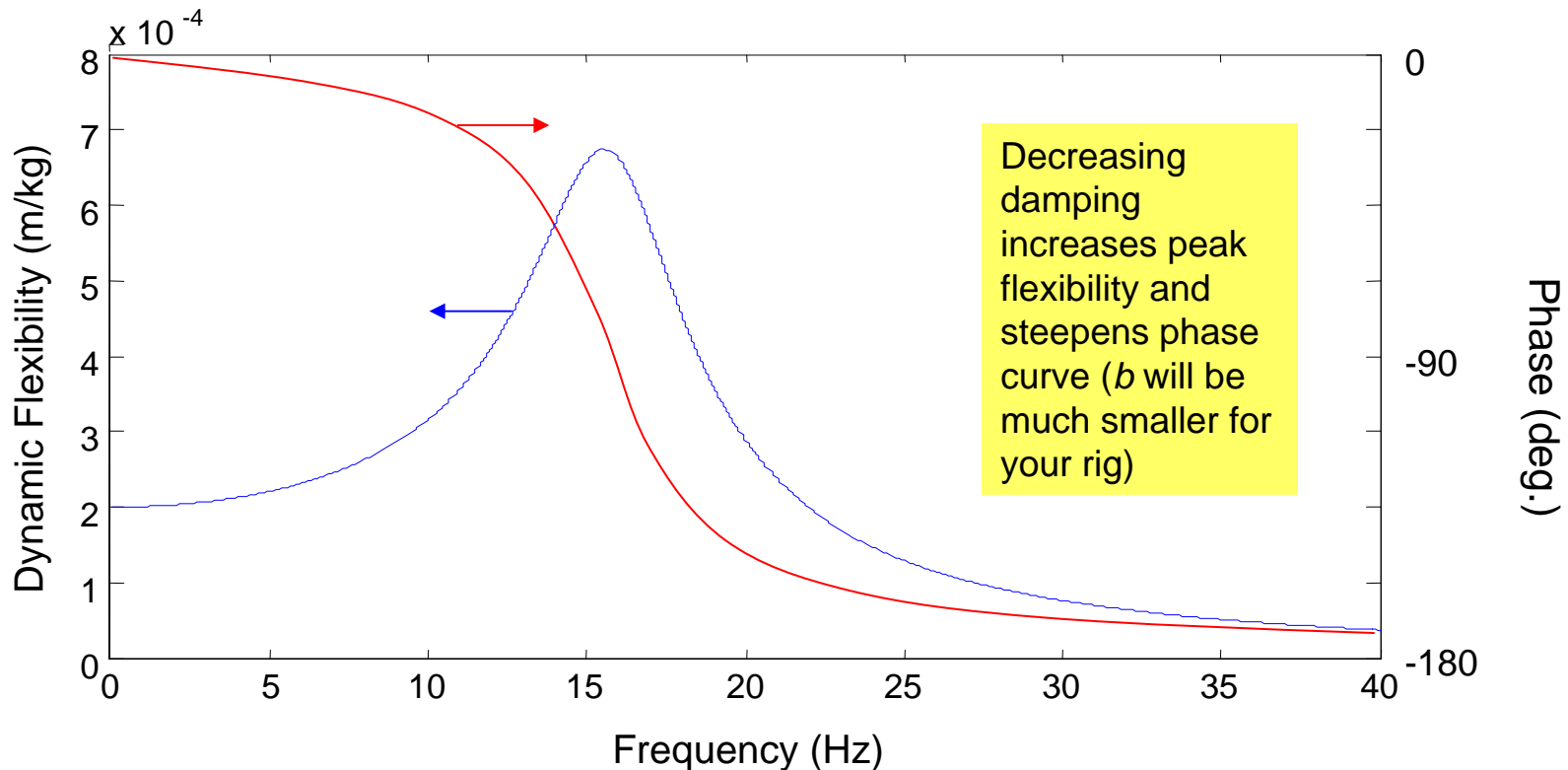
Dynamic flexibility

So, the ratio of the amplitude of the displacement to that of the force is

$$\left| \frac{X}{F} \right| = \frac{1}{\sqrt{(k - m\omega^2)^2 + b^2\omega^2}} = \frac{x_m}{f_m}$$

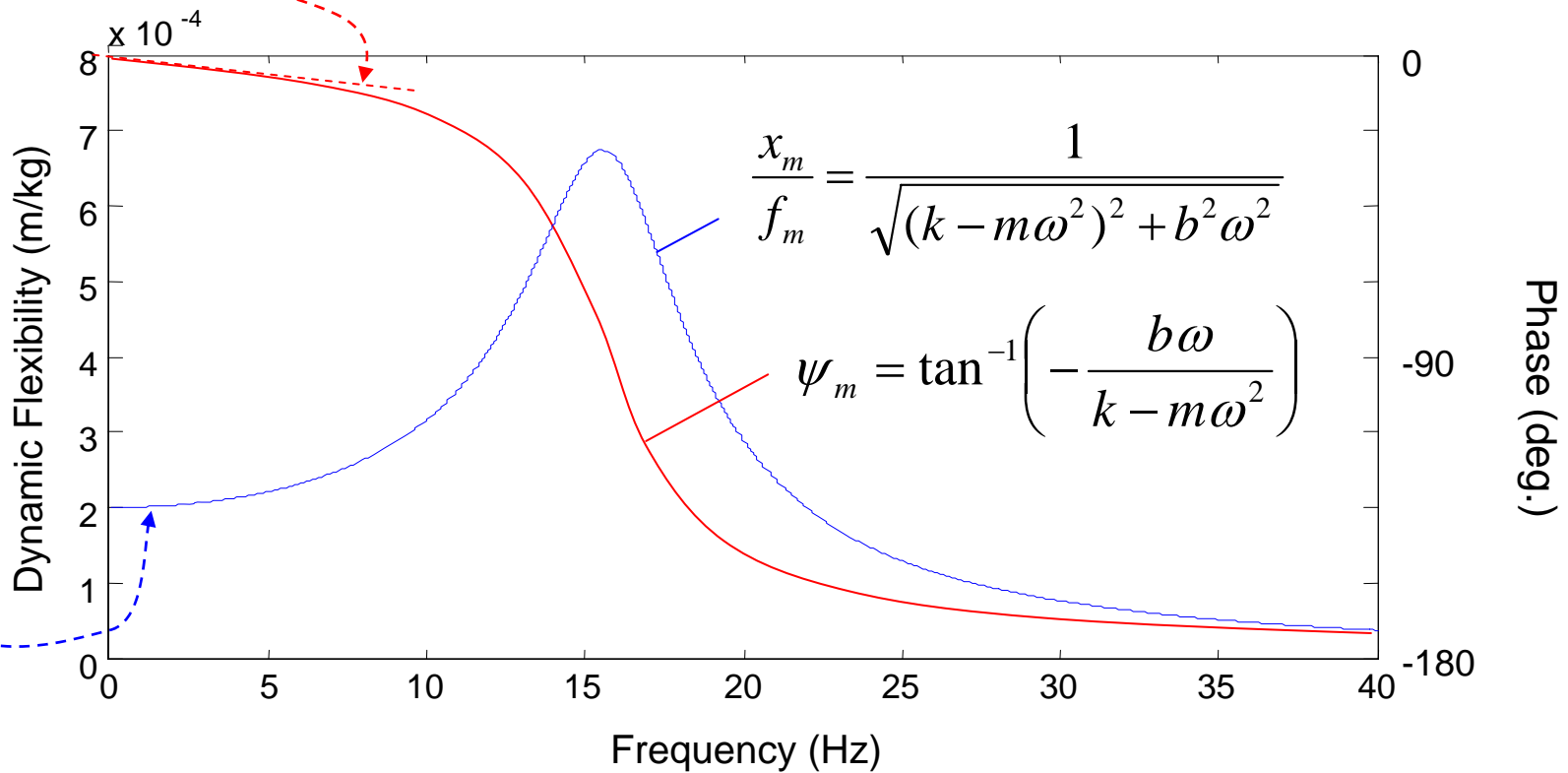
And the phase of the displacement relative to the phase of the force is

$$\arg\left(\frac{X}{F}\right) = \tan^{-1}\left(-\frac{b\omega}{k - m\omega^2}\right) = \psi_x - \psi_f \equiv \psi_m$$



Low frequency behavior

$$\left. \begin{array}{l} \text{(static flexibility)} \quad x_m / f_m = 1/k \\ \text{(in radians)} \quad \psi_m / \omega = -b/k \end{array} \right\} \text{ as } f \rightarrow 0$$



Low Frequency Ideas

$$\left. \begin{array}{l} x_m / f_m = 1/k \\ \psi_m / \omega = -b/k \end{array} \right\} \text{ as } f \rightarrow 0$$

- Can't use shaker at zero frequency (burn out)
- Measure parameters at several low frequencies and check they don't change with frequency (accurate measurements of small phase may be hard).
- Could place structure on side and measure beam displacement due to gravitational force mg . This is truly zero frequency, but need to get m from another measurement. Use add washers of known mass and measure displacement change.

Mass measured independently if the shaker calibration

$$x_m / g = m / k$$

$$x_{m1} / g = (m + m_w) / k$$

Knowing the 2 displacements and the mass of the washers m_w is enough to infer k and m

Peak frequency behavior

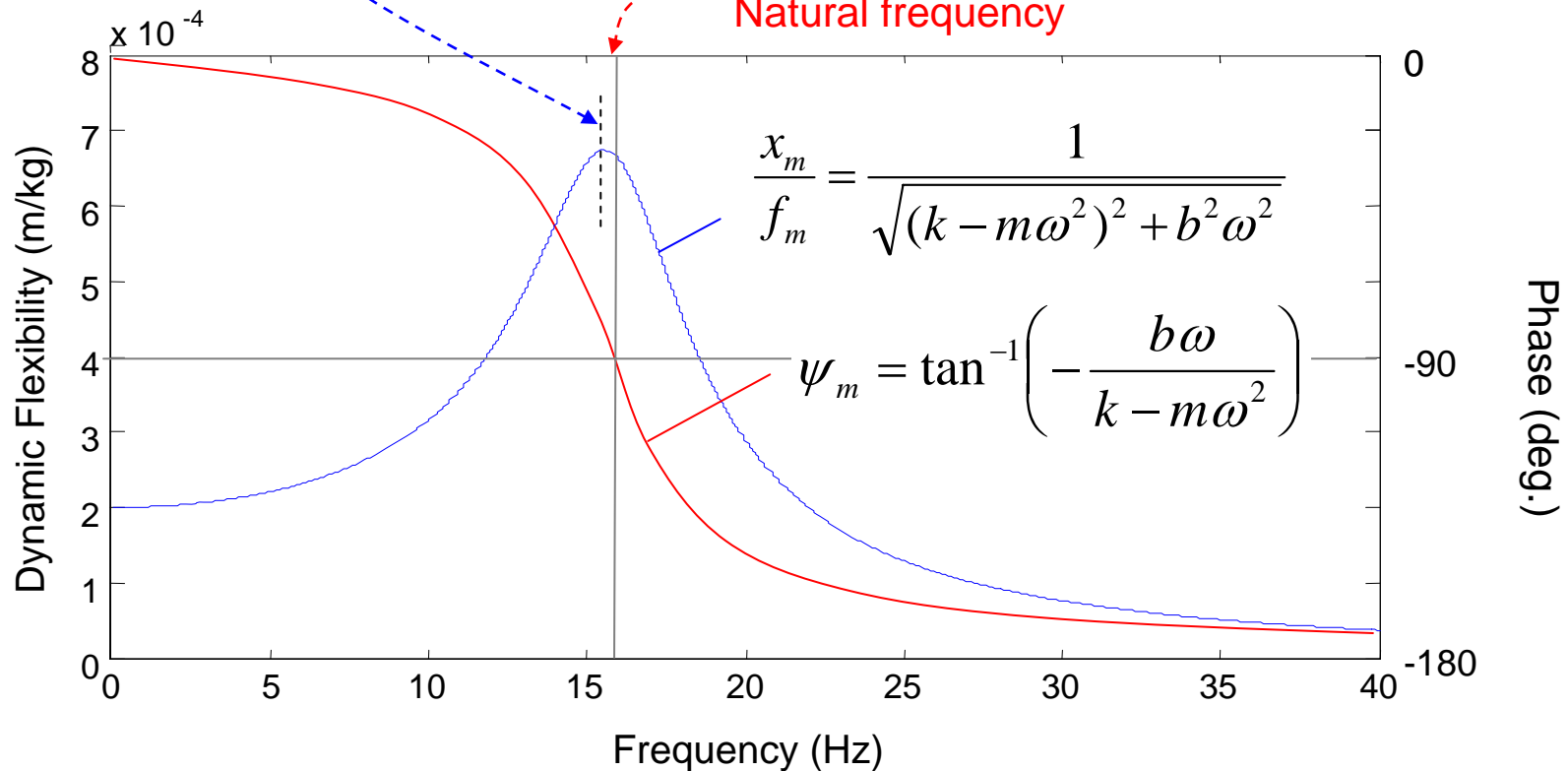
$$\omega = \sqrt{\omega_n^2 - b^2 / m^2} \equiv \omega_r$$

Resonant or damped natural frequency

$$\omega = \sqrt{k / m} \equiv \omega_n$$

$$\rightarrow \begin{cases} \psi_m = -90^\circ \\ x_m / f_m = 1 / (b \omega_n) \end{cases}$$

Natural frequency



Peak Frequency Ideas

$$\omega = \sqrt{\omega_n^2 - b^2 / m^2} \equiv \omega_r$$

$$\omega = \sqrt{k / m} \equiv \omega_n \rightarrow \begin{cases} \psi_m = -90^\circ \\ x_m / f_m = 1 / (b \omega_n) \end{cases}$$

- Lissajous figure (on oscilloscope) is by far the best way to determine when the phase is -90 degrees
- Could measure change in natural frequency when washers are added to separately determine m and k
- Resonant frequency and dynamic flexibility at the natural frequency may drift since you are measuring at the (very sharp) peak of the response

Mass measured independently if the shaker calibration

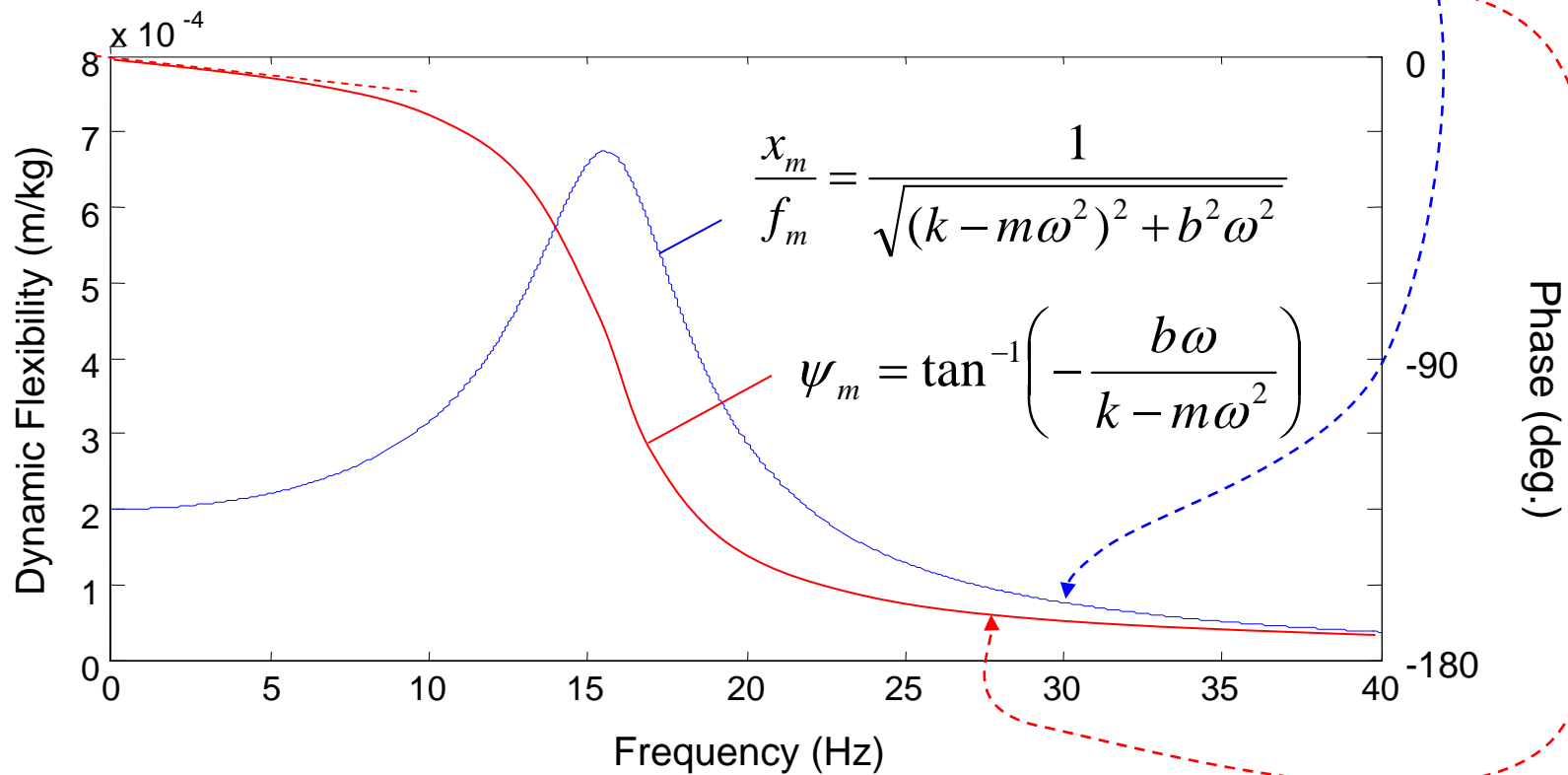
$$\omega_n = \sqrt{k / m}$$

$$\omega_{n1} = \sqrt{k / (m + m_w)}$$

Knowing the 2 natural frequencies and the mass of the washers m_w is enough to infer k and m

High frequency behavior

As $f \rightarrow \infty$ $\left\{ \begin{array}{l} x_m / f_m \approx 1/(m\omega^2) \\ (\psi_m + \pi)\omega \approx b/m \end{array} \right.$ (in radians)

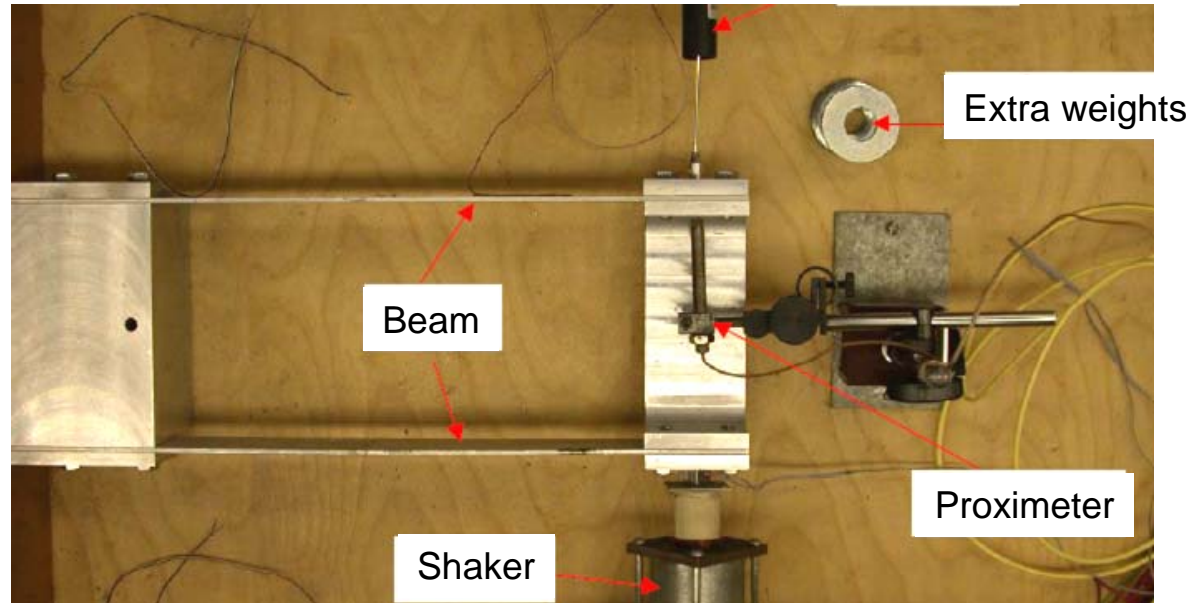


High frequency ideas

$$\text{As } f \rightarrow \infty \quad \left\{ \begin{array}{l} x_m / f_m \approx 1/(m\omega^2) \\ (\psi_m + \pi)\omega \approx b/m \end{array} \right.$$

- Don't know how high is high.
 - Use manual values of parameters to estimate how high frequency needs to be (could take damping from sample logbook)
 - Measure at several high frequencies and make sure results don't change.
- Could use washers again...

Other ideas



- Could just measure entire response function. (Less efficient, but checks theoretical model).
- Check system linearity by checking independence of dynamic flexibility and phase with force amplitude. Is there a limit?
- Check shaker calibration by measuring mass independent of the shaker calibration, and then compare with mass measured using that calibration
- Check proximeter calibration using caliper.
- Add and large rigid cardboard sail to mass. For small vibration amplitudes sail should produce some added mass. For large amplitudes it may add (nonlinear?) damping. Or not? Measure the added mass? Can cardboard sail produce significant force?

Preparing for lab

- Need to come up with your own approach using these schemes and ideas, or any others you can devise.
- Few of the schemes have been tried, so some may not be practical.
- Can use ballpark estimates of parameters (based on the manual or sample logbook, say) and uncertainty analysis, to weed out truly impractical approaches *before* the lab.