# This Week In Lab…

- Introductory Lab Period.
  - Instructions in course manual linked from course homepage www.aoe.vt.edu/aoe3054 (experiments overview section)
  - Also checkout appendix 1 in course manual on electronic logbooks before coming to lab
  - Bring to lab
    - A printed copy of the introductory lab instructions.
    - Your laptop, containing a copy of the logbook template.
- Downloadable Lab Manual

# Next Class

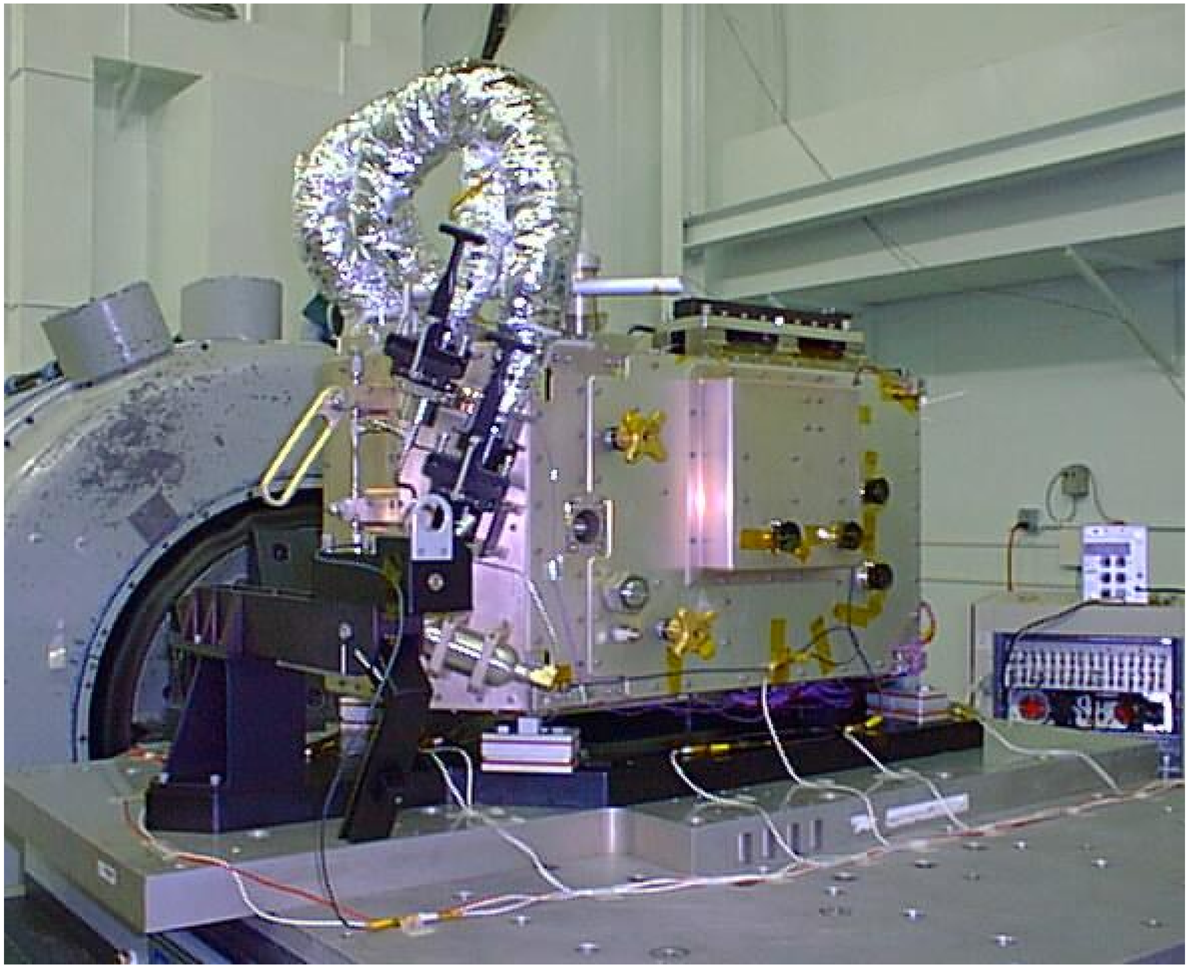
- Monday 1/29
- Writing a logbook and your lab report

# Estimating Experimental Error

(UNCERTAINTY)

- Relevant to everything from now on (required part of every experiment)
- Also covered in
  - ‘Basic Concepts in Experiments’ section of the course manual
  - ‘Estimating Experimental Error’ online class
- Homework on experimental error
  - available on course website (Monday 1/22)
  - Due this Friday 1/26 by 5pm in the ‘In’ box outside my office door (224 E Randolph)

# Testing of Vibration Tolerance of Launch Hardware



# Uncertainty

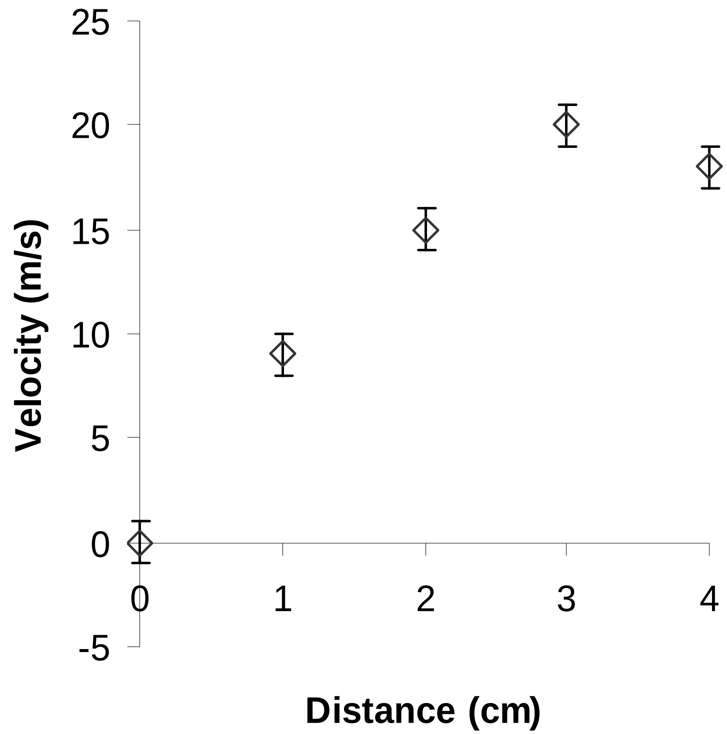
- Concept of an uncertainty interval
- Determining uncertainty
  - in a primary measurement
  - in a result
- Examples
- Using a computer (e.g. Excel) to estimate uncertainty

# Uncertainty Interval

- Definition: *"A band around a measurement chosen so that there is a 95% probability of the true value lying within it"*

The 95% condition is sometimes referred to as "*20:1 odds*". Depending on how critical the information being handled is, higher odds are sometimes specified, e.g. 100:1 or 99%

- Representation:
  - *…the flow speed was measured to be 20±1 m/s*
  - *= 1m/s*
  - *…there is a 5% uncertainty in U*
  - *=0.05*
  - *Error bars...*



# Determining Uncertainty

## *in a <u>Primary Measurement</u>*

• Primary measurement: "                                                                                                    "

E.g. Distance from a ruler, temperature from a thermometer (***<u>not</u>*** velocity from a Pitot probe)

• Make an <u>informed</u> guess based on :

- *digital resolution, smallest division on scales*
- *manufacturers information, calibration info.*
- *Repeated measurements of the same qty:*

Why?

- *comparison between independent measurements of the same quantity*
- *other factors*

    validity of measurement scheme
    operating outside the design/calibration range
    environmental conditions
    blunders

- *experience*

If the fluctuations in the measurement are distributed normally (i.e. they have a Gaussian PDF) then <u>95%</u> of samples will fall within $\pm 2\sigma$ of the mean.

# Determining Uncertainty

## *in a* ***Primary Measurement***

*Example:*


*Uncertainty estimate = ??*

# Determining Uncertainty
## *in a <u>Primary Measurement</u>*

• What you consider an “uncertainty” may not always be an error in the traditional sense. It depends on what you’re hoping to determine from the primary measurement.

*E.g.Suppose you aim to determine the mean value of a fluctuating quantity (like the position of a wing spar, vibrating in flight) by taking a series of samples. Even if there is no measurement error in the samples themselves (i.e. they give you the true instantaneous value), each separate sample is still a fairly poor estimate of the mean. You would probably take the uncertainty in each sample, to be…*

# Determining Uncertainty
## *in a Derived Result*

• Consider an experimental result *R* that is determined from one or more primary measurements *a*, *b*, *c*… by means of a function *f( )*, i.e. *any* derived result.

$$R = f(a, b, c, ...)$$

• The uncertainty in *R* due to each of the primary measurements may be determined by multiplying the uncertainty in that measurement by the sensitivity of *R* to that measurement. Since each of the primary measurements will be independent, the total uncertainty in *R* is thus,

# Example 1: Uncertainty in Velocity from a Pitot-Static

*Determine the uncertainty in velocity measurements made with the Pitot-static probe and digital manometer used in experiment 3*


*__Readings__*

$p_o$-$p$ = 0.22kPa = 220Pa

$p_{atm}$ = 941.1mBar = 94110Pa

$T_{atm}$= 17.0°C = 290.0K

*__Equation__*

$$U = \sqrt{\frac{2(p_o - p)}{(p_{atm} / R / T_{atm})}}$$

*__Result__*

$$U = \sqrt{\frac{2 \times 220}{(94110/287/290.0)}} = \underline{\underline{19.72 m/s}}$$

# *Step 1: Primary Uncertainties*

Manometer ?


Barometer ?


Thermometer ?


| | |
|---|---|
| $p$-$p_o$ | $\delta(p\text{-}p_o)$ = 0.02kPa = 20Pa (eyeball) |
| $p_{atm}$ | $\delta(p_{atm})$ = 0.5mBar = 50Pa (just doesn't feel like 0.1) |
| $T_{atm}$ | $\delta(T_{atm})$ = 0.5$^o$C (its RadioShack after all) |

# Step 2: Partial Derivatives

$$U = \sqrt{\frac{2(p_o - p)}{(p_{atm} / R / T_{atm})}}$$

$$\frac{\partial U}{\partial (p_o - p)} = \tfrac{1}{2}\left(\frac{2(p_o - p)RT_{atm}}{p_{atm}}\right)^{-1/2} \frac{2RT_{atm}}{p_{atm}} = \tfrac{1}{2}\frac{U}{p_o - p} = \tfrac{1}{2}\frac{19.72}{220} = \underline{0.0448}$$

$$\frac{\partial U}{\partial T_{atm}} = \tfrac{1}{2}\frac{U}{T_{atm}} = \tfrac{1}{2}\frac{19.72}{290} = \underline{0.0340}$$

$$\frac{\partial U}{\partial p_{atm}} = -\tfrac{1}{2}\frac{U}{p_{atm}} = -\tfrac{1}{2}\frac{19.72}{94110} = \underline{-0.000105}$$

# Step 3: Combine Uncertainties

$\delta(p-p_o) = 20\text{Pa}$
$\delta(p_{atm}) = 50\text{Pa}$
$\delta(T_{atm}) = 0.5^{o}\text{C}$

$$\frac{\partial U}{\partial(p_o - p)} = \underline{0.0448}$$

$$\frac{\partial U}{\partial T_{atm}} = \underline{0.0340}$$

$$\frac{\partial U}{\partial p_{atm}} = \underline{-0.000105}$$

$$\delta(U) = \sqrt{\left[\frac{\partial U}{\partial(p_o - p)}\delta(p_o - p)\right]^2 + \left[\frac{\partial U}{\partial T_{atm}}\delta(T_{atm})\right]^2 + \left[\frac{\partial U}{\partial p_{atm}}\delta(p_{atm})\right]^2}$$

$$= \sqrt{[0.0448 \times 20]^2 + [0.0340 \times 0.5]^2 + [-0.000105 \times 50]^2}$$

$$= \sqrt{0.803 + 0.00029 + 0.000027} = \underline{0.80 m/s}$$

# *Step 4. Interpretation*

$$\delta(U)=\sqrt{\left[\frac{\partial U}{\partial(p_o-p)}\delta(p_o-p)\right]^2+\left[\frac{\partial U}{\partial T_{atm}}\delta(T_{atm})\right]^2+\left[\frac{\partial U}{\partial p_{atm}}\delta(p_{atm})\right]^2}$$

$$=\sqrt{0.803+0.00029+0.000027}=\underline{0.80\,m/s}$$

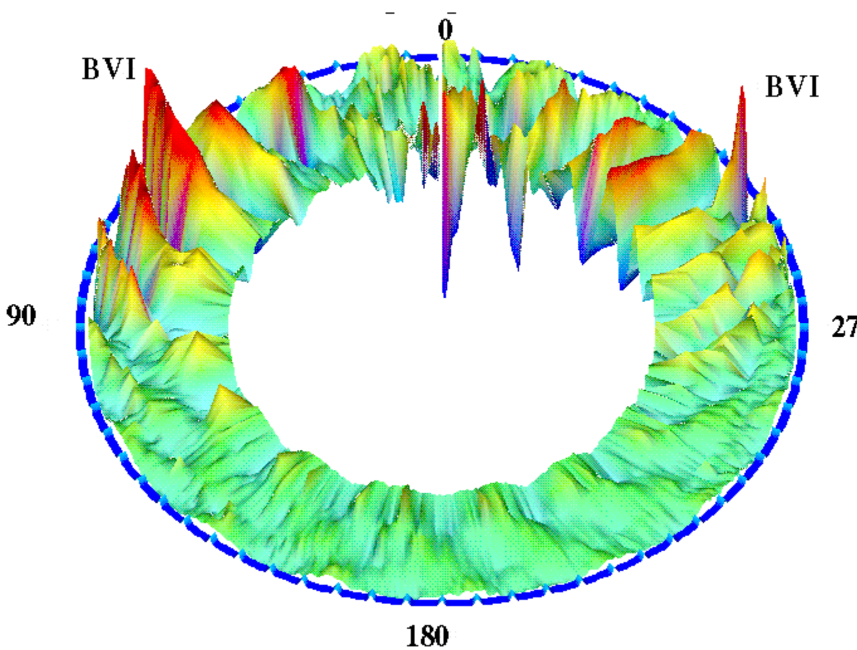
• Result gives information about the relative importance of different sources of error. Here, the uncertainty in velocity is dominated by the . . The coarseness of the temperature measurement is of negligible consequence. This type of information is very valuable, since it tells you where your weakest link is.

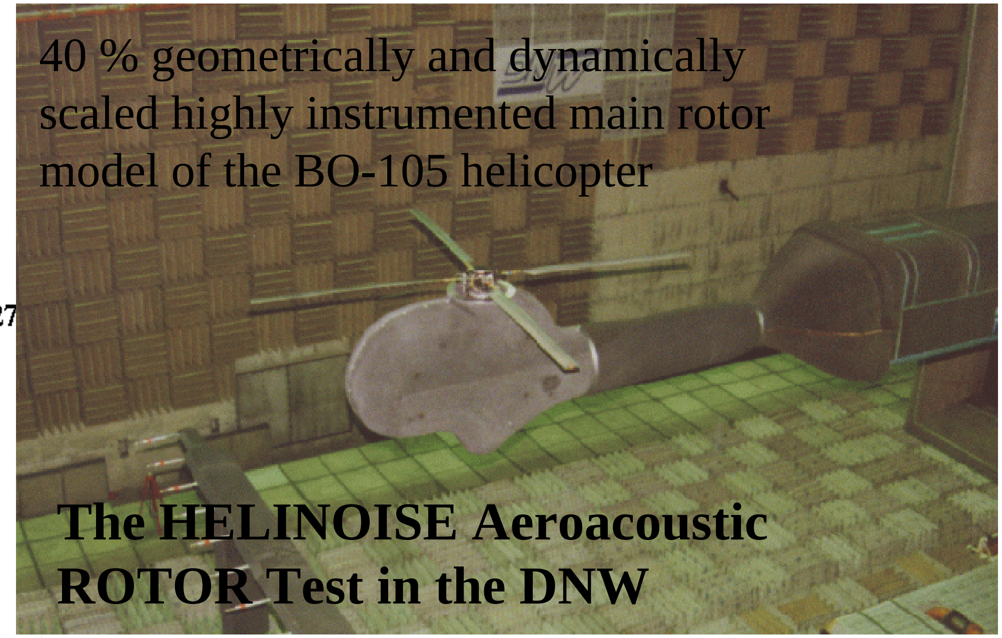
• Since it is (hopefully) a small fraction of the measurement, the accuracy of the uncertainty estimate itself is (e.g. it wouldn't change any conclusions here if the uncertainty were 0.6m/s). This is why we can tolerate informed guesswork in determining the uncertainty in the primary measurements.

# Example 2: Uncertainty in a Statistical Result

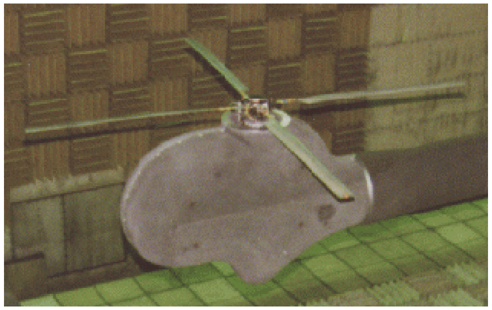
Problem: *The deflection 'y' of a helicopter rotor blade is measured, once per revolution, over N rotor revolutions. The data is used to estimate the mean and standard deviation σ of the deflection. Assuming the error in the individual deflection measurements is negligible, determine the uncertainty in the mean value.*

*Mean* $$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$



# *Step 1: Primary Uncertainties*



$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{y_1 + y_2 + \ldots y_N}{N}$$

Primary measurements: The $N$ samples of $y_i$

# *Step 2: Partial Derivatives*

# *Step 3: Combine Uncertainties*

$$\frac{\partial \bar{y}}{\partial y_i} = \frac{1}{N}$$

$$\delta(y_i) = 2\sigma_y$$

$$\delta(\bar{y}) = \sqrt{\left(\frac{\partial \bar{y}}{\partial y_1}\delta(y_1)\right)^2 + \left(\frac{\partial \bar{y}}{\partial y_2}\delta(y_2)\right)^2 + \ldots \left(\frac{\partial \bar{y}}{\partial y_N}\delta(y_N)\right)^2}$$

$$= \sqrt{\sum_{i=1}^{N}\left(\frac{\partial \bar{y}}{\partial y_i}\delta(y_i)\right)^2} = \sqrt{\sum_{i=1}^{N}\left(\frac{1}{N}2\sigma_y\right)^2}$$

$$= \sqrt{N\left(\frac{1}{N}2\sigma_y\right)^2} =$$

# *Step 4: Interpretation*

- The uncertainty in the mean value goes as the inverse square root of the number of samples.
- Most statistical results behave in this way.

# Determining Uncertainty

*using a computer*

Suppose we write a computer program to determine our result *R* from the primary measurements *a*, *b*, *c*… We have:

$$a\ , b\ , c\ ,... \quad \rightarrow \quad \text{Program} \qquad \rightarrow \quad R$$

or, as before:

The uncertainty in *R* may therefore be calculated using the same expression:

$$\delta(R) = \sqrt{\left[\frac{\partial R}{\partial a}\delta(a)\right]^2 + \left[\frac{\partial R}{\partial b}\delta(b)\right]^2 + \left[\frac{\partial R}{\partial c}\delta(c)\right]^2 + ...}$$

The only problem is determining the partial derivatives.

# Determining Uncertainty

*using a computer*

This can be done by perturbing, in turn, each of the inputs to our program by its uncertainty, and recording the change in output that results. (The modified program that does this is usually called a *jitter* program).

$$a + \delta a, b, c, \ldots \rightarrow \text{Program} \rightarrow R + \frac{\partial R}{\partial a}\delta a \rightarrow \frac{\partial R}{\partial a}\delta a$$

$$a, b + \delta b, c, \ldots \rightarrow \text{Program} \rightarrow R + \frac{\partial R}{\partial b}\delta b \rightarrow \frac{\partial R}{\partial b}\delta b$$

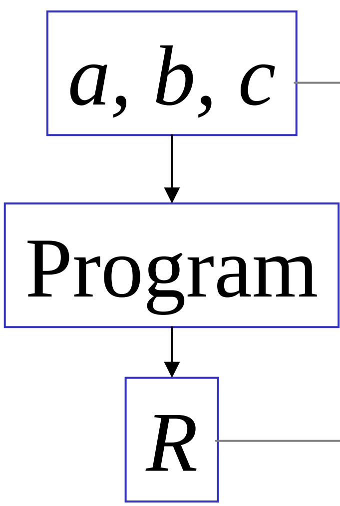
$$a, b, c + \delta c, \ldots \rightarrow \text{Program} \rightarrow R + \frac{\partial R}{\partial c}\delta c \rightarrow \frac{\partial R}{\partial c}\delta c$$

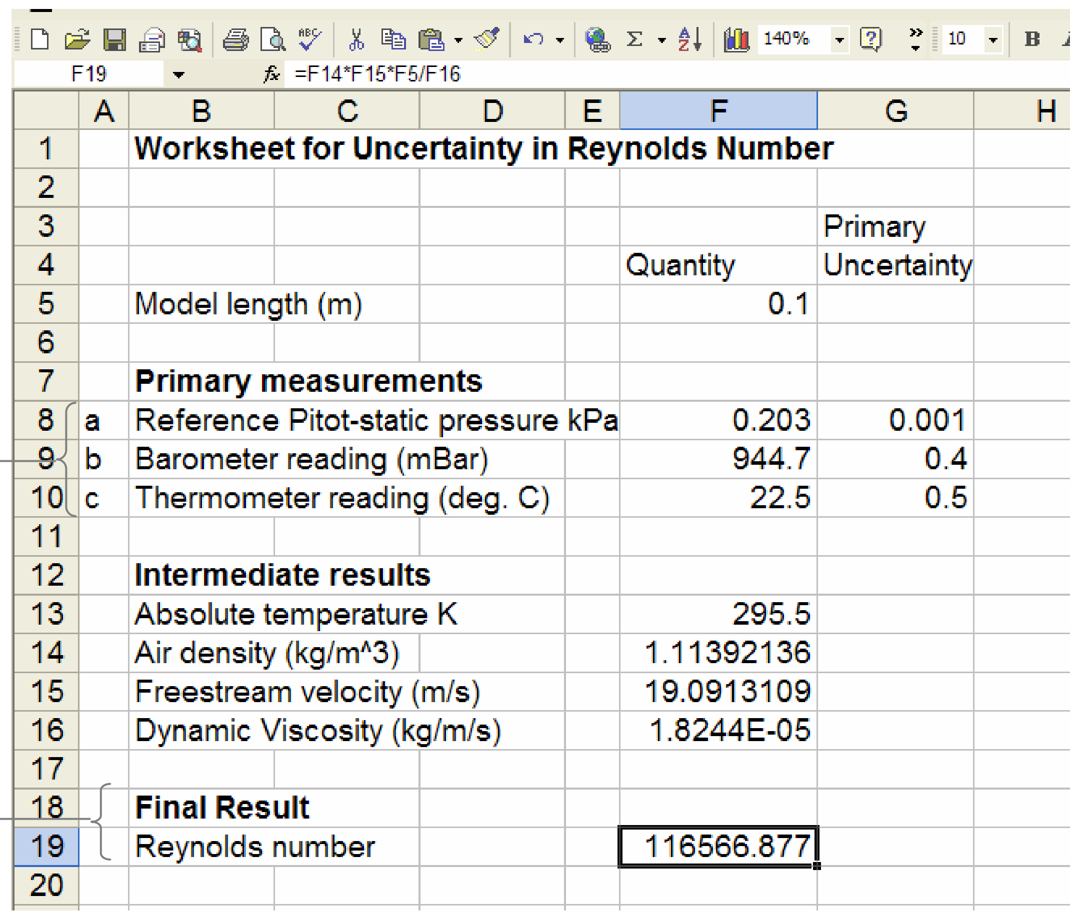
The δ's musn't be too large a fraction of the measured quantity – up to 20% is probably OK. (If δ's are bigger, then just use a smaller fraction and multiply the result accordingly)

# Example

*Computing uncertainty in measured Reynolds number*

$$R = f(a, b, c, \ldots)$$




| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | **Worksheet for Uncertainty in Reynolds Number** | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | Primary | |
| 4 | | | | | | Quantity | Uncertainty | |
| 5 | | Model length (m) | | | | 0.1 | | |
| 6 | | | | | | | | |
| 7 | | **Primary measurements** | | | | | | |
| 8 | a | Reference Pitot-static pressure kPa | | | | 0.203 | 0.001 | |
| 9 | b | Barometer reading (mBar) | | | | 944.7 | 0.4 | |
| 10 | c | Thermometer reading (deg. C) | | | | 22.5 | 0.5 | |
| 11 | | | | | | | | |
| 12 | | **Intermediate results** | | | | | | |
| 13 | | Absolute temperature K | | | | 295.5 | | |
| 14 | | Air density (kg/m^3) | | | | 1.11392136 | | |
| 15 | | Freestream velocity (m/s) | | | | 19.0913109 | | |
| 16 | | Dynamic Viscosity (kg/m/s) | | | | 1.8244E-05 | | |
| 17 | | | | | | | | |
| 18 | | **Final Result** | | | | | | |
| 19 | | Reynolds number | | | | 116566.877 | | |
| 20 | | | | | | | | |

# Example

*Copy calculation into 3 spare columns*

J42

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | **Worksheet for Uncertainty in Reynolds Number** | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | Primary | | | |
| 4 | | | | | | Quantity | Uncertainty | | | |
| 5 | | Model length (m) | | | | 0.1 | | | | |
| 6 | | | | | | | | | | |
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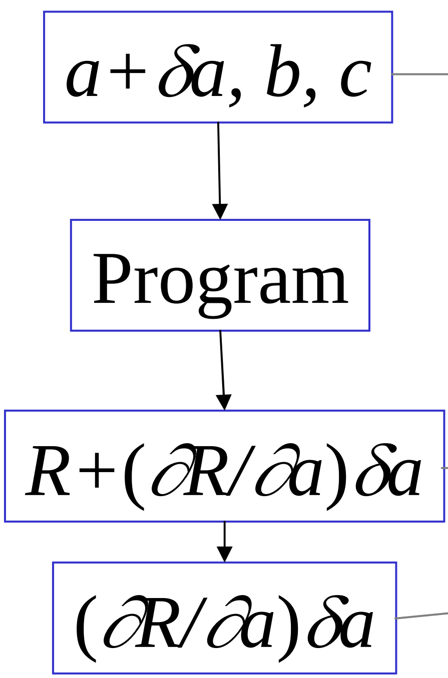
| 10 | c | Thermometer reading (deg. C) | | | | 22.5 | 0.5 | | | |
| 11 | | | | | | | | | | |
| 12 | | **Intermediate results** | | | | | | | | |
| 13 | | Absolute temperature K | | | | 295.5 | | | | |
| 14 | | Air density (kg/m^3) | | | | 1.113921 | | | | |
| 15 | | Freestream velocity (m/s) | | | | 19.09131 | | | | |
| 16 | | Dynamic Viscosity (kg/m/s) | | | | 1.82E-05 | | | | |
| 17 | | | | | | | | | | |
| 18 | | **Final Result** | | | | | | | | |
| 19 | | Reynolds number | | | | 116566.9 | | | | |

# Example




| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | **Worksheet for Uncertainty in Reynolds Number** | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | Primary | | | |
| 4 | | | | | | Quantity | Uncertainty | a+da,b,c | | |
| 5 | | Model length (m) | | | | 0.1 | | 0.1 | 0.1 | 0.1 |
| 6 | | | | | | | | | | |
| 7 | | **Primary measurements** | | | | | | | | |
| 8 | a | Reference Pitot-static pressure kPa | | | | 0.203 | 0.001 | 0.204 | 0.203 | 0.203 |
| 9 | b | Barometer reading (mBar) | | | | 944.7 | 0.4 | 944.7 | 944.7 | 944.7 |
| 10 | c | Thermometer reading (deg. C) | | | | 22.5 | 0.5 | 22.5 | 22.5 | 22.5 |
| 11 | | | | | | | | | | |
| 12 | | **Intermediate results** | | | | | | | | |
| 13 | | Absolute temperature K | | | | 295.5 | | 295.5 | 295.5 | 295.5 |
| 14 | | Air density (kg/m^3) | | | | 1.113921 | | 1.11392 | 1.11392 | 1.11392 |
| 15 | | Freestream velocity (m/s) | | | | 19.09131 | | 19.1383 | 19.0913 | 19.0913 |
| 16 | | Dynamic Viscosity (kg/m/s) | | | | 1.82E-05 | | 1.8E-05 | 1.8E-05 | 1.8E-05 |
| 17 | | | | | | | | | | |
| 18 | | **Final Result** | | | | | | | | |
| 19 | | Reynolds number | | | | 116566.9 | | 116854 | 116567 | 116567 |
| 20 | | | | | | | Change | 286.758 | | |
| 21 | | | | | | | | | | |
| 22 | | | | | | | | | | |

# Example

F21 $f_x$ =SQRT(H20^2+I20^2+J20^2)

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | **Worksheet for Uncertainty in Reynolds Number** | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | Primary | | | |
| 4 | | | | | | Quantity | Uncertainty | a+da,b,c | a,b+db,c | a,b,c+dc |
| 5 | | Model length (m) | | | | 0.1 | | 0.1 | 0.1 | 0.1 |
| 6 | | | | | | | | | | |
| 7 | | **Primary measurements** | | | | | | | | |
| 8 | a | Reference Pitot-static pressure kPa | | | | 0.203 | 0.001 | 0.204 | 0.203 | 0.203 |
| 9 | b | Barometer reading (mBar) | | | | 944.7 | 0.4 | 944.7 | 945.1 | 944.7 |
| 10 | c | Thermometer reading (deg. C) | | | | 22.5 | 0.5 | 22.5 | 22.5 | 23 |
| 11 | | | | | | | | | | |
| 12 | | **Intermediate results** | | | | | | | | |
| 13 | | Absolute temperature K | | | | 295.5 | | 295.5 | 295.5 | 296 |
| 14 | | Air density (kg/m^3) | | | | 1.113921 | | 1.11392 | 1.11439 | 1.11204 |
| 15 | | Freestream velocity (m/s) | | | | 19.09131 | | 19.1383 | 19.0873 | 19.1075 |
| 16 | | Dynamic Viscosity (kg/m/s) | | | | 1.82E-05 | | 1.8E-05 | 1.8E-05 | 1.8E-05 |
| 17 | | | | | | | | | | |
| 18 | | **Final Result** | | | | | | | | |
| 19 | | Reynolds number | | | | 116566.9 | | 116854 | 116592 | 116317 |
| 20 | | | | | | | Change | 286.758 | 24.6755 | -250.368 |
| 21 | | Uncertainty in final result | | | | 381.4752 | | | | |
| 22 | | | | | | | | | | |

$(\partial R/\partial a)\delta a$

$(\partial R/\partial b)\delta b$

$(\partial R/\partial c)\delta c$

$$\delta(R) = \sqrt{\left[\frac{\partial R}{\partial a}\delta(a)\right]^2 + \left[\frac{\partial R}{\partial b}\delta(b)\right]^2 + \left[\frac{\partial R}{\partial c}\delta(c)\right]^2}.$$