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DYNAMICS AND CONTROL PROBLEMS IN THE DEFLECTION OF NEAR-EARTH OBJECTS

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In this report, we investigate the problem of mitigating the threat from an earth-impacting object. We provide a brief historical perspective of the subject, then focus on the problem of deflecting an object, with a unique new viewpoint leading to the development of a fundamental definition of the problem. Within the context of this fundamental deflection problem, we define several typical scenarios, each of which leads to classes of potential optimization problems to be solved. Many deflection strategies have been proposed in the literature, and we describe several of these, with emphasis on a classification that complements the fundamental definition.

INTRODUCTION

It is now widely accepted in the scientific community that the threat of an asteroid or comet colliding with Earth, although small, is very real. In this report, we investigate the problem of mitigating the threat from an asteroid or a comet. Evidently, this dynamics and control problem has not been clearly formulated, and consequently the interrelationships between the space system, the deflection system, and the NEO have not been fully explored. Our formulation of the problem shows the key relationship between the modeling, simulation, and optimization of a space mission for deflecting a NEO.

The problems associated with protecting Earth from the potentially devastating impact of an interplanetary object have received significant attention recently. The extent of interest in this class of problems is seen in the thousands of pages that have been written on the subject, especially as collected in Refs. [7], [4], and [10]. In this report, we are concerned with the dynamics and control issues arising in the subproblem of preventing a predicted collision of a Near-Earth Object (NEO) with Earth. We define a NEO to be a celestial object whose trajectory passes close to the Earth's orbit. This definition includes asteroids and comets.

Other significant subproblems related to NEOs include detection, orbit prediction, and determination of physical properties. Detection is especially important, as only about 200 of the estimated 100,000+ Earth-crossing-asteroids (ECAs) with diameters greater than 0.1 km have been discovered [28], and only about 30 of the estimated 1000+ Earth-crossing short-period comets have been discovered [33]. Similarly, improved orbit prediction is essential if collisions are to be predicted early enough to plan effective prevention missions [42]. Determination of NEO physical properties is important because deflection strategies must consider the probability of fragmentation of an object. Fragmentation of a large NEO into smaller pieces could increase the hazard to Earth [31].

We begin with a brief historical perspective of the NEO problem. We then focus on the problem of deflecting a NEO, leading to a fundamental definition of the problem. Within the

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context of this fundamental deflection problem, we define several typical scenarios, each of which leads to classes of optimization problems to be solved. Many deflection strategies have been proposed in the literature, and we describe several of these, with emphasis on a classification that complements our fundamental definition. This paper is an abbreviated version of a technical report [14]. Some of the issues raised here are examined further in Ref. [25].

HISTORICAL PERSPECTIVE

Because of the public attention paid to the recent collision of a comet with the planet Jupiter (Shoemaker-Levy 9, July 1994, Ref. [22]), there could exist misconception that serious interest in defending Earth against NEO impact originated with that incident. This hazard was discussed by scientists as early as the 1940s (Ref. [21]), and as noted in Ref. [35], a systems engineering study at M.I.T. addressed the problem in 1967. However, today's widespread interest in protection against NEO impacts resulted from the announcement by Alvarez *et al.* in 1980 of substantial evidence that an asteroid collision with Earth was the probable cause of the extinction of the dinosaurs [3]. Most of the papers in the book edited by Gehrels [10] provided brief reviews of the history of this subject. In addition, the recent article by Gehrels [11] gave an excellent overview. What follows is a brief synopsis of material taken from these and other sources.

Studies, Workshops, and Congressional Direction

Following the 1980 report of Alvarez *et al.* [*3*], NASA and JPL cosponsored a 1981 workshop with the goal of determining the probability and risk of NEO impacts with Earth. Participants concluded that detection, orbit determination and prediction, deflection, and exploration are four areas that merit further study. In 1991, the U. S. Congress directed NASA to conduct two workshops on detection and interception. The detection workshop led to the Spaceguard Survey proposal [21]. The interception workshop results were documented in a *Proceedings* [7] and article [8]. Two types of missions were recommended for further study: *exploratory missions* to investigate NEOs and determine their physical properties, and *deflection missions* to divert or destroy threatening NEOs. Participants also concluded that existing and developmental DoD systems should play a significant role in detecting and defending against threatening NEOs.

Clementine I and II

One DoD space mission was especially relevant. The 1994 Clementine mission integrated several elements of DoD/BMDO technology into a system which provided significant scientific data on the lunar terrain and surface composition. Clementine was also intended to perform a fast flyby of the near-Earth asteroid Geographas, but an onboard computer error precluded this phase of the mission. Recommendations for use of similar technologies in NEO and interplanetary exploration were given by Nozette *et al*. [24]. Clementine II is presently in the development phase, with expected launch in 1998 [5].

Other Asteroid And Comet Missions

Several science-oriented space missions to asteroids and comets have been proposed by various national and international space agencies. Cheng *et al.* classified four basic types of missions: *flybys*, *rendezvous*, *landers*, and *sample return* missions [9]. The first three are directly applicable to deflection, whereas the fourth is more closely tied to exploration.

In a *flyby*, the spacecraft passes close to the object but at relatively high speed (*e.g.*, ~10^3 km at ~15 km/s). For science missions this means a short time is available to collect data, whereas for a deflection mission, the large relative velocity may be exploited to enhance the deflection.

For example, an *intercept* mission would be a flyby with relative distance nearly zero. In a *rendezvous* mission, the spacecraft matches position and velocity with the NEO, perhaps establishing an orbit around the object. This has significant benefit for science missions, as well as for deflection missions, since an orbiting spacecraft could determine more about the NEO's physical properties before applying an appropriate deflection strategy.

In a *lander* mission, the spacecraft or a deployable sub-spacecraft lands on the surface of the NEO. For science missions, the lander would normally include scientific instruments and a communication system. For deflection missions, the lander might include, for example, a nuclear weapon, or a robotic system to apply a non-nuclear deflection strategy. In a *sample return* mission, the lander would be able to collect NEO materials and leave the NEO for a return trip to Earth. This type of mission does not appear to be relevant to the deflection problem; however, the possibility does exist of returning an entire NEO to near-Earth space for mining or other science projects. Certainly if this could be accomplished economically for a threatening NEO, it would solve the deflection problem while providing invaluable science data and possibly minerals.

Because of the significant history of solar system exploration, a substantial body of work exists investigating various issues involved in designing these types of space missions. Some of the missions described above have led to significant investigations. Design of space missions is a complicated multidisciplinary field of study, with many different issues to be considered. In the context of NEO deflection missions, several studies have been published. Gurley [13] considered various issues and how they affect mission and vehicle design requirements. For example, the question of whether to divert or destroy the asteroid is a significant factor. Other issues include the use of advanced technology such as propulsion, miniaturization, terminal guidance, and automation. Lau and Hulkower [18] studied the possibility of launching exploratory missions to visit NEOs, but did not consider the problem of deflection. Venetoklis *et al*. [39] studied the application of nuclear rockets to get the deflection system to the NEO, but did not consider the coupling between the space mission and the deflection system. Willoughby *et al* [40] also analyzed the use of nuclear rockets, including using the rocket as the deflection mechanism, with additional fuel to be obtained by mining the asteroid.

Exploratory or deflection missions to comets are likely to be flybys because of the higher velocities of comets relative to the Earth. The flyby of comet Grigg-Skjellerup by the Giotto spacecraft is described by Burnham [6]. Guelman [12] investigated guidance and control for asteroid rendezvous, and Noton [23] investigated problems associated with orbiting a comet. Giotto successfully flew by comet Halley in 1986 and by comet Grigg-Skjellerup in 1992 [29, 30]. The Vega and Suisei spacecraft also flew by comet Halley in 1986 [29]. NASA's comet rendezvous asteroid flyby mission (CRAF) was canceled [9]. The European Space Agency (ESA) has proposed a mission called ROSETTA for comet rendezvous with a soft-landing instrument package [9].

On its way to Jupiter, the Galileo spacecraft made a flyby of the asteroids 951 Gaspra and 243 Ida, obtaining a significant amount of information during this first ever asteroid flyby, including the discovery that Ida possesses a "moon." NASA's Near-Earth Asteroid Rendezvous (NEAR) [9] mission was launched in February 1996. It will rendezvous with asteroid 433 Eros in January 1999. The rendezvous with Eros, which has dimensions of $36 \times 15 \times 13$ km, includes a ~50 km radius orbit around the asteroid for one year. On its way, the NEAR spacecraft flew within 1200 km of the asteroid 253 Mathilde. The Japanese Institute of Space and Astronautical Science (ISAS) has proposed a near-Earth asteroid rendezvous mission that would include a technology demonstration hovering phase with the spacecraft remaining within one foot of the asteroid surface for a period of months [9].

The feasibility of human missions to NEOs increases as space-based activities increase. As pointed out by Jones *et al*. [17], such missions would be excellent training for extended missions to Mars and for possible future NEO diversion missions.

THE FUNDAMENTAL PROBLEM

In establishing the fundamental problem of NEO interception, we assume that a threatening object (*i.e.*, one with sufficiently high kinetic energy) has been detected, its orbit determined and propagated, and that it can been shown with reasonable certainty that it is on a collision course with Earth. Hereafter, we will use the term NEO to mean such objects (since not all NEOs pose a threat). The problem is to alter the NEO's trajectory so as to prevent its collision with Earth. Since there cannot be zero error in orbit determination and propagation, for a successful solution to the problem, one must deflect the object enough so that Earth is outside of the "error cone." In addition, even in an ideal situation of zero error, one may wish to deflect the object so that its closest approach distance is reasonably far away from Earth. For example, if a threatening NEO was determined to be a large asteroid sufficient to cause globally catastrophic climate change, we might wish to deflect it such that its closest approach distance is greater than the cislunar radius. Based on this reasoning, we define the Sphere of Comfort (*SOC*) to be an Earth-centered sphere of predetermined radius which no threatening object is allowed to enter. Obviously, $SOC \geq R_E$, where R_E is the maximum radius of the Earth. Hereafter, we will use the word collision to mean the NEO is inside *SOC* (see Figure 1). The level of threat of the NEO is determined by its kinetic energy at the boundary of *SOC*. Following Ref. [22], we assume that any energy level ≥ 10 Megatons (MT) of TNT would cause a local catastrophe of about the same magnitude as that of the Tunguska blast, and an energy level of 3×10^5 MT is a nominal threshold for a global catastrophe, where 1 MT of TNT is equivalent to 4.2×10^{15} J.

Statement of the Fundamental Problem

Referring to Figure 1, we define the vector **R** as the position vector of the NEO with respect to the Earth, and the vector **x** as the state vector describing the motion of the NEO. Given these definitions, we now state the fundamental problem as that of altering the state vector, $\mathbf{x}$, of the NEO such that $\|\mathbf{R}\| \equiv R \geq SOC \;\; \forall t$ or equivalently, $R_{\min} \geq SOC$ where $R_{\min} = \min_{t \geq t_0} R$ and t_0 is the epoch of detection. Since *R* is continuous and differentiable, $R_{\min}$ satisfies the necessary conditions: $dR/dt = 0$ and $d^2R/dt^2 > 0$.

Elements in Mathematical Modeling

Any approach to solving the fundamental problem ultimately requires specific mathematical models describing the various components of the problem. In this section, we develop mathematical relationships describing the deflection problem. The problem may be decomposed into the coupled system of equations describing the dynamics of the NEO, the dynamics of the spacecraft that delivers the momentum, and the coupling between the control applied to the NEO and the states and parameters involved.

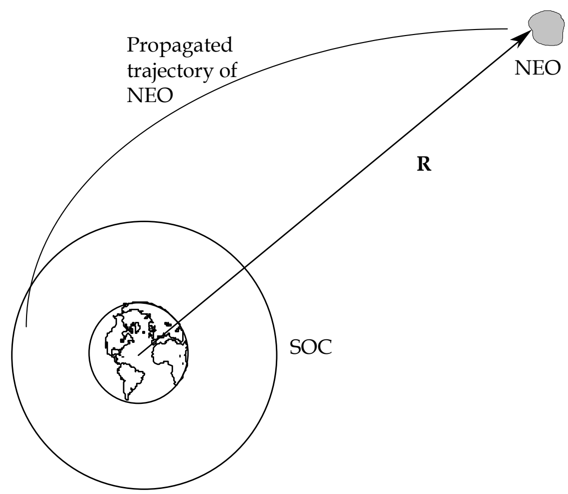

Figure 1. The fundamental problem.

The dynamics of a NEO can be written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathrm{t}; \mathbf{p}_\mathrm{a}) \tag{1}$$

where $\mathbf{x}$ is the state vector describing the NEO's position, velocity, orientation, and angular velocity, $\mathbf{u}$ is the control imparted to the NEO by an appropriate deflection mechanism, $\mathbf{f}(\cdot)$ is the vector representing the forces and moments acting on the NEO, and $\mathbf{p}_a$ is a vector of parameters describing the asteroid, including, for example, mass and composition. These parameters will also affect the control mechanism. An excellent discussion of the important NEO properties and their relationships with various deflection strategies is given by Remo [31] (p. 588, Table X). The approaches to modeling of Eq. (1) are discussed further below.

Normally, the control $\mathbf{u}$ will be delivered by a spacecraft with dynamics governed by a similar system of equations:

$$\dot{\mathbf{y}} = \mathbf{g}(\mathbf{y}, \mathbf{v}, t; \mathbf{p}_s) \tag{2}$$

where $\mathbf{y}$ is the spacecraft state vector, $\mathbf{v}$ is the control on the spacecraft, $\mathbf{g}$ is the vector representing the forces and moments acting on the spacecraft, and $\mathbf{p}_s$ is a vector of parameters describing the spacecraft, including, for example, the payload mass. Normally Eq. (2) will only be valid for $t \geq t_1 \geq t_0$, where t_1 is the mission launch time. The approaches to modeling of Eq. (2) are not discussed further here, as they are well-known.

Since $\mathbf{u}$ is delivered by the spacecraft to the NEO, $\mathbf{u}$ will generally depend on the motion of both the NEO and the spacecraft:

$$\mathbf{u} = \mathbf{h}(\mathbf{x}, \mathbf{y}, t; \mathbf{p}_a, \mathbf{p}_s, \mathbf{p}_d) \tag{3}$$

where $\mathbf{p}_d$ is a parameter vector describing the deflection mechanism capabilities. This equation formalizes the coupling in the dynamics and control of the deflection of the near-Earth object and the guidance of the spacecraft. Note that the parameter vectors $\mathbf{p}_a$, $\mathbf{p}_s$, and $\mathbf{p}_d$ differ in that $\mathbf{p}_\mathrm{a}$ is fixed whereas $\mathbf{p}_\mathrm{s}$ and $\mathbf{p}_d$ include mission design parameters that may be optimized. Since there are various ways of delivering $\mathbf{u}$ to the NEO, there is no one model for $\mathbf{h}(\cdot)$. Consequently, the details of the mathematical models differ for different delivery mechanisms. However, by appropriately classifying the methods of delivery of the control, the repetition of analyses can be avoided. This classification is further elaborated below.

Noting that the distance between the NEO and the Earth may be written as $R(t) = F(\mathbf{x}(t), t)$, the fundamental problem is to select a deflection strategy, and determine a spacecraft control $\mathbf{v}$ and parameter vectors $\mathbf{p}_s$ and $\mathbf{p}_d$ that satisfy the "boundary" condition

$$\min_{t \geq t_0} R \equiv \min_{t \geq t_0} F(\mathbf{x}(t), t) \geq SOC \tag{4}$$

Naturally, there may be many combinations of $\mathbf{v}$, $\mathbf{p}_s$ and $\mathbf{p}_d$ that satisfy Eq. (4). We are especially interested in solutions that are optimal in some sense. An obvious example would be to maximize the miss distance $R_{\min}$, which can be stated mathematically as

$$\max_{\mathbf{u}} \left\{ \min_{t \geq t_0} R \right\} \equiv \max_{\mathbf{u}} \left\{ \min_{t \geq t_0} F(\mathbf{x}(t), t) \right\} \tag{5}$$

The types of relevant optimization problems are discussed further below.

Modeling Issues

The dynamics of the NEO, Eq. (1), are governed by the *n*-body problem and appropriate rotational equations of motion. Consequently, a model for Eq. (1) is not straightforward. For the purposes of accurate prediction, it is important to have accurate state estimation $(\mathbf{x}(t_0))$, dynamical models $(\mathbf{f}(\cdot))$ and state propagation $(\mathbf{x}(t))$. Such accuracies, although important, are not altogether necessary for obtaining solutions to the fundamental problem since solutions obtained with approximate models can be tested for correctness by accurate numerical simulation with higher fidelity models in much the same way as guidance algorithms are designed and tested. Nevertheless, classifying the NEO-scenarios from detection to collision as discussed in the following section simplifies the modeling of $\mathbf{f}(\cdot)$.

The modeling of Eq. (2) parallels that of Eq. (1) but the requirements on fidelity in modeling are not as critical since midcourse guidance can easily account for errors arising from various sources (including modeling). Perhaps the greatest source of difficulty in modeling is Eq. (3), which provides the link between Eqs. (1) and (2). All of the issues implied by Eq. (3) must be addressed: the choice of the control, $\mathbf{u}$, which affects the modeling of Eq. (1), the choice of the arguments of $\mathbf{h}(\cdot)$ which affects the modeling of Eqs. (1) and (2), and of course, the accuracy and tractability of the function, $\mathbf{h}(\cdot)$, itself.

CLASSIFICATION AND MODELING OF NEO DETECTION-TO-COLLISION SCENARIOS

As noted in the previous section, a requirement for a NEO deflection mission arises when a NEO's trajectory is predicted to enter the Earth-centered sphere of comfort (*SOC*). The urgency of the deflection mission, and hence its objectives, will depend on the details of the particular prediction. For example, detection of a large, impacting NEO only a few months before impact would lead to a rapid response mission with the goal of maximizing the miss distance [Eq. (5)], regardless of other costs. Such a scenario would almost certainly lead to the use of nuclear weapons for deflection. On the other hand, detection of the same threatening NEO with decades of lead time would lead to a more conservative approach. Maximizing the miss distance would remain a goal, but minimizing other costs would also be given consideration. Typical analyses of deflection problems have characterized different scenarios as short or long lead time and have used simple rules of thumb to determine the cost, in terms of $\mathbf{\Delta V}$, of achieving a given deflection. Since we are interested in developing classes of optimization problems related to deflection, we need to establish a more definite classification of scenarios.

Our classification is based on the problem of approximating Eq. (1). As noted above, an "exact" representation of Eq. (1) would be complicated (*cf.* Yeomans and Choda [42]) and difficult to use in the development of candidate solutions to the fundamental problem. We envision using approximations in performing mission analysis and then verifying the accuracy of the resulting solutions using a more accurate "truth model." Thus our classification is partially designed to group scenarios where the approximate analysis is expected to be of the same order of accuracy.

One appropriate approximation is to model the NEO as a point mass moving in the gravitational field of a massive body which is fixed in space. This leads to the classical *two-body problem*, with well-known dynamics. In most cases the massive body will be the sun, leading to a *heliocentric* two-body problem. However, whenever the NEO passes close enough to another massive body, such as Earth, the heliocentric two-body model is less useful. In such cases, it may be necessary to use a three-body model, which is significantly more complex than the two-body model. It is possible, however, to obtain reasonable results using *patched conics*.

In the patched conics method, one uses a heliocentric two-body model so long as the body of interest is sufficiently distant from any other massive body, and a *planetocentric* two-body model when the body is sufficiently close to a third body (in this case a planet). The accepted meaning of "sufficiently close to" is based on the *sphere of influence*, which is a planet-centered sphere with radius defined by

$$SOI \cong R_p \left(\frac{M_p}{M_{\text{sun}}} \right)^{2/5} \tag{6}$$

where R_p is the mean radius of the planet's orbit, M_p is the planet's mass, and M_{sun} is the sun's mass. The sphere of influence of the Earth has a radius of roughly 10^6 km, which is more than twice the radius of the moon's orbit. One can advance the argument that a maximum radius for the sphere of comfort *SOC* would be equal to *SOI*, and hence, we will assume that $SOC < SOI$.

Using the patched conics approach, we assume the NEO is in a heliocentric two-body trajectory whenever it is outside the Earth's sphere of influence, and that it is in an Earth-centered two-body trajectory whenever it is inside the Earth's sphere of influence[1]. Since $SOC < SOI$, this means that a threatening NEO must enter the sphere of influence *before* it enters the sphere of comfort. Referring to Figure 2, we denote the time (or epoch) at which the threatening NEO is detected by t_0, the epoch at which it enters the sphere of influence by t^*, and the time at which it enters the sphere of comfort by t_I ($I \Rightarrow$ "Impact"). It is also useful to define the "lead time interval" by $t_L = t_I - t_0$.

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[1] Note that it is possible that the position of the moon may require that the effects of its gravity be taken into account. In this case, the sphere of influence of the moon with respect to the Earth would be required. It is also possible, though unlikely, that a NEO might pass within the *SOI* of another planet on its way to the *SOI* of Earth.

We now classify scenarios into two broad categories: *Unique t* scenarios* and *Multiple t* scenarios*. The unique t^* scenario is the simpler of the two in terms of devising solutions to the fundamental problem since it indicates that once the NEO enters the Earth's *SOI*, it is predicted to enter the *SOC*. In the multiple t^* scenarios, the NEO enters and exits the *SOI* at least once before entering the *SOC*.

Unique t* **Scenarios**

By definition, these scenarios are all characterized by the fact that once the NEO enters the *SOI*, the distance between the NEO and the Earth decreases monotonically. Mathematically, this condition may be expressed as $\dot{R} < 0$ for $t^* < t < t_I$. The detection may occur before or after the NEO enters the *SOI*, and these cases are classified based on whether $t_0 \le t^*$ or $t_0 \ge t^*$.

Scenario U1: t0 ≥ t.* This is the simplest scenario, and the most dangerous. The conditions imply that $R(t_0) < SOI$ and hence the Earth-centered two-body problem (perhaps including lunar effects) is sufficient. Here $\dot{R} < 0$ for $t^* < t < t_I$ can be strengthened to $t_0 < t < t_I$ since $t_0 \ge t^*$. With typical NEO speeds (relative to the Earth) of 10–50 km/s, the warning time is of the order of 7–30 hours.

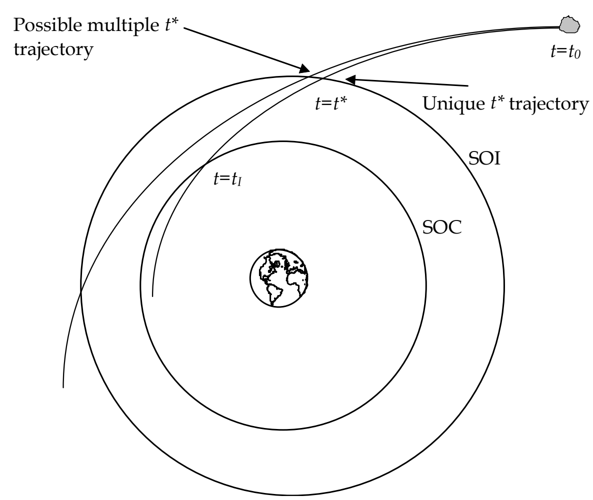

Figure 2. Unique and multiple t^* scenarios.

Scenario U2: t0 ≤ t.* Here $\dot{R}$ may be non-negative over $t_0 < t < t^*$. This condition implies $R(t_0) > SOI$ and hence, using patched conics, leads to a heliocentric two-body problem until $t = t^*$, followed by an Earth-centered two-body problem for $t > t^*$. Alternatively, the three-body problem could be used for a more refined analysis. Three sub-scenarios can be defined by whether impact occurs within one, a few, or many periods of the NEO's orbit (P). Note that for asteroids, the period will normally be on the order of one year, whereas for comets, the period may be significantly longer than one year. Mathematically, the sub-scenarios are characterized by $t_L = O(P)$, $t_L = O(nP)$ with $n > 1$ but "small," and $t_L = O(nP)$ with n "large."

Scenario U2a: $t_L = O(P)$. As in the *U1* scenario, the lead time is typically on the order of months. For this scenario, the heliocentric two-body problem would be used for preliminary mission analysis. Furthermore, since the lead time is relatively short, errors associated with this approximation will be small, and the solution based on the approximation will have reasonable chance of success in the higher fidelity "truth model."

Scenario U2b: $t_L = O(nP)$*; n >1 but small.* This scenario is similar to *U2a* but since the lead time is over several orbits of the NEO, there are two effects that must be considered. First, the errors associated with the approximation will grow, so the solution based on the approximation will be less applicable in the truth model. However, we expect reasonable results which can be refined in relatively straightforward fashion. The second effect is that with advance notice of several orbits, the choice of deflection strategies may be significantly broader. That is, whereas in *U2a*, the deflection may be accomplished using a single nuclear weapon (for example), in *U2b*, a multiple **ΔV** approach may be appropriate.

Scenario U2c: $t_L = O(nP)$*; n >1 and large.* This scenario is similar to *U2b*, and the same two effects are important. However, the even longer lead time means the errors associated with the two-body approximation will be even more significant, with less agreement between the approximation and the truth model. Also, with significantly long lead times, additional deflection strategies may become feasible.

Multiple t* Scenarios

These scenarios do not appear to be as probable as the unique *t** cases; however, they present unique opportunities to apply **ΔV**s near the Earth, perhaps to capture the NEO into an Earth orbit, or to provide a gravity assist to deflect the NEO into a less threatening orbit. The basic scenario is presented in Figure 3. As with the unique *t** scenarios *U2a–U2c*, we classify the multiple *t** scenarios based on the lead time: *Scenario M1a*: $t_L = O(P)$, *Scenario M1b*: $t_L = O(nP)$; $n > 1$ but small, and *Scenario M1c*: $t_L = O(nP)$; $n > 1$ and large.

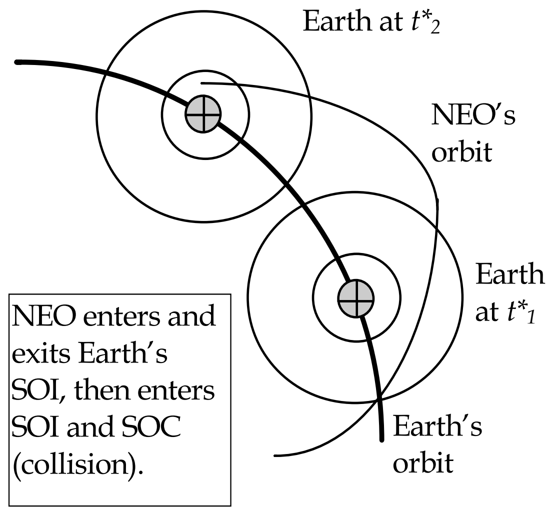

Figure 3. Multiple *t** scenario.

The categories defined here are useful primarily because of how they impact the modeling involved in Eq. (1). In addition, however, these scenarios should prove useful in establishing decision-making criteria and determining effective mitigation strategies.

PROPOSED DEFLECTION DELIVERY STRATEGIES

In this section, we discuss several of the approaches that have been suggested for diverting a threatening NEO. Given the broad international interest, it is not surprising that a large number of alternative deflection strategies have been proposed. One collection of such strategies was included in the chapter on “Assessment of Current and Future Technologies” in Ref. [7]. The various strategies were grouped according to whether the critical technologies are near-term, medium-term, or long-term. An additional grouping was based on effectiveness for distant or close-in NEO interception. For example, nuclear weapons are available now, and would be effective for distant or close-in deflection, whereas solar sails are considered long-term technology and would only be effective if the warning time were on the order of centuries. Several analyses of various methods were included in Ref. [10]. It should be noted, however, that in all studies, the authors used simplified astrodynamics modeling in order to obtain simple formulae for the deflection provided by a given **ΔV**. Furthermore, they typically assumed that the **ΔV** is applied at perihelion of the NEO’s orbit, whereas our development above indicates that this assumption is not always valid.

We consider several deflection strategies in the context of the fundamental problem. Rather than group the strategies according to technological availability or effectiveness, we group them into three broad categories according to how they contribute to the NEO/spacecraft coupling defined in Eq. (3). In so doing, we give valid approximations for Eq. (3) in several of the cases, illustrating how the equations describing the fundamental problem would be formulated. The most effective of the deflection strategies are those based on *cratering*, which provide an impulsive **ΔV** to the NEO. Another category, which includes engines and mass drivers, is based on *continuously* ejecting mass from the NEO, so that these methods provide a continuous acceleration to the NEO instead of an impulsive **ΔV**. The third category includes methods based on *action at a distance*, which in this case is limited to using solar pressure to change the orbit.

Methods Based On Cratering

Many feasible approaches to deflecting a NEO are based on using an explosion to create a crater on the NEO. Under fairly general conditions, a fraction of the ejecta will be moving at a significant velocity relative to the NEO. In the cratering methods, the material is given its exhaust velocity either by a high-speed impact or by an explosion.

Nuclear weapons. Because of the energy released in a nuclear explosion, the use of nuclear weapons for asteroid deflection is evidently the preferred approach, especially if there is a short lead time, or if the asteroid is large. Several researchers have considered the amount of **ΔV** delivered by various applications of nuclear weapons, primarily standoff explosions, surface explosions, and buried explosions, with most researchers offering a comparison of the three approaches. Standoff explosions have the lowest effectiveness of the three, and optimally buried explosions are the most effective. However, analysis of the effectiveness of surface and buried explosions is dependent on certain assumptions about the physical properties of the NEO, whereas analysis of the standoff explosion approach is not. Ahrens and Harris [1] and [2] considered all three cases, as did Shafer *et al* [32]. Simonenko *et al* [34] performed a slightly different analysis, but arrived at essentially the same conclusions. Solem [35] and Solem and Snell [37] used a simplified analysis to reach similar conclusions. In Ref. [37], the special case of short-notice deflection of an asteroid to strike an ocean instead of a continent is considered. Solem [36] proposed using nuclear explosives for the propulsion of a kinetic-energy-based system. Hills [15] proposed using nuclear weapons to capture an asteroid into an orbit around the Earth.

The control **u** provided by a nuclear weapon explosion is complicated and depends on several factors, including the composition and mechanical structure of the NEO, as well as its radiation absorption properties. These parameters are conceptually included in the asteroid parameter vector $\mathbf{p}_a$. In addition the **ΔV** provided depends on the placement and yield of the nuclear device. These parameters would be included in the deflection system parameter vector $\mathbf{p}_d$. We include the function $\mathbf{h}(\cdot)$ for only one case, that of a surface explosion on an asteroid which is small enough so that the *strength regime* is appropriate. Following Ahrens and Harris [2], the scalar ΔV provided is given by

$$M_{\mathrm{NEO}}\Delta V = 3.4\sqrt{YW/\rho} \tag{7}$$

where the mass M_{NEO} (kg), the material strength Y (dyne/cm^2), and the material density ρ (g/cm^3) are asteroid parameters (included in $\mathbf{p}_a$), and the total explosive yield W (Kt) is a payload parameter (included in $\mathbf{p}_d$). Equation (7) defines the magnitude of the **ΔV** provided. The vector control, **u**, depends on the positions of the two objects at the time of detonation. In Figure 4, $\mathbf{x}_r$ is the position vector component of the NEO state vector **x**, and $\mathbf{y}_r$ is the position vector component of the spacecraft state vector **y** at the time of detonation. Defining a unit vector $\mathbf{e}_\mathrm{a}$, by

$$\mathbf{e}_a = \frac{\mathbf{x}_r - \mathbf{y}_r}{\left\|\mathbf{x}_r - \mathbf{y}_r\right\|} \tag{8}$$

the vector $\mathbf{\Delta V}$ may be expressed as

$$\mathbf{\Delta V} = \Delta V \mathbf{e}_a \tag{9}$$

Thus, Eqs. (7)–(9) define the function $\mathbf{h}(\cdot)$ for the case of a surface detonation of a nuclear device on a small asteroid. Clearly there are other situations in which the relationship would differ, including the use of a different model for the effect of the device, or taking into account the fact that the impulsive force might not act through the center of mass, hence inducing a rotation of the asteroid. For larger NEOs, the gravity field of the NEO dominates the dynamics of cratering, and a different analysis for the *gravity regime* is required (*cf.* Ref. [2]). Such questions would need to be addressed in more detailed studies of the particular cases.

Kinetic-energy methods. Perhaps the simplest deflection concept is to impact the NEO with a massive, high-speed projectile. Assuming the projectile "sticks" to the NEO, and the NEO does not fragment, then all the linear momentum of the projectile will be transferred to the new NEO+projectile object. One analysis of this approach, in Ref. [16], considers only the transfer of linear momentum due to a non-elastic collision, and concludes that this approach is more effective than several other non-nuclear methods. However, other researchers have shown that the additional $\mathbf{\Delta V}$ obtained from the mass ejected from the resulting crater exceeds the direct $\mathbf{\Delta V}$ obtained from the impact (Refs. [2], [20], [32], [35], [36], and [37]). Thus the kinetic-energy approach may be viewed as the use of a projectile to construct an impulsive engine using the asteroid's mass as "fuel."

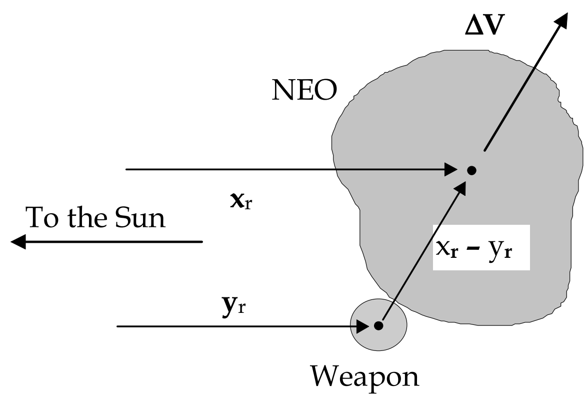

Figure 4. Relative position at detonation.

In Ref. [2], where the nuclear weapons approach is recommended, the treatment of the kinetic energy approach is based on determining the size of an impactor required to provide a 1 cm/s $\mathbf{\Delta V}$ to asteroids of varying size and speed. Simplified analysis of the astrodynamics is used to arrive at this $\mathbf{\Delta V}$ requirement, and the conclusion is that this approach is feasible for asteroids of roughly 0.1 km diameter or smaller. For larger asteroids, based on the large mass required for the projectile, this approach may not be effective if the projectile is the spacecraft. However, it may be possible to deflect a smaller asteroid and use it as a projectile for a larger, threatening NEO. The analysis in Ref. [20] is based on the same cratering models as in Ref. [2] and reaches the same conclusions. These cratering models are empirical and are based on experimental results. In Ref. [32] an analytical approach is taken and similar results are achieved. In Refs. [35], [36], and [37], an *ad hoc* cratering model is used and the authors reach similar conclusions. The analysis in Ref. [36] is unique in that the use of nuclear explosions for propulsion is recommended to obtain the required kinetic energy for the impactor.

In this case, the function $\mathbf{h}(\cdot)$ is similar to the nuclear device example above. The scalar ΔV provided by a kinetic energy impactor on a small asteroid, where the strength regime is applicable, has been developed by Ahrens and Harris [2] as

$$M_{\text{NEO}}\Delta V = m_i v_i \left[1 + 0.16\left(\frac{\rho}{\rho_i}\right)\left(\frac{\rho v_i^2}{Y}\right)^{0.209}\right] \tag{10}$$

where the mass m_i and material density ρ_i are properties of the deflection device ($\mathbf{p}_d$), the mass M_{NEO}, the material density ρ, and the material strength Y are properties of the asteroid ($\mathbf{p}_a$), and the relative velocity v_i is determined from the states of the two bodies immediately before impact ($\mathbf{x}$ and $\mathbf{y}$). Given the scalar ΔV calculated by Eq. (10), the vector $\mathbf{\Delta V}$ for this kinetic energy deflection is given by Eq. (9), with the additional assumption that the relative velocity is in the $\mathbf{e}_a$ direction.

Methods Based On Continuous Mass Ejection

The methods of the previous section are based on ejecting a large quantity of mass over a relatively short time interval, providing a large $\mathbf{\Delta V}$ to the NEO. Such methods can be thought of as special cases of continuous mass ejection. However, since the modeling and analyses involved usually differ significantly, we have separated the two groups. In this section, we consider methods which involve continuously ejecting smaller quantities of mass, perhaps at a higher velocity, which is the principle that leads to rocketry. In these cases the thrust, T, provided by the mechanism is normally expressed as $T = \dot{m}V_e$, where $\dot{m}$ is the rate at which mass is ejected, and V_e is the relative exhaust velocity of the ejected mass. The ejected mass could either be fuel, such as in the case of a conventional rocket motor, or it could be NEO mass in the case of a mass driver.

In either case, the control $\mathbf{u}$ is an acceleration which may be expressed as

$$\mathbf{u} = \frac{\dot{m}V_e}{M_{\text{NEO}}}\mathbf{e} \tag{11}$$

where $\mathbf{e}$ is the vector describing the thrust direction. Note that in many cases, the parameters $\dot{m}$ and V_e would be constants to be optimized in $\mathbf{p}_d$, whereas $\mathbf{e}$ might be a controlled variable to be solved for in an optimization problem (*cf.* Ref. [38]). The thrust could be provided by conventional high or low thrust engines, by mass drivers, or by using a laser or particle beam to vaporize the NEO.

Engines. Ivashkin and Smirnov [16] considered both high and low thrust engines. Willoughby *et al* [40] studied the use of nuclear rockets to perform the deflection mission, including the novel idea of breaking up the NEO by spinning it. For an engine, the fuel could be delivered to the NEO, or it could be obtained by mining the NEO. Lewis [19] suggested that the mineral value of a NEO might be greater than the cost of returning it to Earth; thus one mitigation approach would be to guide the NEO to orbit about Earth.

Mass drivers. The basic principle of a mass driver is to accelerate small pieces of the NEO (Δm) to a significant exhaust velocity (V_e). With each ejected Δm, the NEO's momentum is changed, and the average rate of mass ejection over a time Δt would be used in Eq. (11); *i.e.*, $\dot{m} = \Delta m / \Delta t$. Ahrens and Harris [2] gave an abbreviated analysis of the effectiveness of mass drivers, mainly to illustrate the greater effectiveness of nuclear weapons. A more detailed investigation by Melosh *et al* [20] included analysis of the efficiency of converting solar energy into electricity and electricity into the mechanical energy given to the ejected mass. They reached the interesting conclusion that increasing the speed of mass ejection (larger V_e) can actually decrease the size of asteroid that can be effectively deflected, concluding that the analysis of Ref. [2] was over-simplified.

Beams. The use of laser beams was investigated by Shafer *et al* [32] and by Melosh *et al* [20], who also considered microwave beams and concentrated solar energy. Phipps proposed using a ground-based laser [26]. With these methods, the beams vaporize the NEO's surface layer, resulting in ejected mass, and providing acceleration in the same manner as a mass driver, so the control **u** is essentially given by Eq. (11). One of the systems studied in Ref. [20] uses a solar collector to concentrate sunlight onto a relatively small area of the NEO's surface. This requires the spacecraft to track the NEO for a significant length of time, raising interesting design, dynamics, and control issues.

Methods Based On Solar Pressure

These methods differ considerably from the methods of the two previous sections, as there is no mass ejection involved. Instead, solar pressure is enhanced to yield a small, continuous acceleration to the NEO, thereby changing its orbit. At least two approaches have been suggested: large solar sails, and paint. Melosh *et al* [20] give a brief discussion of the use of solar sails, based on the analysis in the book by Wright [41], pointing out that construction of the large sails required (10 km diameter or more) is currently beyond our capabilities. Ivashkin and Smirnov [16] suggested painting the NEO to increase the effects of solar pressure. In effect, this approach is the same as a small solar sail with fixed orientation relative to the NEO. Since this approach is at the opposite end of the spectrum from the nuclear weapons strategy, we examine in detail how it fits into our development of the fundamental problem.

The application of paint to the NEO is used to increase its absorptivity so that the force due to solar pressure gradually changes the orbit. Clearly this approach would only be effective in the long lead time scenarios (*U2c, M1c*). In this case the models would arise as follows. The true dynamics of the NEO, described by Eq. (1), would require $\mathbf{f}(\mathbf{x},\mathbf{u},t;\mathbf{p}_a)$ to take into account perturbations due to the larger planets, as well as possibly the smaller planets and larger planetoids. In addition, the rotation of the NEO might be significant. However, preliminary analyses for the purposes of determining the optimal mission may use a two-body heliocentric gravity model, taking into account the natural solar pressure acting on the object. The parameter vector $\mathbf{p}_\mathrm{a}$ would include the NEO's mass, m_a, diameter, d_a, and absorptivity, σ_a.

The spacecraft dynamics would take place over a shorter time scale, hence the true dynamics of the spacecraft would require accounting for fewer perturbations, and the two-body heliocentric gravity model would suffice for preliminary mission design, unless a planetary flyby is planned. The spacecraft parameter vector $\mathbf{p}_\mathrm{s}$ would include, for example, the initial mass, m_0, the propulsion system performance characteristics, I_{sp} and $\dot{m}$, and the payload mass m_d. The payload parameter vector $\mathbf{p}_d$ would include, for example, the paint mass, m_p, and the paint absorptivity, σ. The spacecraft mission could be a rendezvous or a lander, after which the paint is applied, perhaps by a robotic system, or it could be an intercept, where the paint might be applied in "water balloon" fashion.

The deflection control mechanism, generally described by Eq. (3), is in this case an almost entirely radial acceleration, which may be expressed as

$$\mathbf{u} = u(\mathbf{x}_\mathrm{r};\, \mathbf{p}_a,\, \mathbf{p}_s,\, \mathbf{p}_d)\, \mathbf{x}_\mathrm{r} \tag{12}$$

where $\mathbf{x}_\mathrm{r}$ is the position vector from the sun to the NEO (*i.e.*, part of the state vector $\mathbf{x}$). The dependence of $\mathbf{h}(\cdot)$ on the spacecraft state $\mathbf{y}$ is in this case simply that the control $\mathbf{u}$ is zero until the paint is applied at $t = t_2$, and is described as above for $t \geq t_2$.

MISSION OPTIMIZATION

As discussed above, the development of solutions to the fundamental problem involves designing a space mission with spacecraft parameters $\mathbf{p}_s$, deflection payload parameters $\mathbf{p}_d$, and control $\mathbf{v}$, such that Eq. (4) is satisfied. Furthermore, since it is human nature to achieve things optimally, we want to solve the fundamental problem for any scenario by means of an optimal space mission to the NEO. Naturally, the first question that arises is *what do we mean by optimal?* The optimality criteria might depend on the scenarios as well as on a personal point of view. In an unconstrained world, optimality could mean intercepting the NEO in minimum time and achieving the maximum deflection with minimal expenditure of energy while minimizing cost. It is apparent that these are conflicting optimality criteria. Nevertheless, it is useful to formulate and solve the optimization problem with separate and simple optimality criteria since it provides extremal bounds on what is achievable. One can then weigh the optimality criteria appropriately to decide on the "best" course of action.

The optimal threat mitigation problem is unique when compared to traditional optimization problems encountered in space missions, because the astrodynamics of the NEO is coupled with the astrodynamics of the spacecraft, and the choice of the payload[2] is also coupled to the astrodynamics of the NEO. To demonstrate this coupling more clearly, consider a scenario in which the NEO is an ECA (see Figure 5) and suppose we have a sufficiently long warning time — of the order of many NEO orbital periods (Scenario *U2c*, for example). The minimum $\mathbf{\Delta V}$-point for deflecting the ECA is its perihelion, P1 (Ref. [2]). In addition, the longer the warning time, the smaller the required minimum $\mathbf{\Delta V}$. Further, let the optimality criterion be the minimization of lift-off mass, m_1 — the motivation for this optimality criterion being that its minimization could lead to delivering "more bang for the buck." In this regard, we will use the term cost interchangeably with lift-off mass. Note that $\mathbf{u} = \mathbf{\Delta V}$ and the challenge in modeling is to determine $\mathbf{h}(\cdot)$ in order that one can formally write $\mathbf{u} = \mathbf{h}(\mathbf{x}, \mathbf{y}, t; \mathbf{p}_a, \mathbf{p}_s, \mathbf{p}_d)$; of course, the choice of the arguments of $\mathbf{h}(\cdot)$ is dependent on the type of $\mathbf{\Delta V}$ delivery mechanism carried by the spacecraft (*i.e.*, the payload).

Now, at first glance, this scenario suggests that carrying the minimum necessary payload (mass) to achieve $R_{\min} \geq SOC$ would be optimal ($m\mathbf{\Delta V}$ on the NEO = $\mathbf{F}\Delta t$ delivered by the payload)[3]. This analysis is not strictly correct since it ignores the "trajectory cost;" *i.e.,* the propellant mass required to go to the point of application of the minimum momentum change (P1). In other words, it is quite possible for the trajectory cost to be prohibitively large to visit the minimum $\mathbf{\Delta V}$ -point whereas it may be cheaper overall to intercept the NEO elsewhere. For example, a space flight to the perihelion requires at least one $\mathbf{\Delta V}$ (on the spacecraft) in addition to the sum of $\mathbf{\Delta V}$s required to put the spacecraft in a heliocentric orbit of 1 AU. Thus, although delivering a $\mathbf{\Delta V}$ to the ECA at P2 or P3 may be costlier than delivering it at P1, the $\mathbf{\Delta V}$ required for the spacecraft itself to go to P2 or P3 may be substantially less so as to be cost effective.

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[2] The payload could be the entire spacecraft as in the case of kinetic energy delivery mechanisms

[3] This is based on the assumption that the smaller the "impulse," $\mathbf{F}\Delta t$, delivered by the payload, the smaller its mass will be for a given payload.

The above scenario demonstrates a coupling between the spacecraft trajectory and the ECA trajectory for a given optimization criterion, viz., lift-off mass. If we add the complexity of determining the best payload as well, then this requires an accurate model that maps the momentum delivery capability of a payload to mass. This is rather difficult to model since the delivery of momentum can be achieved by several means and the resulting analysis is different for each. However, since the choices are finite, we can simplify the overall problem by solving the spacecraft design and mission problem for each payload and thereafter choosing the payload/design combination that yields the minimal cost. Repeated solving of the problem for each payload may be reduced by classifying payloads with this objective in mind. As demonstrated above, the same cannot be said of the coupling between the spacecraft trajectory design and the astrodynamics of the NEO.

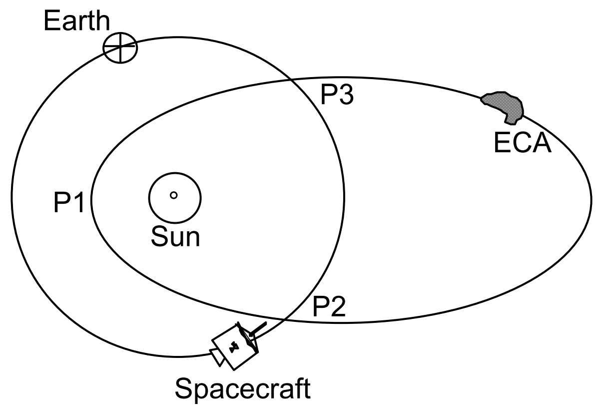

Figure 5. Earth-crossing asteroid intercept scenario.

Optimality Criteria

Equations (1)–(4) address the fundamental problem. The optimization problem is to obtain an optimal solution to this problem. More precisely, the objective is to determine the spacecraft control $\mathbf{v}(\cdot)$ and parameters $\mathbf{p}_s$, and payload parameters $\mathbf{p}_d$, where the optimality criterion is the minimization of a given performance measure. The optimality criterion may be classified under two broad categories: i) ignorable mission cost, and ii) significant mission cost. The former category is widely discussed in the literature (Part VII of Ref. [10]) where the performance measure is stated simply as minimizing $\mathbf{\Delta V}$ imparted to the NEO to achieve the boundary condition of Eq. (4) with $SOC = R_E$. Usually, these are simple calculations in the sense that the astrodynamical details are ignored. We formalize this optimization criterion as a Lagrange problem :

$$J[\mathbf{u}] = \int_{t_2}^{t_3} \Phi(\mathbf{x}, \mathbf{u}, t)\, dt \tag{13}$$

where t_3 denotes the end of the NEO-deflection maneuver. As discussed above, there may be cases where trajectory cost is significant, so that the performance measure must include this cost as well. In these situations, the optimality criterion may be stated as

$$J[\mathbf{v}, \mathbf{h}] = \int_{t_1}^{t_2} \Psi(\mathbf{y}, \mathbf{v}, t)\, dt + \int_{t_2}^{t_3} \Phi(\mathbf{x}, \mathbf{h}, t)\, dt \tag{14}$$

Besides the obvious differences between Eqs. (13) and (14), it is important to note that in the former, we write $J = J[\mathbf{u}]$ to emphasize the lack (or ignorance) of coupling indicated by Eqs. (1)–(3). On the other hand, by writing $J = J[\mathbf{v}, \mathbf{h}]$ in Eq. (14), we emphasize that the measure of performance is based on the trajectory of the spacecraft $\mathbf{v}(\cdot) \mapsto J$ and the interaction of the payload (which depends on parameters describing the payload, $\mathbf{p}_d$) with the trajectory of the NEO:

$$\left(\mathbf{x}, \mathbf{y}, t; \mathbf{p}_a, \mathbf{p}_s, \mathbf{p}_d\right) \mapsto \mathbf{u} \mapsto J \tag{15}$$

Clearly, unless $\mathbf{h}(\cdot)$ satisfies some fairly unique conditions, the two integrals of Eq. (14) are coupled in a way that implies that minimization of the individual integrals will not result in the minimization of J. Thus, one can regard Eq. (13) as a special case of Eq. (14).

Finally, we identify a class of problems that do not immediately fit the format of Eq. (14). Based on the fundamental problem, we can formulate a maximin problem

$$\max_{\mathbf{u}}\left\{\min_{t \geq t_0} R\right\} \equiv \max_{\mathbf{u}}\left\{\min_{t \geq t_0} F(\mathbf{x}(t), t)\right\} \tag{16}$$

which is somewhat different from classical maximin problems in the sense that t is not bounded from above. In any case, this problem requires us to perform the optimization indicated by Eq. (16) under the constraints of Eqs. (1)–(3). The additional constraint that is important is that $\mathbf{p}_d \in D$ (*i.e.*, a given amount of payload) which could be mapped to a finite allowable control authority, $\mathbf{u} \in U$ [see Eq. (15)].

CONCLUSIONS

The problem of deflecting an asteroid or comet on a collision course with the Earth poses significant dynamics and control issues that have not been adequately addressed in the literature. Although a significant number of approaches have been suggested and analyzed in previous studies, the dynamics and control analyses have all been based on simplified astrodynamics models. More importantly, assumptions about the optimal application of suggested deflection strategies are not strictly justified. In this report, we have developed a formulation of the fundamental dynamics and control problem which makes clear the coupling between the astrodynamics of the NEO and the astrodynamics of the spacecraft which delivers the deflection mechanism. Our categorization of collision scenarios, spacecraft mission analysis, and deflection mechanisms is then based on this formulation. The coupling leads to a class of optimization problems that is distinct from those which have been considered in previous analyses of the problem.

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