High Speed Flywheels for Integrated Energy Storage and Attitude Control

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Abstract

We present new results on attitude control of spacecraft with flywheels, where the wheels are also used to store energy as "mechanical batteries." A brief review of the literature of this concept is given. The nonlinear equations of motion for a gyrostat model are given in dimensionless form, and recognized as a noncanonical Hamiltonian system. Decomposition of the space of internal torques separates the attitude control function from the energy storage function. A class of control laws is developed to execute large-angle rotational maneuvers while simultaneously performing energy storage and extraction operations.

1 Introduction

The power engineering community has been investigating high performance kinetic energy storage systems since the 1960s, with significant research and development activity throughout the subsequent decades. Three conferences were devoted entirely to flywheels.[1–3] Rockwell[4] conducted a systems study on flywheels for energy storage, and discussed some space applications. Hagen and Erdman[5] (1976) gave a literature review with over 400 citations on flywheels, and Mallon and Kuhn[6]'s 1979 survey included over 500 citations. Kirk[7] and Kirk and Studer[8] provided concise treatment of the issues associated with flywheel energy storage. Davis and Csomor[9] characterized the advantages of flywheels in a variety of applications.

The list of proposed applications includes primary storage for automobiles, electromechanical actuators for aerospace vehicles, uninterruptible power supplies for critical facilities such as hospitals and computer centers, and secondary battery replacement for satellites (see, for example, Appendix G of Ref.[4]). In this last application, it is also possible that the flywheels can perform the functions of some attitude control sensors and actuators. Two NASA workshops were devoted to this idea in 1983–1984.[10,11] (An additional workshop was held in 1988 on magnetic suspension systems.[12]) This multiple use application is the focus of this paper. We have two goals in this paper: to provide a review of the relevant literature, and to present analysis of the exact, nonlinear equations of motion for a gyrostat-based model of a spacecraft using flywheels for integrated energy storage and attitude control.

We first describe the basic concept of integrated power and attitude control, commonly referred to as IPACS,[13] including a detailed literature review. Most investigations of IPACS focus on general design issues; where attitude control results are provided, these are usually based on linearized equations of motion. Herein the "exact" nonlinear equations of motion are used to develop a particular class of nonlinear control laws. A decomposition of the N-dimensional space of the control torques is particularly useful. The control is an open-loop maneuver to control the rotors so that the trajectory remains near the set of equilibria where the spacecraft platform has zero angular velocity.[14,15] The result is a large-angle maneuver that maintains small angular velocity while simultaneously allowing energy storage and extraction.

2 Literature Review

The Integrated Power and Attitude Control System (IPACS) concept has been studied since the 1960s, but was particularly popular during the 1980s. Most spacecraft use chemical batteries (NiCd, NiH_2, usually) to store excess energy generated by the solar panels during periods of exposure to the sun.[16] During eclipse, the batteries are used to provide power for the spacecraft subsystems. The batteries are recharged when the spacecraft exits the eclipse. The primary problem with this approach is the cycle life of batteries and the additional power system mass required to control the charge and discharge cycles.

The use of flywheels instead of batteries to store energy on spacecraft was suggested as early as 1961 (Roes[17]), when a 17 W hr/kg composite flywheel spinning at 10 to 20 thousand RPM on magnetic bearings was proposed. The configuration included two counter-rotating flywheels, and the author did not mention the possibility of using the momentum for attitude control. However, since many spacecraft use flywheels (momentum wheels, control moment gyros, *etc.*) to control attitude, the integration of these two functions is naturally of interest. Numerous studies of this integration have been conducted.

Adams[18] investigated the use of a particular class of flexible filament flywheels, focusing on energy and momentum storage capabilities. Anderson and Keckler[19] originated the term "IPACS" in 1973, and Rockwell studied the use of an integrated system (Cormack[20]). Keckler and Jacobs[21] presented a description of the concept. Will *et al.*[22] investigated the IPACS concept and performed simulations of (linearized) equations of motion. Notti *et al.*[13,23] performed an extensive systems study and investigated linear control laws for the attitude control. Their study included trade studies on the use of momentum wheels, control moment gyros, and counter-rotating pairs. Rodriguez *et al.*[24] also performed a system study for NASA. Gross[25] conducted a study for Boeing, and summarized his findings in a workshop presentation.[11] Anand *et al.*[26,27] discussed the system design issues associated with using magnetic bearings, as did Downer *et al.*[28] Flatley studied a tetrahedron array[29] of four momentum wheels, and considered the issues associated with simultaneously torquing the wheels for attitude control and for energy storage.

Odea *et al.*[30] included simultaneous attitude determination in their study of a

Combined Attitude, Reference, and Energy Storage (CARES) system, focusing on technology-related issues with the system design. Oglevie and Eisenhaure[31,32] performed a system level study of IPACS systems, and Ref.[31] included a substantial list of references to earlier work. Olmsted[33] presented the technology-related issues associated with a particular flywheel design. Studer and Rodriguez[34] discussed the criteria associated with optimal design of an integrated Attitude Control and Energy Storage (ACES) system. Van Tassel and Simon[35] and Olszewski[36,37] described the key technologies involved in IPACS. Rockwell conducted a system study for NASA (Santo *et al.*[38]), focusing on space station applications.

The IPACS concept has been studied extensively over the past couple of decades. The enabling technologies involved are primarily composite materials and magnetic bearings. In the 1990s it appears that these technologies have reached the maturity required to allow the on-orbit evaluation of IPACS systems.

3 Gyrostat Model

The model we study is shown in Fig. 1, consisting of a rigid body $\mathcal{B}$ containing N "batteries" denoted $\mathcal{B}_j, j = 1 \ldots N$. Each battery contains one or two rigid, axisymmetric rotors, or flywheels, with spin axis $\vec{\mathbf{a}}_j$ in the body frame (not necessarily fixed). The reason for allowing each battery to contain either one or two flywheels is to allow for the consideration of two distinct configurations. In one configuration, each battery consists of a pair of counter-rotating flywheels spinning about the same axis. Of course, in this configuration, some differential speed control must be provided if attitude control is to be available. In the second configuration, a redundant set of non-collinear flywheels is used. In this configuration, the axial torques must be coordinated to provide both power and attitude control. The latter approach is taken in this paper.

The reference axes $\vec{\mathbf{b}}_i$ are fixed in the body, but are not necessarily the system

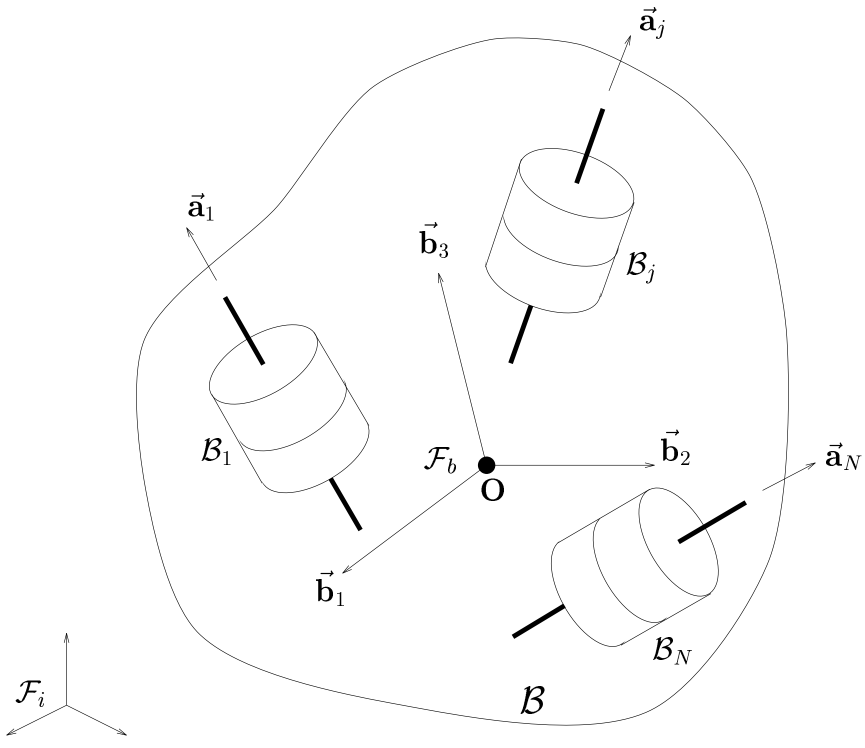

Figure 1: Model N-Battery Gyrostat.

principal axes. In case the flywheel axes $\vec{\mathbf{a}}_j$ are fixed in the body frame, the model is a gyrostat,[39] and it is possible to develop understanding of the nonlinear dynamics to a significant degree (see, for example, Refs.[14,15] and references cited there). This is an excellent class of models for studying the motion of spacecraft with reaction wheels or momentum wheels, where deviation of the flywheel axes is normally assumed negligible. For control moment gyros, or IPACS using gimballed magnetic bearings, the gyrostat model is not appropriate.

The rotational equations of motion for a gyrostat with N flywheel batteries may be expressed as

$$\dot{\mathbf{h}} = \mathbf{h}^{\times}\mathbf{J}^{-1}(\mathbf{h} - \mathbf{A}\mathbf{h}_a) + \mathbf{g}_e \tag{1}$$

$$\dot{\mathbf{h}}_a = \mathbf{g}_a \tag{2}$$

where $\mathbf{h}$ is the system angular momentum, $\mathbf{h}_a$ is the $N \times 1$ matrix of the axial angular momenta of the batteries, $\mathbf{g}_e$ is the 3×1 matrix of external torques, $\mathbf{g}_a$ is the $N \times 1$ matrix of the internal axial torques applied by the platform to the batteries, $\mathbf{A}$ is the $3 \times N$ matrix containing the axial vectors of the batteries, and $\mathbf{J}$ is an inertia-like matrix defined as

$$\mathbf{J} = \mathbf{I} - \mathbf{A}\mathbf{I}_s\mathbf{A}^T \tag{3}$$

Here $\mathbf{I}_s = \text{diag}\{I_{s1}, \cdots, I_{sN}\}$ is an $N \times N$ matrix with the axial moments of inertia of the batteries on the diagonal. The angular velocity may be written as

$$\boldsymbol{\omega} = \mathbf{J}^{-1}(\mathbf{h} - \mathbf{A}\mathbf{h}_a) \tag{4}$$

Observe that $\boldsymbol{\omega}$ may also be written as ∇H, where the ∇ is with respect to $\mathbf{h}$, and H is a Hamiltonian function. Thus Eq. (1) is in noncanonical Hamiltonian form.[40,41] This formulation is especially useful for identifying relative equilibrium motions and for characterizing their stability.

4 Internal Torque Decomposition

Here we develop a decoupling of the axial torques based on the decomposition of $\mathbf{A}$ into its row and null spaces. We use the singular value decomposition (svd) of $\mathbf{A}$, given by

$$\mathbf{A} = \mathbf{U}_A\mathbf{S}_A\mathbf{V}_A^T \tag{5}$$

where $\mathbf{U}_A \in \mathbf{R}^{3\times 3}$ and $\mathbf{V}_A \in \mathbf{R}^{N\times N}$ are orthonormal matrices, and $\mathbf{S}_A$ is a $3 \times N$ matrix with the singular values of $\mathbf{A}$ on the diagonal.[42] It is common to write $\mathbf{S} = [\mathbf{S}_1 \,|\, \mathbf{0}]$, and $\mathbf{V}_A = [\mathbf{V}_1 \,|\, \mathbf{V}_2]$, where $\mathbf{S}_1 = \text{diag}\,\{s_1, s_2, s_3\}$, $\mathbf{V}_1 \in \mathbf{R}^{N\times 3}$ and $\mathbf{V}_2 \in \mathbf{R}^{N\times N-3}$. The s_i are the singular values of $\mathbf{A}$, and $\mathbf{V}_1$ and $\mathbf{V}_2$ are orthonormal bases for the row and null spaces of $\mathbf{A}$, respectively.

Now, we note that the axial angular momenta of the rotors, $\mathbf{h}_a$, can be written as

$$\mathbf{h}_a = \mathbf{h}_a^r + \mathbf{h}_a^n \tag{6}$$

where the superscripts r and n denote row and null, respectively. Note that, since $\mathbf{A}\mathbf{h}_a^n = \mathbf{0}$, $\mathbf{h}_a^n$ contributes nothing to the dynamics described by Eqs. (1). Thus it is natural to use $\mathbf{h}_a^r$ for attitude control and $\mathbf{h}_a^n$ for energy storage.

We let $\boldsymbol{\eta}_a = \mathbf{V}_1^T\mathbf{h}_a$ and $\dot{\boldsymbol{\eta}}_a = \boldsymbol{\gamma}_a = \mathbf{V}_1^T\mathbf{g}_a$ represent the three-dimensional rotor momenta and torques associated with attitude control, and let $\boldsymbol{\mu}_a = \mathbf{V}_2^T\mathbf{h}_a$ and $\dot{\boldsymbol{\mu}}_a = \boldsymbol{\pi}_a = \mathbf{V}_2^T\mathbf{g}_a$ represent the components associated with energy storage. In this way, the axial torques $\mathbf{V}_1\boldsymbol{\gamma}_a$ and $\mathbf{V}_2\boldsymbol{\mu}_a$ independently control the attitude and energy storage.

5 Rotational Maneuvers

In this section, we develop a generalization of the stationary-platform maneuvers developed in Ref.,[15] making use of the decomposition developed in the previous section. For gyrostats with 3 or more rotors, any orientation change is possible, provided that the wheels' saturation speeds are sufficiently large and that the wheels are not coplanar. In order to perform simultaneous energy storage and attitude control, 4 or more rotors are required.

The basic idea is to control the rotors such that the angular momentum remains near the set of equilibria for which the platform is stationary. That is, $\boldsymbol{\omega} = \mathbf{0}$, and since $\|\mathbf{h}\| = 1$, this leads to the following stationary-platform conditions:

$$\mathbf{h} = \mathbf{A}\mathbf{h}_a \tag{7}$$

$$\mathbf{h}_a^T\mathbf{A}^T\mathbf{A}\mathbf{h}_a = 1 \tag{8}$$

The second condition defines an ellipsoid in a three-dimensional subspace of $\mathbf{R}^N$.

For stationary-platform maneuvers, we assume initial conditions on $\mathbf{h}$ and $\mathbf{h}_a$ that satisfy Eqs. (7–8). The initial and desired final stationary-platform equilibria define two points on the ellipsoid defined by Eq. (8). The spinup torques $\mathbf{g}_a$ are chosen such that the condition on $\mathbf{h}_a$ is satisfied throughout the maneuver. Differentiation of Eq. (8) with respect to time shows that any $\mathbf{g}_a$ which is orthogonal to $\mathbf{A}^T\mathbf{A}\mathbf{h}_a$ will yield such a maneuver. Thus the torque vector $\mathbf{g}_a$ must lie in the tangent space of the ellipsoid. For the $N = 2$ case, the ellipsoid is a simple ellipse, and the trajectory in $\mathbf{h}_a$ space simply traces the ellipse.[14] For the $N = 3$ case, there are infinitely many choices for the trajectory, since there are infinitely many curves connecting any two points on the ellipsoid.[15] For $N \geq 4$, not only are there infinitely many choices, but there are also torques that lie in the null space of $\mathbf{A}$ and therefore do not affect the attitude motion at all. We are therefore interested in choosing the torques that satisfy the stationary-platform condition and are in the row space of $\mathbf{A}$.

The stationary-platform condition can be written in terms of $\boldsymbol{\eta}_a$ using the singular value decomposition of $\mathbf{A}^T\mathbf{A}$, which is similar to the svd of $\mathbf{A}$. Specifically, $\mathbf{A}^T\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$, with $\mathbf{U} = \mathbf{V} = \mathbf{V}_A$, and the singular values of $\mathbf{A}^T\mathbf{A}$ are the squares of the singular values of $\mathbf{A}$. Thus, Eq. (8) can be written as

$$\boldsymbol{\eta}_a{}^T\mathbf{S}_1\boldsymbol{\eta}_a = 1 \tag{9}$$

Now we can define a maneuver that begins at $\boldsymbol{\eta}_a(0) = \mathbf{V}_1^T\mathbf{h}_a(0)$, and ends at $\boldsymbol{\eta}_a(t_f) = \mathbf{V}_1^T\mathbf{h}_a(t_f)$. As in Ref.,[15] we choose the control torque to lie in the plane spanned by these two points. To this end, we define a rotation matrix, $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3]$, where

$$\begin{aligned} \mathbf{c}_1 &= \boldsymbol{\eta}_{ao}/\|\boldsymbol{\eta}_{ao}\| && (10)\\ \mathbf{c}_2 &= \mathbf{c}_3^{\times}\mathbf{c}_1 && (11)\\ \mathbf{c}_3 &= \boldsymbol{\eta}_{ao}^{\times}\boldsymbol{\eta}_{af}/\|\boldsymbol{\eta}_{ao}^{\times}\boldsymbol{\eta}_{af}\| && (12) \end{aligned}$$

Then let $\boldsymbol{\nu} = \mathbf{C}^T\boldsymbol{\eta}_a$ and Eq. (9) becomes

$$\boldsymbol{\nu}^T\mathbf{C}^T\mathbf{S}_1^2\mathbf{C}\boldsymbol{\nu} = 1 \tag{13}$$

and, since $\nu_3 = 0$ by choice, this can be written as

$$\begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = 1 \tag{14}$$

where the d_{ij} are the corresponding components of the symmetric matrix $\mathbf{C}^T\mathbf{S}_1^2\mathbf{C}$.

Recalling that averaging results of Ref.[14] are only valid if the momentum transfer torques are "small," we introduce a small parameter ϵ to scale the internal torques. For the trajectory to follow the ellipse defined by Eq. (14), it is sufficient to take

$$\begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \epsilon \begin{bmatrix} -d_{12} & -d_{22} \\ d_{12} & d_{12} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{15}$$

or

$$\dot{\boldsymbol{\nu}} = \epsilon \begin{bmatrix} -d_{12} & -d_{22} & 0 \\ d_{12} & d_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{\nu} = \epsilon \mathbf{E} \boldsymbol{\nu} \tag{16}$$

which leads to

$$\dot{\boldsymbol{\eta}}_a = \boldsymbol{\gamma}_a = \epsilon \mathbf{C}\mathbf{E}\mathbf{C}^T \boldsymbol{\eta}_a \tag{17}$$

or

$$\dot{\mathbf{h}}_a = \mathbf{g}_a = \epsilon \mathbf{V}_1 \mathbf{C}\mathbf{E}\mathbf{C}^T \mathbf{V}_1^T \mathbf{h}_a \tag{18}$$

This is a control law which yields a stationary-platform maneuver. Since $\mathbf{C}$, $\mathbf{E}$, and $\mathbf{V}_1$ are constant matrices depending only on $\mathbf{A}$ and the initial and final values of $\mathbf{h}_a$, this is a constant coefficient linear system of equations. Since $\mathbf{h}_a$ lies on the ellipsoid, it is evident that the torques are bounded, and are $\mathcal{O}(\epsilon)$. Equation (18) can be solved in closed form and is decoupled from the platform dynamics. Thus the stationary-platform maneuver is an easy-to-implement open-loop maneuver which is useful in three ways: the platform angular velocity is small throughout the maneuver, the motor torque is small throughout the maneuver, and the control is independent of the energy storage and extraction torques effected by the control $\dot{\boldsymbol{\mu}}_a = \boldsymbol{\pi}_a$.

6 Conclusions

A cluster of 4 or more high-speed flywheels can be used simultaneously to provide attitude control and energy storage. With four or more non-coplanar wheels, a natural decoupling is possible using the singular value decomposition. This decoupling allows the internal axial torques driving the rotors to perform attitude control and energy storage operations. The relative simplicity of the decomposition should make it possible to use this approach in implementing integrated power and attitude control systems.

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