AOE 5984: Homework Assignment 4
Due Wednesday October 15, 2003

This assignment involves implementation of a PID controller for a spacecraft which is unstable in pitch motion.

A spacecraft is in a 6700 km circular orbit about a spherical Earth, and its principal moments of inertia are $\mathbf{I} = \text{diag}\ \{40, 30, 50\}$ kg m^2. The nominal gravity gradient attitude is intended to be such that the body frame is aligned with the orbital frame; *i.e.*, $\mathcal{F}_b \equiv \mathcal{F}_o$. Note that this configuration is unstable.

Determine appropriate proportional, integral, and derivative gains for feedback control of the linear system such that $\omega_n = \omega_p$ (defined below), $\zeta = 0.7$, and $T = 10/(\zeta\omega_n)$. The frequency ω_p is defined to be the natural frequency of the open-loop pitch motion of a spacecraft in the same orbit, but with $\mathbf{I} = \text{diag}\ \{40, 50, 30\}$ kg m^2, which is the stable motion you simulated in Homework 2.

Recall that for the second-order differential equation

$$\ddot{x} + k_o x = -k_p x - k_I \int_{0^-}^{t} x(\tau)\, d\tau - k_d \dot{x}$$

the closed-loop characteristic equation is

$$s^3 + k_d s^2 + k_p s + k_I = 0$$

and that we want this polynomial to factor as

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)(s + 1/T) = 0$$

where ω_n is the natural frequency, ζ is the damping ratio, and T is the integral time constant. Relating the gains to these parameters leads to

$$\begin{aligned} k_p &= \omega_n^2 + 2\zeta\omega_n/T \\ k_I &= \omega_n^2/T \\ k_d &= 2\zeta\omega_n + 1/T \end{aligned}$$

Use these gains in the following simulations:

1. linear pitch equation, 5° initial condition error, no disturbance
2. linear pitch equation, 135° initial condition error, 1 N m disturbance torque
3. nonlinear pitch equation, 5° initial condition error, no disturbance
4. nonlinear pitch equation, 135° initial condition error, 1 N m disturbance torque
5. nonlinear equations, 5° initial condition error, no disturbance
6. nonlinear equations, 135° initial condition error, 1 N m disturbance torque

Provide appropriate graphs to discuss the similarities and differences between the results for the different cases.