Mission Analysis for Attitude Dynamics and Control

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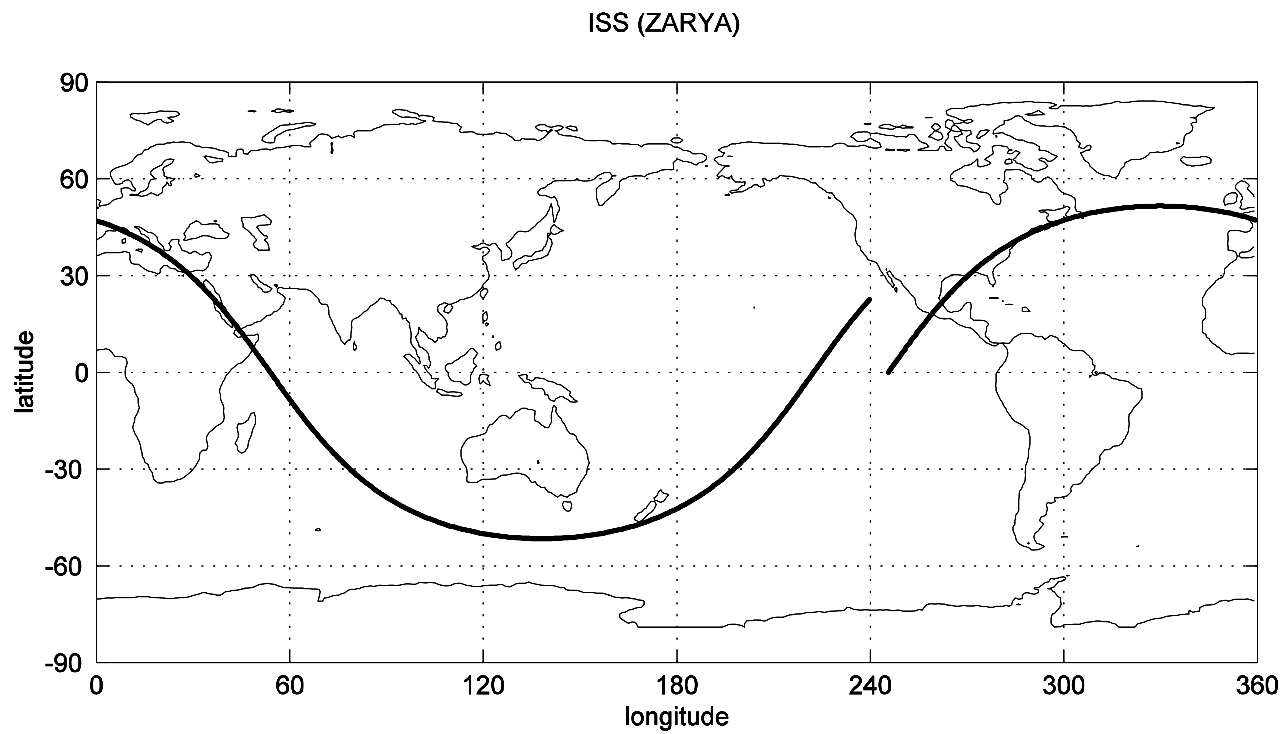
Sub-Satellite Point, Ground Track

- As satellite orbits the Earth, the sub-satellite point (SSP) traces a ground track

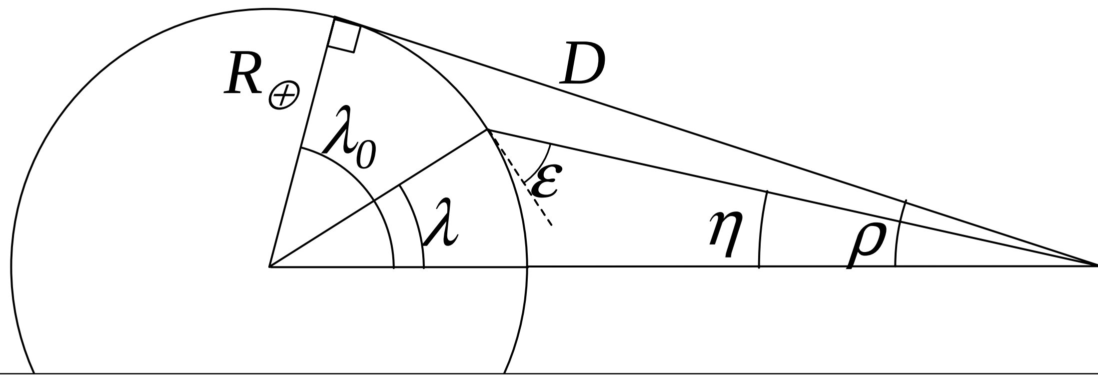


Algorithm for SSP, Ground Track

- Compute position vector in ECI
- Determine Greenwich Sidereal Time θ_g at epoch, θ_{g0}
- Latitude is $\delta_s = \sin^{-1}(r_3/r)$
- Longitude is $L_s = \tan^{-1}(r_2/r_1) - \theta_{g0}$

- Propagate position vector in "the usual way"
- Propagate GST using $\theta_g = \theta_{g0} + \omega_\oplus(t-t_0)$
  where $\omega_\oplus$ is the angular velocity of the Earth
- Notes:
  http://www.aoe.vt.edu/~chall/courses/aoe4134/sidereal.pdf
  http://aa.usno.navy.mil/data/docs/WebMICA_2.html
  http://tycho.usno.navy.mil/sidereal.html

Geometry of Earth-Viewing



- Given altitude H, we can state
  $\sin\rho = \cos\lambda_0 = R_\oplus / (R_\oplus + H)$
  $\rho + \lambda_0 = 90°$
- For a target with known position vector, λ is easily computed
  $\cos\lambda = \cos\delta_s \cos\delta_t \cos\Delta L + \sin\delta_s \sin\delta_t$
- Then $\tan\eta = \sin\rho \sin\lambda / (1 - \sin\rho \cos\lambda)$
- And $\eta + \lambda + \varepsilon = 90°$ and $D = R_\oplus \sin\lambda / \sin\eta$

Error Sources

Table 2.1: Sources of Pointing and Mapping Errors[2]

Spacecraft Position Errors		
ΔI	In- or along-track	Displacement along the spacecraft's velocity vector
ΔC	Cross-track	Displacement normal to the spacecraft's orbit plane
ΔR_S	Radial	Displacement toward the center of the Earth (nadir)
Sensing Axis Orientation Errors (in polar coordinates about nadir)		
$\Delta\eta$	Elevation	Error in angle from nadir to sensing axis
$\Delta\phi$	Azimuth	Error in rotation of the sensing axis about nadir
Other Errors		
ΔR_T	Target altitude	Uncertainty i the altitude of the observed object
ΔT	Clock error	Uncertainty in the real observation time

Error Budgets

Table 2.2: Pointing and Mapping Error Formulas[2]

Source	Magnitude	Magnitude of Mapping Error (km)	Magnitude of Pointing Error (rad)	Direction of Error
Attitude Errors:				
Azimuth	$\Delta\phi$ (rad)	$\Delta\phi D \sin\eta$	$\Delta\phi \sin\eta$	Azimuthal
Nadir Angle	$\Delta\eta$ (rad)	$\Delta\eta D / \sin\varepsilon$	$\Delta\eta$	Toward nadir
Position Errors:				
In-track	ΔI (km)	$\Delta I (R_T / R_S) \cos H$	$(\Delta I / D) \sin Y_I$	Parallel to ground track
Cross-track	ΔC (km)	$\Delta C (R_T / R_S) \cos G$	$(\Delta C / D) \sin Y_C$	Perpendicular to ground track
Radial	ΔR_S (km)	$\Delta R_S \sin\eta / \sin\varepsilon$	$(\Delta R_S / D) \sin\eta$	Toward nadir
Other Errors:				
Target altitude	ΔR_T (km)	$\Delta R_T / \tan\varepsilon$	—	Toward nadir
S/C Clock	ΔT (s)	$\Delta T V_e \cos\ \text{lat}$	$\Delta T (V_e / D) \cos\ \text{lat} \sin J$	Parallel to Earth's equator

$\sin H = \sin\lambda \sin\phi$

$\sin G = \sin\lambda \cos\phi$

$V_e = 464$ m/s (Earth rotation velocity at equator)

$\cos Y_I = \cos\phi \sin\eta$

$\cos Y_C = \sin\phi \sin\eta$

$\cos J = \cos\phi_E \cos\varepsilon$, where ϕ_E = azimuth relative to East