

# Chapter 1

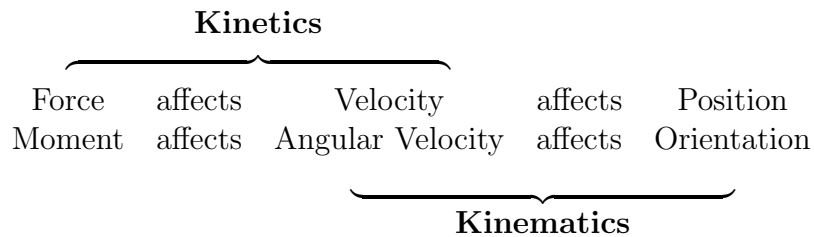
## Introduction

Spacecraft dynamics and control is a rich subject involving a variety of topics from mechanics and control theory. In a first course in dynamics, students learn that the motion of a rigid body can be divided into two types of motion: *translational* and *rotational*. For example, the motion of a thrown ball can be studied as the combined motion of the mass center of the ball in a parabolic trajectory and the spinning motion of the ball rotating about its mass center. Thus a first approximation at describing the motion of a ball might be to model the ball as a point mass and ignore the rotational motion. While this assumption may give a reasonable approximation of the motion, in actuality the rotational motion and translational motion are coupled and must be studied together to obtain an accurate picture of the motion. The motion of the mass center of a curve ball is an excellent example of how the rotational and translational motions are coupled.<sup>1</sup> The ball does not follow the parabolic trajectory predicted by analysis of the translational motion, because of the unbalanced forces and moments on the spinning ball.

The study of spacecraft dynamics is similar to the study of baseball dynamics. One first gains an understanding of the translational motion of the mass center using particle dynamics techniques, then the rotational motion is studied. Thus the usual study of spacecraft dynamics begins with a course in orbital dynamics, usually in the junior or senior year, and is followed by a course in attitude dynamics in the senior year or the first year of graduate study. In this book I assume that the student has had a semester of orbital dynamics, but in fact I only use circular orbits, so that a student with some appreciation for dynamics should be able to follow the development without the orbital dynamics background. The basics of orbital dynamics are included in Appendix A, and the text by Bate, Mueller, and White<sup>2</sup> is an affordable introduction to the required material.

Another important way to decompose dynamics problems is into *kinematics* and *kinetics*. For translational motion, kinematics is the study of the change in position for a given velocity, whereas kinetics is the study of how forces cause changes in velocity. For rotational motion, kinematics is the study of the change in orientation

for a given angular velocity, and kinetics is the study of how moments cause changes in the angular velocity. Translational kinematics is relatively easy to learn, since it only involves the motion of a point in three-dimensional space. Rotational kinematics, however, is usually more difficult to master, since it involves the orientation of a reference frame in three-dimensional space.



In this chapter, I begin by describing how attitude dynamics and control arises in the operation of spacecraft. This discussion is followed by a description of the fundamental attitude control concepts that are in widespread use. Finally, I give an overview of the textbook.

## 1.1 Attitude dynamics and control in operations

Essentially all spacecraft include one or more subsystems intended to interact with or observe other objects. Typically there is one primary subsystem that is known as the payload. For example, the primary mirror on the Hubble Space Telescope is one of many instruments that are used to observe astronomical objects. The communications system on an Intelsat satellite is its payload, and the infrared sensor on board a Defense Support System (DSP) satellite is its payload. In each case, the payload must be pointed at its intended subject with some accuracy specified by the “customer” who purchased the spacecraft. This accuracy is typically specified as an angular quantity; e.g., 1 degree, 10 arcseconds, or 1 milliradian. The attitude control system designer must design the attitude determination and control subsystem (ADCS) so that it can meet the specified accuracy requirements.

It costs more than \$10,000 to put a kilogram of mass into low-Earth orbit (LEO), and even more to put it into geostationary orbit (GEO).<sup>3</sup> A typical spacecraft masses about 500 kg, and costs tens to several hundred millions of dollars to design, manufacture, test, and prepare for launch. All of this money is spent to purchase the mission capability of the spacecraft. The attitude control system, propulsion system, launch vehicle, and so forth, are only there so that the mission may be performed effectively. If the mission can be accomplished without an ADCS, then that mass can be used to increase the size of the payload, decrease the cost of launch, or in some other way improve the performance or reduce the cost. The bottom line is that the payload and its operation are the *raison d’être* for the spacecraft. This focus justifies

our spending a little time on describing how the ADCS fits into the operations of the payload.

There are many spacecraft payloads, but most fit into one of just two categories: communications and remote sensing. On a communications satellite, the payload comprises the radio transceivers, multiplexers, and antennas that provide the communications capability. Historically, most communications satellites have been in the geostationary belt, and have been either dual-spin or three-axis stabilized. More recently, a host of LEO commsats has been put into orbit, including the 66-satellite Iridium constellation, the 36-satellite OrbComm constellation, and the planned 48-satellite GlobalStar constellation. The Iridia use hydrazine propulsion for three-axis stabilization, whereas the OrbComms are gravity-gradient stabilized. The GlobalStar spacecraft are three-axis stabilized, using momentum wheels, magnetic torquers, and thrusters. In commsats, the mission of the ADCS is to keep the spacecraft pointed accurately at the appropriate ground station. The more accurate the ADCS, the more tightly focused the radio beam can be, and the smaller the power requirements will be. However, ADCS accuracy carries a large price tag itself, so that design trades are necessary.

There are two basic types of remote sensing satellites: Earth-observing and space-observing. An Earth-observing spacecraft could be nadir-pointing, it could be scanning the land or sea in its instantaneous access area, or it could selectively point to and track specific ground targets. In the first case, a passive gravity-gradient stability approach might suffice, whereas in the second and third cases, an active ADCS would likely be required, using some combination of momentum wheels, magnetic torquer rods, or thrusters.

A space-observing system could simply point away from the Earth, with an additional requirement to avoid pointing at the sun. This “anti-Earth” pointing is essentially the ADCS requirement for the CATSAT mission being built by the University of New Hampshire\*. The CATSAT ADCS uses momentum wheels and magnetic torquer rods. More complicated space-observing systems require both large-angle slewing capability and highly accurate pointing control. The Hubble Space Telescope is a well-known example. It uses momentum wheels for attitude control, and performs large-angle maneuvers at about the same rate as the minute-hand of a clock. The HST does not use thrusters because the plume would contaminate the sensitive instruments.

One problem with momentum wheels, reaction wheels, and control moment gyros is *momentum buildup*: external torques such as the gravity gradient torque and solar radiation pressure will eventually cause the wheel to reach its maximum speed, or the CMG to reach its maximum gimbal angle. Before this saturation occurs, the spacecraft must perform an operation called *momentum unload*, or *momentum dump*: external torques are applied, using thrusters or magnetic torquer rods, that cause the

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\*Cooperative Astrophysics and Technology SATellite. See Refs. 4 and 5, and the website <http://www.catsat.sr.unh.edu/>.

ADCS to decrease the wheels' speeds or the CMGs' gimbal angles. Depending on the spacecraft, this type of "maneuver" may be performed as often as once per orbit.

Another ADCS operation involves keeping the spacecraft's solar panels pointing at the sun. For example, when the HST is pointing at a particular target, it still has a degree of freedom allowing it to rotate about the telescope axis. This rotation can be used to orient the solar panel axis so that it is perpendicular to the direction to the sun. Then the panels are rotated about the panel axis so that they are perpendicular to the sun direction. This maneuver is known as *yaw steering*.

## 1.2 Overview of attitude dynamics concepts

The attitude of a spacecraft, i.e., its orientation in space, is an important concept in spacecraft dynamics and control. Attitude motion is approximately decoupled from orbital motion, so that the two subjects are typically treated separately. More precisely, the orbital motion does have a significant effect on the attitude motion, but the attitude motion has a less significant effect on the orbital motion. For this reason orbital dynamics is normally covered first, and is a prerequisite topic for attitude dynamics. In a third course in spacecraft dynamics, the coupling between attitude and orbital motion may be examined more closely. In this course, we focus on the attitude motion of spacecraft in circular orbits, with a brief discussion of the attitude motion of simple spacecraft in elliptic orbits.

Operationally, the most important aspects of attitude dynamics are attitude determination, and attitude control. The reason for formulating and studying the dynamics problem is so that these operational tasks can be performed accurately and efficiently. Attitude determination, like orbit determination, involves processing observations ("obs") to obtain parameters for describing the motion. As developed in Appendix A, we can determine the orbit of the satellite if we have the range ( $\rho$ ), range rate ( $\dot{\rho}$ ), azimuth ( $Az$ ), azimuth rate ( $\dot{Az}$ ), elevation ( $El$ ), and elevation rate ( $\dot{El}$ ) from a known site on the Earth. The deterministic algorithm to compute the six orbital elements from these six measurements is well-known and can be found in most astrodynamics texts. Of course, due to measurement noise, the use of only six measurements is not practical, and statistical methods are normally used, incorporating a large number of observations.

Similarly, we can determine the attitude, which can be described by three parameters such as Euler angles, by measuring the directions from the spacecraft to some known points of interest. For example, suppose a spacecraft has a Sun sensor and an Earth sensor. The two sensors provide vector measurements of the direction from the spacecraft to the sun ( $\vec{v}_s$ ) and to the Earth ( $\vec{v}_e$ ). These are normally unit vectors, so each measurement provides two pieces of information. Thus the two measurements provide four known quantities, and since it only takes three variables to describe attitude, the problem is overdetermined, and statistical methods are required (such as least squares). Actually there is a deterministic method that discards some of the

Table 1.1: Attitude Control Concepts

Concept	Passive/ Active	Internal/ External	Environmental
Gravity Gradient	P	E	Y
Spin Stabilization	A	I	N
Dual-Spin	A	I	N
Momentum Wheels	A	I	N
Control Moment	A	I	N
Gyros			
Magnetic	A	E	Y
Torquer Rods			
Thrusters	A	E	N
Dampers	P or A	I	N

measurements and we develop it in Chapter 4. A wide variety of attitude determination hardware is in use. The handbook edited by Wertz (Ref. 6) provides a wealth of information on the subject. The more recent text by Sidi (Ref. 7) is notable for its appendices on hardware specifications.

Controlling the attitude of a spacecraft is also accomplished using a wide variety of hardware and techniques. The choice of which to use depends on the requirements for pointing accuracy, pointing stability, and maneuverability, as well as on other mission requirements such as cost and lifetime. All attitude control concepts involve the application of torques or moments to the spacecraft. The various methods can be grouped according to whether these torques are passive or active, internal or external, and whether the torques are environmental or not. A reasonably complete list of concepts is shown in Table 1.1.

The most fundamental idea in the study of attitude motion is the *reference frame*. Throughout the book we work with several different reference frames, and you must become familiar and comfortable with the basic concept. As a preview, let us consider an example where four different reference frames are used. In Fig. 1.1, we show three reference frames useful in describing the motion of a spacecraft in an equatorial orbit about the Earth. One of the reference frames whose origin is at the center of the Earth,  $\mathbf{O}$ , is an inertial reference frame with unit vectors  $\hat{\mathbf{I}}$ ,  $\hat{\mathbf{J}}$ , and  $\hat{\mathbf{K}}$ . This frame is usually referred to as the *Earth-centered inertial* (ECI) frame. The  $\hat{\mathbf{I}}$  axis points in the direction of the vernal equinox, and the  $\hat{\mathbf{I}} \hat{\mathbf{J}}$  plane defines the equatorial plane. Thus the  $\hat{\mathbf{K}}$  axis is the Earth's rotation axis, and the Earth spins about  $\hat{\mathbf{K}}$  with angular velocity  $\omega_{\oplus} = 2\pi$  radians per sidereal day. In vector form, the angular velocity of the Earth is  $\vec{\omega}_{\oplus} = \omega_{\oplus} \hat{\mathbf{K}}$ .

The other frame centered in the Earth is an *Earth-centered, Earth-fixed* (ECEF)

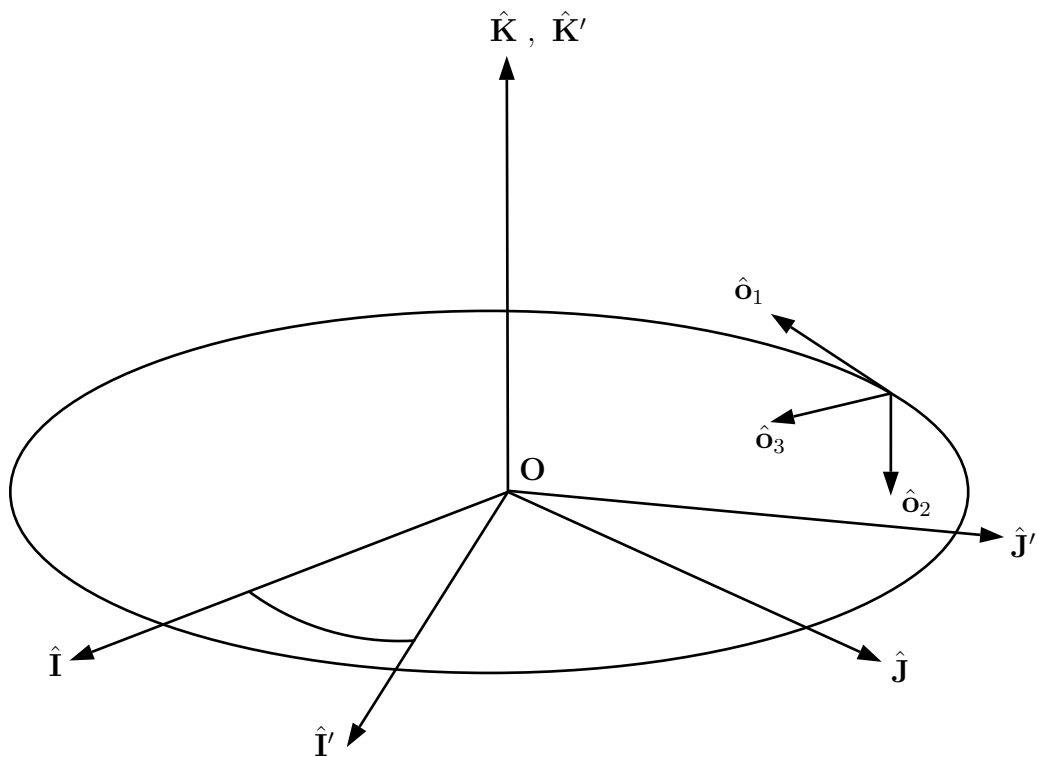


Figure 1.1: Earth-Centered Inertial (ECI =  $\{\hat{\mathbf{I}}, \hat{\mathbf{J}}, \hat{\mathbf{K}}\}$ ), Earth-Centered Earth-Fixed (ECEF =  $\{\hat{\mathbf{I}}', \hat{\mathbf{J}}', \hat{\mathbf{K}}'\}$ ), and Orbital Reference Frames ( $\mathcal{F}_o = \{\hat{\mathbf{o}}_1, \hat{\mathbf{o}}_2, \hat{\mathbf{o}}_3\}$ ) for an Equatorial Orbit

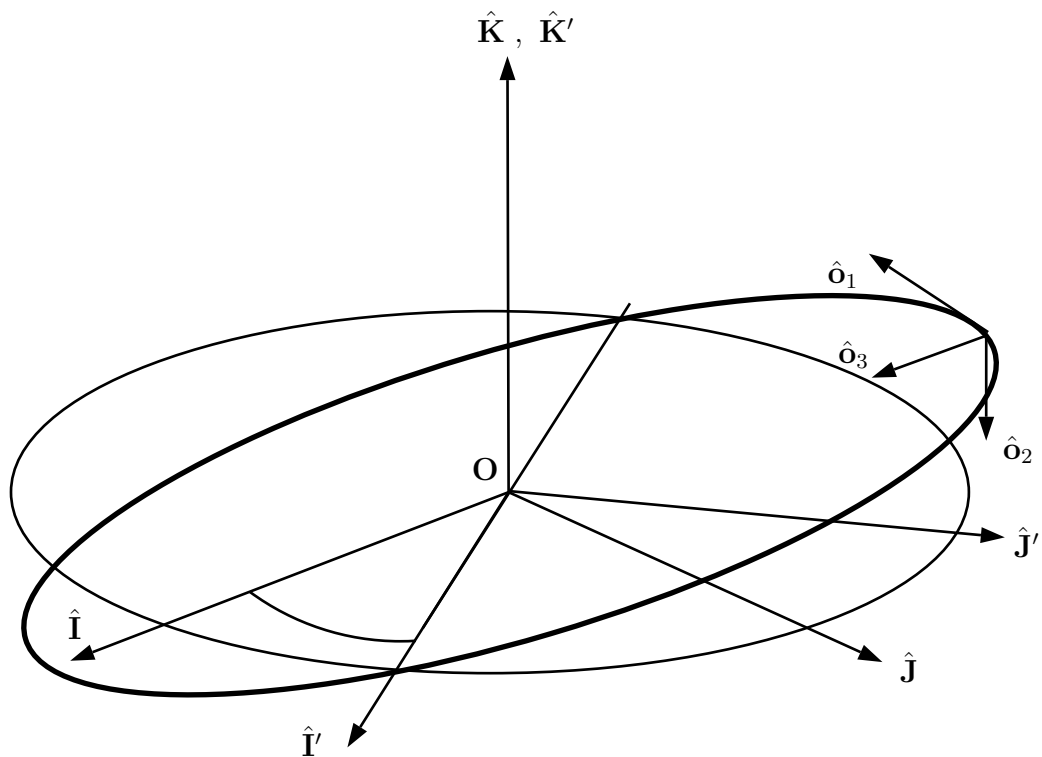


Figure 1.2: Earth-Centered Inertial, Earth-Centered Earth-Fixed, and Orbital Reference Frames for an Inclined Orbit

frame which rotates with respect to the ECI frame with angular velocity  $\vec{\omega}_{\oplus} = \omega_{\oplus} \hat{\mathbf{K}}$ . This frame has unit vectors represented by  $\hat{\mathbf{I}}'$ ,  $\hat{\mathbf{J}}'$ , and  $\hat{\mathbf{K}}'$ . Note that  $\hat{\mathbf{K}}$  is the same in both the ECI and ECEF frames. The importance of the ECEF frame is that points on the surface of the Earth, such as ground stations and observation targets, are fixed in this frame. However, since the frame is rotating, it is not an inertial reference frame.

The other frame in the figure has its origin at the mass center of the spacecraft. This point is assumed to be in an orbit (circular or elliptical) about the Earth, thus its motion is given. As drawn in the figure, this orbit is also an equatorial orbit, so that the orbit normal is in the  $\hat{\mathbf{K}}$  direction. The origin of this frame is accelerating and so it is not inertial. This frame is called the *orbital frame* because its motion depends only on the orbit. The unit vectors of the orbital frame are denoted  $\hat{\mathbf{o}}_1$ ,  $\hat{\mathbf{o}}_2$ , and  $\hat{\mathbf{o}}_3$ . The direction pointing from the spacecraft to the Earth is denoted by  $\hat{\mathbf{o}}_3$ , and the direction opposite to the orbit normal is  $\hat{\mathbf{o}}_2$ . The remaining direction,  $\hat{\mathbf{o}}_1$  is defined by  $\hat{\mathbf{o}}_1 = \hat{\mathbf{o}}_2 \times \hat{\mathbf{o}}_3$ . In the case of a circular orbit,  $\hat{\mathbf{o}}_1$  is in the direction of the spacecraft velocity vector. For those familiar with aircraft attitude dynamics, the three axes of the orbital frame correspond to the roll, pitch, and yaw axes, respectively. This reference frame is non-inertial because its origin is accelerating, and because the frame is rotating. The angular velocity of the orbital frame with respect to inertial space is  $\vec{\omega}_o = -\omega_o \hat{\mathbf{o}}_2$ . The magnitude of the orbital angular velocity is constant only if the orbit is circular, in which case  $\omega_o = \sqrt{\mu/r^3}$ , where  $\mu$  is the gravitational parameter, and  $r$  is the orbit radius (see Appendix A). If the orbit is not circular, then  $\omega_o$  varies with time. Note well that  $\vec{\omega}_o$  is the angular velocity of the orbital frame with respect to the inertial frame and is determined by the translational, or orbital, dynamics.

Another reference frame of interest is shown in Fig. 1.3 in relation to the orbital frame. This frame is the body-fixed frame, with basis vectors  $\hat{\mathbf{b}}_1$ ,  $\hat{\mathbf{b}}_2$ , and  $\hat{\mathbf{b}}_3$ . Its origin is at the spacecraft mass center, just as with the orbital frame. However, the spacecraft body, or platform, is in general not aligned with the orbital frame. *The relative orientation between these two reference frames is central to attitude determination, dynamics, and control.* The relative orientation between the body frame and the orbital frame is determined by the satellite's rotational dynamics, which is governed by the kinetic and kinematic equations of motion. The primary purpose of this text is to develop the theory and tools necessary to solve problems involving the motion of the body frame when the orbit is known.

### 1.3 Overview of the textbook

Most textbooks on this subject begin with some treatment of kinematics and then proceed to a study of a variety of dynamics problems, with some control problems perhaps included. Our approach is similar, but our aim is to spend more time up front, both in motivation of the topics, and in developing an understanding of how to describe and visualize attitude motion. To this beginning, we have an introductory

Figure 1.3: Orbital and Body Reference Frames

chapter on Space Mission Analysis that will hopefully help readers to develop an appreciation for how attitude dynamics fits into the overall space mission. For a more traditional course, this chapter could be read quickly or even skipped entirely.

Chapter 3 introduces attitude kinematics, developing the classical topics in some detail, and introducing some new topics that may be used in a second reading. Chapter 4 covers the important topic of attitude determination. This topic is not usually covered in an introductory course, but I believe that mastery of this subject enhances the student's appreciation for the remaining material. In Chapter 5 we develop the standard equations of motion and relevant results for rigid body dynamics. These topics lead directly to Chapter 6 on satellite attitude dynamics, where we apply basic dynamics principles to a variety of problems. In Chapter 7 we introduce and develop equations of motion for the gyroscopic instruments that are used as sensors in spacecraft attitude control systems. These are used in some simple examples, before proceeding to Chapter 8 where more rigorous development of attitude control problems is presented.

## 1.4 References and further reading

Dynamics and control of artificial spacecraft has been the subject of numerous texts and monographs since the beginning of the space age. Thomson's book,<sup>8</sup> originally published in 1961, is one of the earliest, and is currently available as a Dover reprint. The book remains a valuable reference, despite its age. Wertz's handbook<sup>6</sup> is perhaps the best reference available on the practical aspects of attitude determination and control. The text by Kaplan<sup>9</sup> treats a wide range of topics in both orbital and attitude dynamics. Kane, Likins, and Levinson<sup>10</sup> present a novel approach to satellite dynamics, using Kane's equations. Hughes' book<sup>11</sup> focuses on modeling and analysis of attitude dynamics problems, and is probably the best systematic and rigorous treatment of these problems. Rimrott's book<sup>12</sup> is similar to Hughes,<sup>11</sup> but uses scalar notation, perhaps making it more accessible to beginning students of the subject. Wiesel<sup>13</sup> covers rigid body dynamics, as well as orbital dynamics and basic rocket dynamics. Agrawal's book<sup>14</sup> is design-oriented, but includes both orbital and attitude dynamics. The brief book by Chobotov<sup>15</sup> covers many of the basics of attitude dynamics and control. Especially useful are the "Recommended Practices" given at the end of each chapter. The books by Griffin and French,<sup>16</sup> Fortescue and Stark,<sup>17</sup> Larson and Wertz,<sup>18</sup> and Pisacane and Moore<sup>19</sup> are all design-oriented, and so present useful information on the actual implementation of attitude determination and control systems, and the interaction of this subsystem with the overall spacecraft. The chapter on attitude in Pisacane and Moore gives an in-depth treatment of the fundamentals, whereas the relevant material in Larson and Wertz is more handbook-oriented, providing useful rules of thumb and simple formulas for sizing attitude determination and control systems. Bryson<sup>20</sup> treats a variety of spacecraft orbital and attitude control problems using the linear-quadratic regulator technique. Sidi<sup>7</sup> is a practice-oriented

text, providing many detailed numerical examples, as well as current information on relevant hardware. The new book by Wie<sup>21</sup> furnishes a modern treatment of attitude dynamics and control topics. Finally, an excellent source of information on specific spacecraft is the *Mission and Spacecraft Library*, located on the World Wide Web at <http://leonardo.jpl.nasa.gov/msl/home.html>.

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## 1.5 Exercises

1. What types of attitude control concepts are used by the following spacecraft? If you can, tell what types of sensors and actuators are used in each case.
  - (a) Explorer I
  - (b) Global Positioning System
  - (c) Hubble Space Telescope
  - (d) Intelsat IV
  - (e) Iridium
  - (f) OrbComm
  - (g) Starshine
  - (h) Cassini
  - (i) HokieSat

2. What companies manufacture the following attitude control actuators? List some of the performance characteristics for at least one specific component from each type of actuator.
  - (a) momentum wheels
  - (b) control moment gyros
  - (c) magnetic torquer rods
  - (d) damping mechanisms
  - (e) hydrazine thrusters
3. What companies manufacture the following attitude determination sensors? List some of the performance characteristics for at least one specific component from each type of sensor.
  - (a) Earth horizon sensors
  - (b) magnetometers
  - (c) rate gyros
  - (d) star trackers
  - (e) sun sensors
4. Which control strategies rely on naturally occurring fields, and what fields are they?
5. For a circular orbit, what are the directions of the position, velocity, and orbital angular momentum vectors in terms of the orbital frame's base vectors? Support your answer with a reasonably accurate sketch.
6. Repeat Exercise 5 for an elliptical orbit.
7. Make a sketch of the reference frames missing from Fig. 1.3.