Molniya Orbit Example using f and g

This is a two-line element set for the Molniya 1-91 satellite:

```
MOLNIYA 1-91
1 25485U 98054A   00300.78960173  .00000175  00000-0  40203-2 0  6131
2 25485  63.1706 206.3462 7044482 281.6461  12.9979  2.00579102 15222
```

Let's find the position and velocity at epoch and 30 minutes later.

The orbital elements are

$$\begin{aligned} a &= 26559 \text{ km} & e &= 0.704482 \\ i &= 63.1706^\circ & \Omega &= 206.346^\circ \\ \omega &= 281.646^\circ & \nu_0 &= 78.2265^\circ \end{aligned}$$

All but a and ν_0 are by inspection. The semimajor axis comes from the mean motion (2.00579102 revs/day), and ν_0 is obtained by first solving Kepler's equation for E (with $M = 12.9979^\circ$) and then calculating ν. We also need to know that $t - T = 1,555.244$ seconds (calculated from the definition of M_0).

The position and velocity vectors at epoch are

$$\begin{aligned} \vec{\mathbf{r}}_0 &= -10515.45\vec{\mathbf{I}} - 5235.37\vec{\mathbf{J}} + 49.17\vec{\mathbf{K}} \\ \vec{\mathbf{v}}_0 &= -2.10305\vec{\mathbf{I}} - 4.18146\vec{\mathbf{J}} + 5.563290\vec{\mathbf{K}} \end{aligned}$$

where units are in km and km/s.

Initially $f = 1$, $g = 0$, $\dot{f} = 0$, and $\dot{g} = 1$. Note that this satisfies the identity $1 = f\dot{g} - \dot{f}g$.

Now we want to calculate f and g and their derivatives at $t = t_0 + 1800$ seconds. The steps are:

1. Calculate M: $M = 28.04134^\circ$

2. Solve Kepler's equation for E: $E = 64.45895^\circ$

3. Calculate the desired expressions; $f = 0.757079$, $g = 1684.296$, $\dot{f} = -2.13588 \times 10^{-4}$, and $\dot{g} = 0.84569$.

and

4. Substitute them into the expressions for $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$

$$\begin{aligned} \vec{\mathbf{r}} &= -11503.18\vec{\mathbf{I}} - 11,006.39\vec{\mathbf{J}} + 9407.45\vec{\mathbf{K}} \\ \vec{\mathbf{v}} &= 0.467452\vec{\mathbf{I}} - 2.41800\vec{\mathbf{J}} + 4.69432\vec{\mathbf{K}} \end{aligned}$$