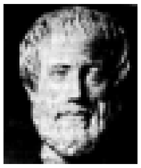
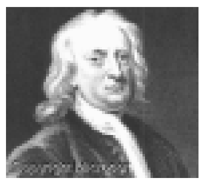
A Brief History of Orbital Mechanics



- Aristotle (384-322 BC)
- Ptolemy (87-150 AD)
- Nicolaus Copernicus (1473-1543)
- Tycho Brahe (1546-1601)
- Johannes Kepler (1571-1630)
- Galileo Galilei (1564-1642)
- Sir Isaac Newton (1643-1727)



Kepler's Laws

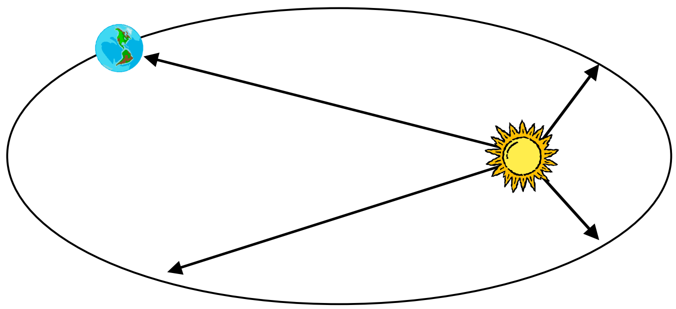
I. The orbit of each planet is an ellipse with the Sun at one focus.

II. The line joining the planet to the Sun sweeps out equal areas in equal times.

III. The square of the period of a planet is proportional to the cube of its mean distance to the sun.

Kepler's First Two Laws

I. The orbit of each planet is an ellipse with the Sun at one focus.



II. The line joining the planet to the Sun sweeps out equal areas in equal times.

Kepler's Third Law

III. The square of the period of a planet is proportional to the cube of its mean distance to the sun.

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Here T is the period, a is the semimajor axis of the ellipse, and m is the gravitational parameter (depends on mass of central body)

$$\mu_\oplus = GM_\oplus = 3.98601\times10^5\ \mathrm{km^3s^{-2}}$$

$$\mu_\mathrm{sun} = GM_\mathrm{sun} = 1.32715\times10^{11}\ \mathrm{km^3s^{-2}}$$

Earth Satellite Orbit Periods

Orbit	Altitude (km)	Period (min)
LEO	300	90.52
LEO	400	92.56
MEO	3000	150.64
GPS	20232	720
GEO	35786	1436.07

You should be able to do these calculations! Don't forget to add the radius of the Earth to the altitude to get the orbit radius.

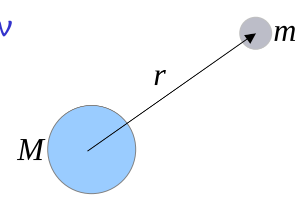
Newton's Laws

- Kepler's Laws were based on observation data: "curve fits"
- Newton established the theory
  - Universal Gravitational Law

$$F_g = -\frac{GMm}{r^2}$$

  - Second Law

$$\vec{\mathbf{F}} = m\ddot{\vec{\mathbf{r}}}$$



Universal Gravitational Constant

$$G = 6.672 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$$

Elliptical Orbits

AOE

- Planets, comets, and asteroids orbit the Sun in ellipses
- Moons orbit the planets in ellipses
- Artificial satellites orbit the Earth in ellipses
- To understand orbits, you need to understand ellipses (and other conic sections)
- But first, let's study circular orbits
  - a circle is a special case of an ellipse

Circular Orbits

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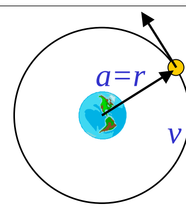
- The speed of a satellite in a circular orbit depends on the radius



$$v_c = \sqrt{\frac{\mu}{r}}$$

- If an orbiting object at a particular radius has a speed $< v_c$, then it is in an elliptical orbit with lower energy
- If an orbiting object at radius r has a speed $> v_c$, then it is in a higher-energy orbit which may be elliptical, parabolic, or hyperbolic

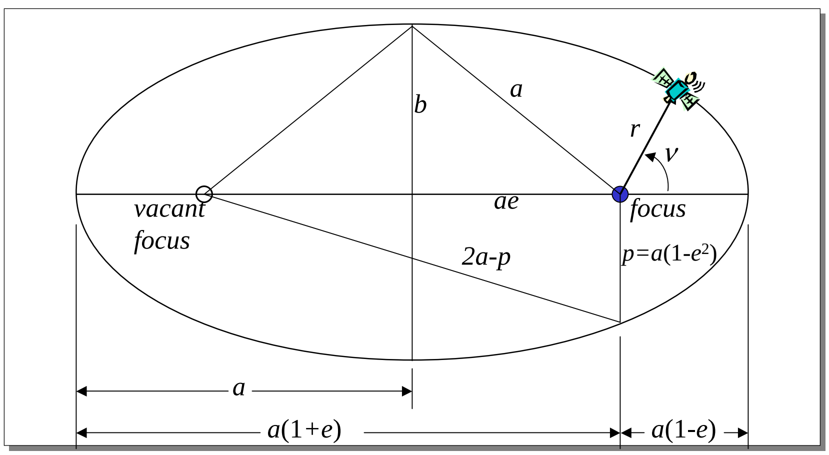
The Energy of an Orbit

- Orbital energy is the sum of the kinetic energy, $mv^2/2$, and the potential energy, $-\mu m/r$
- Customarily, we use the specific mechanical energy, E (i.e., the energy per unit mass of satellite)

$$E = \frac{v^2}{2} - \frac{\mu}{r} \quad \Leftrightarrow \quad E = \frac{-\mu}{2a}$$

- From this definition of energy, we can develop the following facts
  - $E<0$ ⇔ orbit is elliptical or circular
  - $E=0$ ⇔ orbit is parabolic
  - $E>0$ ⇔ orbit is hyperbolic

Properties of Ellipses



Facts About Elliptical Orbits

- Periapsis is the closest point of the orbit to the central body
  - $r_p = a(1-e)$
- Apoapsis is the farthest point of the orbit from the central body
  - $r_a = a(1+e)$
- Velocity at any point is
  - $v = (2E+2\mu/r)^{1/2}$
- Escape velocity at any point is
  - $v = (2\mu/r)^{1/2}$