Visibility Prediction Algorithm

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Given the orbital elements a, e, i, Ω, ω, and ν, for an Earth-orbiting satellite at epoch t_0, and the latitude L and longitude λ_E of a ground station, determine when the satellite will next be "visible" from the ground station. Here "visible" means that the Z component of the range vector $\vec{\boldsymbol{\rho}}$, expressed in SEZ coordinates, is positive; *i.e.*, $\rho_Z > 0$ is the stopping condition.

1. Determine the local sidereal time at epoch of the ground station, θ_0. This may involve determining the Greenwich sidereal time θ_{g0-} at a previous epoch t_{0-} and then computing $\theta = \theta_{g0-} + \omega_\oplus(t_0 - t_{0-}) + \lambda_E$, where $\omega_\oplus$ is the angular velocity of the Earth.
2. Select a time step, Δt. For most low-Earth orbit (LEO) spacecraft, $\Delta t = 60$ s is a reasonable time step. This means, however, that the final answer will only be accurate to within a minute. Of course, once this "coarse" solution is found, a more refined answer can be found using a smaller step size and restricting the calculations to the Δt s prior to the time found in the iteration.
3. Compute the time of periapsis passage, T. This value is needed in computing the mean anomaly and solving Kepler's equation.
4. Compute the rotation matrix $\mathbf{R}^{ip}$ that takes vectors from the perifocal frame (PQW) to the Earth-centered inertial frame (IJK).
5. Repeat the following steps until the stopping condition ($\rho_Z > 0$) is met[1]:
   (a) Update time: $t \leftarrow t + \Delta t$
   (b) Update sidereal time: $\theta \leftarrow \theta + \omega_\oplus \Delta t$
   (c) Update true anomaly: Solve Kepler's equation and compute the new true anomaly, ν.
   (d) Compute the position vector $\vec{\mathbf{r}}$ in the PQW frame. The only orbital element that changes is ν.
   (e) Rotate the position vector into the IJK frame. Depends on Ω, i, and ω, none of which change, so the rotation matrix $\mathbf{R}^{ip}$ is constant throughout the algorithm.
   (f) Rotate the position vector into the SEZ frame. Depends on L and θ. Since θ changes with time, this rotation matrix depends on time.
   (g) Compute ρ_Z as $\rho_Z = r_Z - R_\oplus$, where $R_\oplus$ is the radius of the Earth.
   (h) Check $\rho_Z > 0$. If it's positive, then you're done[2] and the time of next visibility is t.

[1] You may want to go through this iteration once with the initial conditions, *i.e.*, skipping steps (a) – (c), just to make sure that the spacecraft is not already visible from the site.

[2] If a greater accuracy is required, you can restart the algorithm with the values from the last iteration before $\rho_Z > 0$ was satisfied; *i.e.*, restart with $t - \Delta t$, $\theta - \omega_\oplus \Delta t$, $\nu(t - \Delta t)$, and using a smaller Δt. Another possible finesse is to develop this algorithm so that it returns ρ_Z as a function of t and then use the secant method to find a zero of this function. This will probably only work if the initial guess is sufficiently close to a solution.