ASSESSMENT OF LAUNCH VEHICLE ADVANCES TO ENABLE HUMAN MARS EXCURSIONS†

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Abstract—A mathematical model is developed for the assessment of the launch mass of a vehicle designed for a human mission to Mars. The mission involves six stages: (i) ascent from Earth surface to low Earth orbit, (ii) outgoing trip from low Earth orbit to low Mars orbit, (iii) descent and landing on Mars, (iv) ascent from Mars surface to low Mars orbit, (v) return trip from low Mars orbit to low Earth orbit, (vi) descent and landing on Earth. The basic objective is to minimize the launch mass while containing the total flight time.

The mathematical model includes two parts: interplanetary flight and planetary flight. The interplanetary flight model is based on the restricted four-body scheme and covers the spacecraft transfer from a low Earth orbit to a low Mars orbit and back. The planetary flight model concerns the spacecraft ascent from Earth surface to low Earth orbit and from Mars surface to low Mars orbit. The sequential gradient-restoration algorithm is employed to solve optimal trajectory problems of interplanetary flight in mathematical programming format and optimal trajectory problems of planetary flight in optimal control format.

The planetary flight study shows that, due to the large gravitational constant of Earth, it is best to assemble the spacecraft in low Earth orbit and launch it from there, rather than from the Earth surface. To reduce the ratio of outgoing LEO mass to return LEO mass, it is best to design the spacecraft as consisting of three modules: Earth return module, habitation module, Mars excursion module.

The interplanetary flight study shows that, for minimum energy LEO–LMO–LEO transfer, the total characteristic velocity is $11.30 \text{ km/s}$. The round-trip time is 970 days, including a stay of 454 days on Mars while waiting for an optimal return date. For a fast transfer mission with a stay of 30 days on Mars, the round-trip time can be reduced to less than half at the cost of nearly doubling the characteristic velocity, thereby resulting into a mass ratio 10 times higher than that of a minimum energy mission, if chemical propellants are used.

To decrease the total mass ratio, use of advanced techniques is indispensable. First, aerobraking techniques can contribute considerably to the reduction of mass ratios: excess velocity on arrival to Mars (outgoing trip) and excess velocity on arrival to Earth (return trip) can be depleted via aerobraking maneuvers instead of propulsive maneuvers. Second, the development of engine/propellant combinations with high specific impulse can be another key factor for reducing the mass ratio. Third, cargo transportation can be used: equipment and propellant not required for the outgoing trip can be sent before the crew leaves Earth via a cargo spacecraft using a low-thrust engine having high specific impulse. Numerical computation shows that, if both aerobraking techniques and cargo transportation techniques are employed, the mass ratio for a minimum energy mission can be brought down by a factor of 5, while the mass ratio for a fast transfer mission can be brought down by a factor of 20.

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To sum up, the mathematical model developed for a launch vehicle can help the engineer to assess proper development directions. Numerical results are highly dependent on certain factors characterizing hardware and propellant such as engine specific impulse, spacecraft structural factor and aerobraking structural factor. At this time, we must look at a round trip Earth–Mars–Earth by humans as a formidable undertaking. This paper merely indicates some useful directions.

1. INTRODUCTION

Manned missions to Mars have been dreamed for a long time, but so far only robotic missions have been executed, since these are considerably less complicated. Over the past 40 years, a wide variety of spacecraft, engine, and trajectory design options for Earth-to-Mars missions have been proposed [1–8]. In this paper, a mathematical model is developed for the assessment of the initial launch mass of a vehicle designed for human mission to Mars. The mission includes the following branches: (i) ascent from Earth surface to low Earth orbit, (ii) outgoing trip from low Earth orbit to low Mars orbit, (iii) descent and landing on Mars, (iv) ascent from Mars surface to low Mars orbit, (v) return trip from low Mars orbit to low Earth orbit, and (vi) descent and landing on Earth.

The model includes two parts: interplanetary flight and planetary flight. The interplanetary flight model is based on the restricted four-body scheme and covers the spacecraft transfer from a low Earth orbit to a low Mars orbit and back. The restricted four-body scheme is adopted to achieve increased accuracy with respect to the patched conics scheme [5,6,8]. The planetary flight model concerns the spacecraft ascent from Earth surface to low Earth orbit and from Mars surface to low Mars orbit.

The basic objective is to minimize the initial launch mass while containing the total flight time. In this paper, the feasibility point of view is taken; mission analysis is simplified by assuming that the Earth and Mars orbits around Sun are circular. While the eccentricity of the Earth orbit is relatively small, that of the Mars orbit is relatively high. Therefore, if the precision point of view is taken, it is clear that the eccentricities of the Earth and Mars orbits around Sun must be considered.

This paper is organized as follows. Section 2 provides the mathematical model for interplanetary flight; Section 3 provides the mathematical model for planetary flight; Section 4 presents mass ratio estimates; Section 5 gives planetary, orbital, and spacecraft data; Section 6 contains numerical results on optimal interplanetary flight and optimal planetary flight, which are discussed for various types of engines and different mission options, with and without aerobraking and/or cargo; finally, Section 7 contains the conclusions.

1.1. Algorithm

The sequential gradient-restoration algorithm (SGRA) is employed to solve optimal trajectory problems of interplanetary flight in mathematical programming problem format and optimal trajectory problems of planetary flight in optimal control format.

SGRA is an iterative technique which involves a sequence of two-phase cycles, each cycle including a gradient phase and a restoration phase. In the gradient phase, the augmented performance index (performance index augmented by the constraints weighted via appropriate Lagrange multipliers) is decreased, while avoiding excessive constraint violation. In the restoration phase, the constraint error is decreased, while avoiding excessive change in the performance index. In a complete gradient-restoration cycle, the performance index is decreased, while the constraints are satisfied to a preselected accuracy. Thus, a succession of feasible suboptimal solutions is generated, each new solution being an improvement over the previous one from the point of view of the performance index.

The sequential gradient-restoration algorithm was developed by Miele et al. during the period 1968–1986 for both mathematical programming problems [9] and optimal control problems [10,11]. It has proven to be a powerful tool for solving optimal trajectory problems of atmospheric and space flight. Applications and extensions of this algorithm have been reported in the US, Japan, Germany, and other countries around the world; in particular, a version of this algorithm is currently used at NASA-JSC under the code name SEGRAM, developed by McDonnell Douglas Technical Service Company [12].

2. INTERPLANETARY FLIGHT MODEL

Let LEO denote a low Earth orbit, and let LMO denote a low Mars orbit. The interplanetary flight model refers to the spacecraft transfer from LEO to LMO for the outgoing trip and from LMO to LEO for the return trip.

We employ the restricted four-body scheme (spacecraft, Earth, Mars, Sun) and the following assumptions: (A1) Sun is fixed in space; (A2)
Earth and Mars are subject to Sun gravity; (A3) the spacecraft is subject to the gravitational attractions of Earth, Mars, and Sun along the entire trajectory; (A4) the eccentricity of the Earth and Mars orbits around Sun is neglected, implying circular planetary motions; (A5) the inclination of the Mars orbital plane vis-a-vis Earth orbital plane is neglected, implying planar spacecraft motion; (A6) LEO and LMO are circular orbits; (A7) for both the outgoing and return trips, two impulses are applied, one at LEO and one at LMO; and (A8) if necessary, a midcourse impulse is applied.

Having adopted the restricted four-body scheme, five motions must be considered: the inertial motions of Earth, Mars, and spacecraft with respect to Sun; the relative motions of the spacecraft with respect to Earth and Mars. To study these motions, we employ three coordinate systems: Sun coordinate system (SCS), Earth coordinate system (ECS), and Mars coordinate system (MCS).

In this paper, the inertial motions of Earth, Mars, and spacecraft are described in Sun coordinates, while the boundary conditions are described in Earth coordinates for LEO and Mars coordinates for LMO. In each coordinate system, a position vector is defined via the radial distance \( r \) and phase angle \( \phi \), while a velocity vector is defined via the velocity modulus \( V \) and local path inclination \( \gamma \). Let E, M, S denote the centers of Earth, Mars, Sun; let P denote the spacecraft; let \( t \) denote the time. For the SCS/ECS and ECS/SCS coordinate transformations, and for the SCS/MCS and MCS/SCS coordinate transformations, see [13,14].

2.1. System equations

Below, we give the system equations for Earth, Mars, and spacecraft.

2.1.1. Earth. Subject to the Sun gravitational attraction, the motion of Earth (subscript E) around Sun is described by the following differential equations for the radial distance \( r_E \), phase angle \( \phi_E \), velocity \( V_E \), and path inclination \( \gamma_E \) with respect to the local horizon:

\[
\text{(SCS)} \quad \frac{dr_E}{dt} = V_E \sin \gamma_E, \quad (1a)
\]
\[
\frac{d\phi_E}{dt} = \left(\frac{V_E}{r_E}\right) \cos \gamma_E, \quad (1b)
\]
\[
\frac{dV_E}{dt} = -\left(\frac{\mu_S}{r_E^3}\right) \sin \gamma_E, \quad (1c)
\]
\[
\frac{d\gamma_E}{dt} = \left(\frac{V_E}{r_E} - \frac{\mu_S}{V_E r_E^2}\right) \cos \gamma_E, \quad (1d)
\]

where \( \mu_S \) is the Sun gravitational constant. Neglecting the orbital eccentricity, we approximate the Earth trajectory with a circle; hence,

\[
r_E = \text{const}, \quad (2a)
\]
\[
\phi_E = \omega_E t + \text{const}, \quad \omega_E = \sqrt{\frac{\mu_S}{r_E^3}}, \quad (2b)
\]
\[
V_E = \sqrt{\frac{\mu_S}{r_E}}, \quad (2c)
\]
\[
\gamma_E = 0, \quad (2d)
\]

where \( \omega_E \) is the angular velocity of Earth around Sun.

2.1.2. Mars. Subject to the Sun gravitational attraction, the motion of Mars (subscript M) around Sun is described by the following differential equations for the radial distance \( r_M \), phase angle \( \phi_M \), velocity \( V_M \), and path inclination \( \gamma_M \) with respect to the local horizon:

\[
\text{(SCS)} \quad \frac{dr_M}{dt} = V_M \sin \gamma_M, \quad (3a)
\]
\[
\frac{d\phi_M}{dt} = \left(\frac{V_M}{r_M}\right) \cos \gamma_M, \quad (3b)
\]
\[
\frac{dV_M}{dt} = -\left(\frac{\mu_S}{r_M^3}\right) \sin \gamma_M, \quad (3c)
\]
\[
\frac{d\gamma_M}{dt} = \left(\frac{V_M}{r_M} - \frac{\mu_S}{V_M r_M^2}\right) \cos \gamma_M, \quad (3d)
\]

where \( \mu_S \) is the Sun gravitational constant. Neglecting the orbital eccentricity, we approximate the Mars trajectory with a circle; hence,

\[
r_M = \text{const}, \quad (4a)
\]
\[
\phi_M = \omega_M t + \text{const}, \quad \omega_M = \sqrt{\frac{\mu_S}{r_M^3}}, \quad (4b)
\]
\[
V_M = \sqrt{\frac{\mu_S}{r_M}}, \quad (4c)
\]
\[
\gamma_M = 0, \quad (4d)
\]

where \( \omega_M \) is the angular velocity of Mars around Sun.

2.1.3. Spacecraft. Subject to the gravitational attractions of Sun, Earth, and Mars along the entire trajectory, the motion of the spacecraft (subscript P) around Sun is described by the following differential equations for the radial distance \( r_P \), phase angle \( \phi_P \), velocity \( V_P \), and path inclination \( \gamma_P \) with respect to the local horizon:

\[
\text{(SCS)} \quad \frac{dr_P}{dt} = V_P \sin \gamma_P, \quad (5a)
\]
\[
d\phi_P/dt = (V_P/r_P) \cos \gamma_P, \quad (5b)
\]
\[
dV_P/\!dt = -(\mu_S/r_P^2) \sin \gamma_P
+ (\mu_E/r_{PE}^2) \cos (\psi_{PE} - \gamma_P)
+ (\mu_M/r_{PM}^2) \cos (\psi_{PM} - \gamma_P), \quad (5c)
\]
\[
d\gamma_P/\!dt = (V_P/r_P - \mu_S/r_P^2) \cos \gamma_P
+ (\mu_E/V_{PE}^2) \sin (\psi_{PE} - \gamma_P)
+ (\mu_M/V_{PM}^2) \sin (\psi_{PM} - \gamma_P).
\quad (5d)
\]

Here, \( \mu_S, \mu_E, \mu_M \) are the gravitational constants of Sun, Earth, and Mars; \( r_{PE} \) and \( r_{PM} \) are the radial distances of the spacecraft from Earth and Mars; \( \psi_{PE} \) and \( \psi_{PM} \) are the inclinations of the Earth gravity direction \( PE \) and Mars gravity direction \( PM \) with respect to the local horizon. The distances \( r_{PE} \) and \( r_{PM} \) are given by

\[
r_{PE} = \sqrt{[(x_P - x_E)^2 + (y_P - y_E)^2]}, \quad (6a)
\]
\[
r_{PM} = \sqrt{[(x_P - x_M)^2 + (y_P - y_M)^2]}, \quad (6b)
\]

with

\[
x_P = r_P \cos \phi_P, \quad x_E = r_E \cos \phi_E,
\]
\[
x_M = r_M \cos \phi_M, \quad (7a)
\]
\[
y_P = r_P \sin \phi_P, \quad y_E = r_E \sin \phi_E,
\]
\[
y_M = r_M \sin \phi_M. \quad (7b)
\]

The gravitational inclinations \( \psi_{PE} \) and \( \psi_{PM} \) are given by

\[
\psi_{PE} = -\pi/2 - k_{PE} \cos^{-1}\left\{ [x_P(x_P - x_E) \\
+ y_P(y_P - y_E)]/r_{PE} \right\}, \quad (8a)
\]
\[
\psi_{PM} = -\pi/2 - k_{PM} \cos^{-1}\left\{ [x_P(x_P - x_M) \\
+ y_P(y_P - y_M)]/r_{PM} \right\}, \quad (8b)
\]

with

\[
k_{PE} = \text{sign}[x_P(y_P - y_E) - y_P(x_P - x_E)], \quad (9a)
\]
\[
k_{PM} = \text{sign}[x_P(y_P - y_M) - y_P(x_P - x_M)]. \quad (9b)
\]

**Remark.** In eqns (1)–(9), the \( \phi \)-angles are measured starting from the radial direction and are positive counterclockwise. The \( \gamma \)-angles and \( \psi \)-angles are measured starting from the local horizon and are positive clockwise.

### 2.2. Boundary conditions for the outgoing trip

#### 2.2.1. Departure from LEO.

In the Earth coordinate system, the spacecraft conditions at departure from LEO (time \( t = t_0 \)) are given by

\[
\begin{align*}
\text{(ECS)} & \quad r_{PE}(t_0) = r_{LEO}, \quad (10a) \\
& \quad \phi_{PE}(t_0) = \text{free}, \quad (10b) \\
& \quad V_{PE}(t_0) = V_{LEO} + \Delta V_{LEO}(t_0), \quad (10c) \\
& \quad \gamma_{PE}(t_0) = 0. \quad (10d)
\end{align*}
\]

Relative to Earth \( r_{PE}, \phi_{PE}, V_{PE}, \gamma_{PE} \) are the radial distance, phase angle, velocity, and path inclination of the spacecraft; \( V_{LEO} \) is the spacecraft velocity in the low Earth orbit prior to application of the tangential, accelerating velocity impulse; \( \Delta V_{LEO}(t_0) \) is the acceleratong velocity impulse at LEO; \( V_{PE}(t_0) \) is the spacecraft velocity after application of the accelerating velocity impulse.

#### 2.2.2. Midcourse impulse.

For the outgoing trip of minimum energy trajectories, there is no need of a midcourse impulse. On the other hand, for the outgoing trip of fast transfer trajectories, a midcourse impulse is needed. In the Sun coordinate system, the spacecraft conditions at midcourse impulse (time \( t = t_1 \)) are given by

\[
\begin{align*}
\text{(SCS)} & \quad r_{P}(t_1+) = r_{P}(t_1-), \quad (11a) \\
& \quad \phi_{P}(t_1+) = \phi_{P}(t_1-), \quad (11b) \\
& \quad V_{P}(t_1+) = V_{P}(t_1-) + \Delta V_{MID}(t_1), \quad (11c) \\
& \quad \gamma_{P}(t_1+) = \gamma_{P}(t_1-), \quad (11d)
\end{align*}
\]

where \( \Delta V_{MID}(t_1) \) is the midcourse impulse at \( t_1 \). Relative to Sun, \( r_{P}(t_1+), \phi_{P}(t_1+), V_{P}(t_1+), \gamma_{P}(t_1+) \) are the radial distance, phase angle, velocity, and path inclination of the spacecraft after application of the velocity impulse; \( r_{P}(t_1-), \phi_{P}(t_1-), V_{P}(t_1-), \gamma_{P}(t_1-) \) are the radial distance, phase angle, velocity, and path inclination of the spacecraft before application of the velocity impulse.

#### 2.2.3. Arrival to LMO.

In the Mars coordinate system, the spacecraft conditions at arrival to LMO (time \( t = t_2 \)) are given by

\[
\begin{align*}
\text{(MCS)} & \quad r_{PM}(t_2) = r_{LMO}, \quad (12a)
\end{align*}
\]
\[
\phi_{PM}(t_2) = \text{free}, \quad (12b)
\]
\[
V_{PM}(t_2) = V_{LMO} + \Delta V_{LMO}(t_2), \quad (12c)
\]
\[
V_{LMO} = \sqrt{(\mu_m/r_{LMO})},
\]
\[
\gamma_{PM}(t_2) = 0. \quad (12d)
\]

Relative to Mars, \( r_{PM}, \phi_{PM}, V_{PM}, \gamma_{PM} \) are the radial distance, phase angle, velocity, and path inclination of the spacecraft; \( V_{LMO} \) is the spacecraft velocity in the low Mars orbit after application of the tangential, decelerating velocity impulse; \( \Delta V_{LMO}(t_2) \) is the decelerating velocity impulse at LMO; \( V_{PM}(t_2) \) is the spacecraft velocity before application of the decelerating velocity impulse.

### 2.3. Boundary conditions for the return trip

#### 2.3.1. Departure from LMO

In the Mars coordinate system, the spacecraft conditions at departure from LMO (time \( t = t_3 \)) are given by

\[
(MCS) \quad r_{PM}(t_3) = r_{LMO}, \quad (13a)
\]
\[
\phi_{PM}(t_3) = \text{free}, \quad (13b)
\]
\[
V_{PM}(t_3) = V_{LMO} + \Delta V_{LMO}(t_3), \quad (13c)
\]
\[
V_{LMO} = \sqrt{(\mu_m/r_{LMO})}, \quad (13d)
\]
\[
\gamma_{PM}(t_3) = 0. \quad (13d)
\]

Formally, eqns (13) can be obtained from eqns (12) by simply replacing the time \( t = t_2 \) with the time \( t = t_3 \). However, there is a difference of interpretation: \( V_{LMO} \) is now the spacecraft velocity in the low Mars orbit after application of the tangential, accelerating velocity impulse; \( \Delta V_{LMO}(t_3) \) is the accelerating velocity impulse at LMO; \( V_{PM}(t_3) \) is the spacecraft velocity after application of the accelerating velocity impulse.

#### 2.3.2. Arrival to LEO

In the Earth coordinate system, the spacecraft conditions at arrival to LEO (time \( t = t_4 \)) are given by

\[
(ECS) \quad r_{PE}(t_4) = r_{LEO}, \quad (14a)
\]
\[
\phi_{PE}(t_4) = \text{free}, \quad (14b)
\]
\[
V_{PE}(t_4) = V_{LEO} + \Delta V_{LEO}(t_4), \quad (14c)
\]
\[
V_{LEO} = \sqrt{(\mu_E/r_{LEO})}, \quad (14d)
\]
\[
\gamma_{PE}(t_4) = 0. \quad (14d)
\]

Formally, eqns (14) can be obtained from eqns (10) by simply replacing the time \( t = t_0 \) with the time \( t = t_4 \). However, there is a difference of interpretation: \( V_{LEO} \) is now the spacecraft velocity in the low Earth orbit after application of the tangential, decelerating velocity impulse; \( \Delta V_{LEO}(t_4) \) is the decelerating velocity impulse at LEO; \( V_{PM}(t_2) \) is the spacecraft velocity before application of the decelerating velocity impulse.

### 2.4. Minimum energy problem

For a round-trip Mars mission, the study of a minimum energy trajectory can be treated as a mathematical programming problem involving the following performance index, constraints, and parameters.

#### 2.4.1. Performance index

For a minimum energy trajectory, the waiting time \( r_{STAY} = t_3 - t_2 \) on LMO is free. There is no need of a midcourse impulse to enforce any particular requirement concerning the relative position of Mars vis-a-vis Earth at spacecraft arrival to LMO; hence, we assume that \( \Delta V_{MID}(t_3) = 0 \).

The most obvious performance index is the characteristic velocity

\[
\Delta V = \Delta V_{LEO}(t_0) + \Delta V_{LMO}(t_2) + \Delta V_{LMO}(t_3) + \Delta V_{LEO}(t_4), \quad (15)
\]

which is the sum of all the velocity impulses, excluding the midcourse impulse. For the outgoing trip, \( \Delta V_{LEO}(t_0) \) is the accelerating velocity impulse at LEO and \( \Delta V_{LMO}(t_2) \) is the decelerating velocity impulse at LMO; for the return trip, \( \Delta V_{LMO}(t_3) \) is the accelerating velocity impulse at LMO and \( \Delta V_{LEO}(t_4) \) is the decelerating velocity impulse at LEO.

Note that minimizing \( \Delta V \) is the same as minimizing the mass ratio \( m(t_0)/m(t_4) \), namely, the ratio of spacecraft mass at LEO departure to spacecraft mass at LEO return; see Section 4 for details.

#### 2.4.2. Constraints

For the outgoing trip, the spacecraft trajectory can be determined via the system equations (1)–(9), if the spacecraft state at departure from LEO, Mars/Earth inertial phase angle difference at departure from LEO, and flight time are given. For the return trip, the spacecraft trajectory can be determined via the system equations (1)–(9), if the spacecraft state at departure from LMO, Mars/Earth inertial phase angle difference at departure from LMO, and flight time are given. Because the departure conditions (10),
(13) and system equations (1)–(9) can be satisfied trivially, the only constraints to be enforced are the arrival conditions (12) and (14) of the outgoing and return trips. Since eqns (12b) and (14b) involve free quantities, we are left with six scalar constraints, namely,

\[(\text{MCS}) \quad r_{PM}(t_2) = r_{LMO}, \quad (16a)\]

\[V_{PM}(t_2) = \sqrt{\left(\mu_M/r_{LMO}\right)} + \Delta V_{LMO}(t_2), \quad (16b)\]

\[\gamma_{PM}(t_2) = 0, \quad (16c)\]

\[(\text{ECS}) \quad r_{PE}(t_4) = r_{LEO}, \quad (17a)\]

\[V_{PE}(t_4) = \sqrt{\left(\mu_E/r_{LEO}\right)} + \Delta V_{LEO}(t_4), \quad (17b)\]

\[\gamma_{PE}(t_4) = 0. \quad (17c)\]

### 2.4.3. Parameters

Under Assumptions (A1)–(A7), the following parameters can be used to control a round-trip minimum energy trajectory:

- **Velocity impulses** \(\Delta V_{LEO}(t_0), \Delta V_{LMO}(t_2), \Delta V_{LMO}(t_3), \Delta V_{LEO}(t_4)\)

- **Transfer/stay times** \(\tau_{OUT} = t_2 - t_0, \tau_{STAY} = t_3 - t_2, \tau_{RET} = t_4 - t_3\)

**Mars/Earth inertial phase angle difference at departure from LEO**

\[\Delta \phi(t_0) = \phi_M(t_0) - \phi_E(t_0), \quad (18c)\]

**Spacecraft/Earth relative phase angle at departure from LEO** \(\phi_{PE}(t_0)\)

\[\quad (18d)\]

**Spacecraft/Mars relative phase angle at departure from LMO** \(\phi_{PM}(t_1)\)

\[\quad (18e)\]

We are in the presence of a mathematical programming problem involving \(n = 10\) parameters, \(q = 6\) constraints, and hence \(n - q = 4\) degrees of freedom. These degrees of freedom must be saturated in such a way that the performance index (15) is minimized.

**Remark.** The Mars/Earth inertial phase angle difference at departure from LMO and the total round-trip time can be determined from the relations

\[\Delta \phi(t_3) = \Delta \phi(t_0) - (\omega_E - \omega_M)(\tau_{OUT} + \tau_{STAY}), \quad (19a)\]

\[\tau_{TOT} = t_4 - t_0 = \tau_{OUT} + \tau_{STAY} + \tau_{RET}. \quad (19b)\]

Note that these quantities are known if the parameters (18) are known. Also note that, once the parameters (18) are known, the round trip trajectory can be computed using the system equations (1)–(9).

### 2.5. Fast transfer problem

For a round-trip Mars mission, the study of a fast transfer trajectory can be reduced to a mathematical programming problem involving the following performance index, constraints, and parameters.

**2.5.1. Performance index.** To generate a fast transfer trajectory, the waiting time on Mars \(\tau_{STAY} = t_3 - t_2\) must have a small value compared to that of the minimum energy trajectory; in turn, this requires the application of a midcourse impulse \(\Delta V_{MID}(t_1)\) to enforce the requirement that Mars be ahead of Earth at spacecraft arrival to LMO.

The most obvious performance index is once more the characteristic velocity

\[\Delta V = \Delta V_{LEO}(t_0) + \Delta V_{MID}(t_1) + \Delta V_{LMO}(t_2)\]

\[+ \Delta V_{LMO}(t_3) + \Delta V_{LEO}(t_4), \quad (20)\]

which is the sum of all the velocity impulses, including the midcourse impulse.

**2.5.2. Constraints.** The arrival conditions (16) and (17) are the only constraints to be enforced, since the departure and midcourse conditions (10), (11), (13) and system equations (1)–(9) can be satisfied trivially. Therefore, we are left with six scalar constraints, namely,

\[(\text{MCS}) \quad r_{PM}(t_2) = r_{LMO}, \quad (21a)\]

\[V_{PM}(t_2) = \sqrt{\left(\mu_M/r_{LMO}\right)} + \Delta V_{LMO}(t_2), \quad (21b)\]

\[\gamma_{PM}(t_2) = 0, \quad (21c)\]

\[(\text{ECS}) \quad r_{PE}(t_4) = r_{LEO}, \quad (22a)\]

\[V_{PE}(t_4) = \sqrt{\left(\mu_E/r_{LEO}\right)} + \Delta V_{LEO}(t_4). \quad (22b)\]
\[ \gamma_{PE}(t_4) = 0. \quad (22c) \]

These conditions are the same as those of the minimum energy trajectory.

For the fast transfer trajectory, two additional constraints must be considered via the specification of the waiting time and total round-trip time,

\[ \tau_{STAY} = \text{given}, \quad (23a) \]

\[ \tau_{TOT} = \tau_{OUT} + \tau_{STAY} + \tau_{RET} = \text{given}, \quad (23b) \]

where \( \tau_{TOT} \) is the total round-trip time.

The particular values \( \tau_{STAY} = 30 \) days and \( \tau_{TOT} = 446 \) days are chosen so as to reduce the stay time on Mars to less than 10% and the total round-trip time to less than 50% of the corresponding values for the minimum energy trajectory, while containing the total characteristic velocity of the fast transfer trajectory within twice the value of the total characteristic velocity of the minimum energy trajectory.

**Remark.** Once the parameters (24) are known, the round-trip trajectory can be computed using the system equations (1)–(9).

### 3. PLANETARY FLIGHT MODEL

The planetary flight model refers to the spacecraft ascent from Earth surface to LEO or from Mars surface to LMO. We employ the following assumptions: (A1) the flight takes place in a vertical plane over a spherical planet, Earth or Mars; (A2) the planet rotation is neglected; (A3) the gravitational field is central and obeys the inverse square law; (A4) the thrust is directed along the spacecraft reference line; hence, the thrust angle of attack is the same as the aerodynamic angle of attack; (A5) the dependence of thrust on altitude and velocity is disregarded; and (A6) the spacecraft is controlled via the angle of attack and power setting.

#### 3.1. System equations

Subject to the planet gravitational attraction and to aerodynamic and propulsive forces, the motion of the spacecraft with respect to planet is described by the following differential equations for the altitude \( h \), velocity \( V \), path inclination \( \gamma \), and mass \( m \) (see [15,16]):

\[ \frac{dh}{dt} = V \sin \gamma, \quad (25a) \]

\[ \frac{dV}{dt} = (\beta T/m) \cos \alpha - D/m - g \sin \gamma, \quad (25b) \]

\[ \frac{d\gamma}{dt} = (\beta T/mV) \sin \alpha + (L/mV) + (V/r - g/V) \cos \gamma, \quad (25c) \]

\[ \frac{dm}{dt} = -\beta T/ISP g_E. \quad (25d) \]

The quantities appearing on the right-hand side of eqns (25) are the reference thrust \( T \), power setting \( \beta \), drag \( D \), lift \( L \), radial distance \( r \), local acceleration of gravity \( g \), Earth sea-level acceleration of gravity \( g_E \), angle attack \( \alpha \), and engine specific impulse \( ISP \).

The following relations tie the radius \( r \) and local gravitational acceleration \( g \) to the altitude above the planet surface:

\[ r = R_E + h, \quad (26a) \]

or

\[ r = R_M + h, \quad (27a) \]

\[ g = \mu_E/r^2 = \mu_E/(R_E + h)^2 \quad (26b) \]

\[ g = \mu_M/r^2 = \mu_M/(R_M + h)^2, \quad (27b) \]
where eqns (26) apply for ascent from Earth surface and eqns (27) apply for ascent from Mars surface. The aerodynamic forces are given by

\[ D = (1/2)C_D(\alpha, M)\rho(h)SV^2, \]  
\[ L = (1/2)C_L(\alpha, M)\rho(h)SV^2, \]  

where \( C_D \) is the drag coefficient, \( C_L \) is the lift coefficient, \( \rho \) is the air density, and \( S \) is a reference surface area. While the air density \( \rho \) is a function of the altitude \( h \), the aerodynamic coefficients \( C_D \) and \( C_L \) are known functions of angle attack \( \alpha \) and Mach number \( M = V/a(h) \), where \( a \) is the speed of sound.

The angle of attack \( \alpha \) and power setting \( \beta \) are subject to the two-sided inequality constraints

\[ \alpha_L \leq \alpha \leq \alpha_U, \]  
\[ \beta_L \leq \beta \leq \beta_U. \]  

In addition to the control inequality constraints (29), some path constraints are imposed on tangential acceleration \( a_t \), dynamic pressure \( q \), and heating rate \( Q \) per unit time and unit surface area,

\[ a_t(h, V, \dot{\gamma}, m, \alpha, \beta) \leq a_{TU}, \]  
\[ q(h, V) \leq q_U, \]  
\[ Q(h, V) \leq Q_U. \]  

See [16] for details concerning the left-hand and right-hand sides of inequalities (30).

### 3.2. Boundary conditions

For vertical launch from rest, the initial conditions (time \( t = 0 \)) are

\[ h(0) = 0, \]  
\[ V(0) = 0, \]  
\[ \gamma(0) = \pi/2, \]  
\[ m(0) = m_0. \]  

The final conditions (time \( t = \tau \)) require level flight with circular velocity; hence,

\[ h(\tau) = h_{\text{LEO}} = r_{\text{LEO}} - R_E, \]  
\[ V(\tau) = V_{\text{LEO}} = \sqrt{\mu_E/r_{\text{LEO}}}, \]  

\( \gamma(\tau) = 0 \) (32c)

or

\[ h(\tau) = h_{\text{LMO}} = r_{\text{LMO}} - R_M, \]  
\[ V(\tau) = V_{\text{LMO}} = \sqrt{\mu_M/r_{\text{LMO}}}, \]  

\( \gamma(\tau) = 0 \) (33c)

where eqns (32) apply for ascent from Earth surface and eqns (33) apply for ascent from Mars surface.

### 3.3. Optimal control problem

The maximum payload problem can be formulated as follows:

\[ \max \quad I = m(\tau), \]
\[ \text{s.t.} \quad (25)-(33). \]

The unknowns of the optimal control problem include the state variables \( h(t), V(t), \gamma(t), m(t) \), control variables \( \alpha(t), \beta(t) \), and parameter \( \tau \).

The sequential gradient restoration algorithm (SGRA) in optimal control format is employed to solve the above maximum payload problem [10,11].

### 3.4. Remark

The solutions of the optimal control problem (34) depend strongly on two principal parameters: spacecraft structural factor \( \epsilon \) and engine specific impulse \( I_{sp} \). The former is the ratio of structural mass to sum of structural mass and propellant mass, while the latter is the thrust per unit reference weight flow; see Section 4. Typical values are \( \epsilon = 0.1 \) and \( I_{sp} = 450 \) s. With these particular values, a single-stage configuration is feasible for the ascent from Mars surface to LMO. On the other hand, the ascent from Earth surface to LEO cannot be done at this time with a single-stage configuration and requires at least a two-stage configuration (see [16]).

### 4. Mass Ratio Estimates

A round-trip human mission to Mars involves multiple stages: (i) ascent from Earth surface to LEO, (ii) outgoing trip LEO to LMO, (iii) descent from LMO to Mars surface, (iv) ascent from Mars surface to LMO, (v) return trip LMO to LEO, and (vi) descent from LEO to Earth surface.

Each stage (i)–(vi) can be executed by means of rockets with chemical propulsion. In some
stage, chemical propulsion can be replaced in part or in toto by electrical (ion) propulsion or thermal-nuclear propulsion. In some stage, aero-braking techniques can be employed in part or in toto to save propellant mass.

4.1. Propulsive stage

Let \( m_i \) denote the initial mass of a stage; let \( m_f \) denote the final mass. By definition, the following relations hold:

\[
\begin{align*}
  m_i &= m_s + m_p + m_*, \\
  m_f &= m_s + m_*,
\end{align*}
\]

(35a)

(35b)

where \( m_s \) is the structural mass, \( m_p \) is the propellant mass, and \( m_* \) is the payload mass. Let \( \varepsilon \) denote the structural factor, the ratio of structural mass to sum of structural mass and propellant mass,

\[
\varepsilon = m_s/(m_s + m_p).
\]

(36)

The final mass to initial mass ratio can be written as \[ (37) \]

\[
\eta = m_f/m_i = \exp(-\Delta V/g_E I_{SP}),
\]

(37)

where \( \Delta V \) is the characteristic velocity, \( I_{SP} \) is the engine specific impulse, and \( g_E \) is the gravitational acceleration at sea level on Earth.

For propulsive stages, the propulsive mass ratio can be written as

\[
\eta = m_i/m_*.
\]

(38)

Upon simple transformations in light of eqns (35)–(37), eqn (38) becomes

\[
\eta = (1 - \varepsilon)/(\delta - \varepsilon),
\]

(39)

where \( \delta \) is given by eqn (37).

4.2. Aerobraking stage

Let \( m_i \) denote the initial mass of a stage; let \( m_f \) denote the final mass; let \( m_{AB} \) denote the aerobraking mass. By definition, the following relations hold:

\[
\begin{align*}
  m_i &= m_{AB} + m_*, \\
  m_f &= m_*,
\end{align*}
\]

(40a)

(40b)

where \( m_{AB} \) is the aerobraking mass (thermal protection mass). Let \( \varepsilon_{AB} \) denote the structural factor, the ratio of aerobraking mass to sum of aerobraking mass and payload mass,

\[
\varepsilon_{AB} = m_{AB}/(m_{AB} + m_*).
\]

(41)

For aerobraking stages, the aerobraking mass ratio can be written as

\[
\eta_{AB} = m_i/m_*.
\]

(42)

Upon simple transformations in light of eqns (40) and (41), eqn (42) becomes

\[
\eta_{AB} = 1/(1 - \varepsilon_{AB}).
\]

(43)

5. PLANETARY, ORBITAL, AND SPACECRAFT DATA

5.1. Planetary data

The gravitational constants for Sun, Earth, and Mars are given by

\[
\begin{align*}
  \mu_S &= 1.327E11, \\
  \mu_E &= 3.986E05, \\
  \mu_M &= 4.283E04 \text{ km}^3/\text{s}^2.
\end{align*}
\]

(44)

Earth and Mars travel around Sun along orbits with average radii

\[
\begin{align*}
  r_E &= 1.496E08, \\
  r_M &= 2.279E08 \text{ km}.
\end{align*}
\]

(45a)

The associated average translational velocities and angular velocities are given by

\[
\begin{align*}
  V_E &= 29.78, \\
  V_M &= 24.13 \text{ km/s}, \\
  \omega_E &= 0.986, \\
  \omega_M &= 0.524 \text{ deg/day}.
\end{align*}
\]

(45b)

(45c)

In particular, the angular velocity difference between Earth and Mars is

\[
\Delta \omega = \omega_E - \omega_M = 0.462 \text{ deg/day}.
\]

(46)

5.2. Orbital data

For the outgoing trip, the spacecraft is to be transferred from a low Earth orbit to a low Mars orbit; for the return trip, the spacecraft is to be transferred from a low Mars orbit to a low Earth orbit. The radii of the terminal orbits are

\[
\begin{align*}
  r_{LEO} &= 6841, \\
  r_{LMO} &= 3597 \text{ km},
\end{align*}
\]

(47a)

corresponding to the altitudes

\[
\begin{align*}
  h_{LEO} &= 463, \\
  h_{LMO} &= 200 \text{ km},
\end{align*}
\]

(47b)

since the Earth and Mars surface radii are given by

\[
\begin{align*}
  R_E &= 6378, \\
  R_M &= 3397 \text{ km}.
\end{align*}
\]

(47c)

The circular velocities (subscript \( c \)) at LEO and LMO are given by

\[
\begin{align*}
  (V_c)_{LEO} &= 7.633, \\
  (V_c)_{LMO} &= 3.451 \text{ km/s},
\end{align*}
\]

(47d)
and the corresponding escape velocities (subscript $^*$) are

$$(V_e)^{\text{LEO}} = 10.795, \quad (V_e)^{\text{LMO}} = 4.880 \text{ km/s.} \quad (47e)$$

### 5.3. Spacecraft data

Major parameters are the engine specific impulse and the spacecraft structural factor.

**Specific impulse.** Three types of engines (chemical engine, nuclear engine, ion engine) are considered. The assumed values for the specific impulse are $[3,5,17,18]$

- $I_{sp} = 450 \text{ s}$, chemical engine, \hspace{2cm} (48a)
- $I_{sp} = 900 \text{ s}$, nuclear engine, \hspace{2cm} (48b)
- $I_{sp} = 3000 \text{ s}$, ion engine. \hspace{2cm} (48c)

**Structural factor.** For propulsive stages, the structural factor is assumed to be $[3,5]$

$$\varepsilon = 0.10.$$ \hspace{2cm} (49)

For aerobraking stages, the structural factor is assumed to be $[3]$

$$\varepsilon_{AB} = 0.15.$$ \hspace{2cm} (50)

### 6. COMPUTATIONAL RESULTS AND ANALYSIS

#### 6.1. Optimal interplanetary trajectories

Two optimal interplanetary trajectories are presented in this paper: minimum energy trajectory and fast transfer trajectory. The results obtained via the sequential gradient restoration algorithm are shown in Figs. 1–6 and Tables 1–4.

Figures 1–3 refer to the minimum energy trajectory, while Figs. 4–6 refer to the fast transfer trajectory. Each set of figures includes three parts: (i) the round-trip interplanetary trajectory going from Earth to Mars and returning from Mars to Earth; (ii) for the outgoing trip, the near-planet trajectory at departure from Earth and arrival to Mars;
Outgoing trip: Departure

Outgoing trip: Arrival

Fig. 2. Minimum energy trajectory in near-planet space, Earth or Mars coordinates. Outgoing trip: $\Delta V_{\text{LEO}} = 3.55 \text{ km/s}$, $\Delta V_{\text{LMO}} = 2.10 \text{ km/s}$, $\Delta V_{\text{OUT}} = 5.65 \text{ km/s}$, Mars entry velocity $V_{\text{PM}} = 5.55 \text{ km/s}$.

Return trip: Departure

Return trip: Arrival

Fig. 3. Minimum energy trajectory in near-planet space, Mars or Earth coordinates. Return trip: $\Delta V_{\text{LMO}} = 2.10 \text{ km/s}$, $\Delta V_{\text{LEO}} = 3.55 \text{ km/s}$, $\Delta V_{\text{RET}} = 5.65 \text{ km/s}$, $\Delta V_{\text{TOT}} = 11.30 \text{ km/s}$, Earth entry velocity $V_{\text{PE}} = 11.18 \text{ km/s}$.

Table 4. Mars/Earth inertial phase angle differences $\Delta \phi$ (deg) at departure and arrival for round-trip Mars mission

<table>
<thead>
<tr>
<th></th>
<th>Minimum energy solution</th>
<th>Fast transfer solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outgoing trip</td>
<td>$\Delta \phi(t_0)$: +43.9</td>
<td>+164.6</td>
</tr>
<tr>
<td></td>
<td>$\Delta \phi(t_2)$: -75.1</td>
<td>+45.8</td>
</tr>
<tr>
<td>Return trip</td>
<td>$\Delta \phi(t_1)$: +75.0</td>
<td>+32.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta \phi(t_4)$: -43.9</td>
<td>-41.4</td>
</tr>
</tbody>
</table>

(iii) for the return trip, the near-planet trajectory at departure from Mars and arrival to Earth.

Tables 1–4 list the main data for a Mars mission, specifically: transfer/stay times (Table 1), characteristic velocities (Table 2), relative-to-planet velocities at departure and arrival (Table 3), and Mars/Earth inertial phase angle differences at departure and arrival (Table 4).

For a minimum energy mission, the total flight time is 970.0 days, including 257.9 days for the outgoing trip, 454.3 days waiting in LMO, and 257.8 days for the return trip. The total characteristic velocity is 11.30 km/s, including 5.65 km/s for the outgoing trip and 5.65 km/s for the return trip. There is no midcourse velocity impulse.

For a fast transfer mission, the total flight time is 446.0 days, including 256.9 days for the outgoing
Fig. 4. Fast transfer trajectory in interplanetary space, Sun coordinates. Outgoing trip time = 256.9 days, stay time in Mars = 30.0 days, return trip time = 159.1 days, total round-trip time = 446.0 days.

The minimum energy trajectory has a mirror property (see [13]), which is a geometrical relation between the outgoing and return trips (Fig. 1): the trajectory of the return trip can be obtained from that of the outgoing trip by means of a three-step procedure including rotation, reflection, and inversion. An important byproduct of the mirror property is the fact that Mars is behind Earth by 75.1° at spacecraft arrival to Mars, but ahead of Earth by 75.0° at spacecraft departure from Mars. The mismatch of $360 - 150.1 = 209.9°$ in Earth/Mars inertial phase angle difference explains the need for a long waiting time in Mars (446 days).

Another property of the minimum energy trajectory is the asymptotic parallelism property (see [14]). Asymptotic parallelism means that the trajectory of the spacecraft in near-planet space (departure from or arrival to Earth/Mars) is parallel to the planet velocity at the end of the first day on departure and at the beginning of the last day on arrival; this is hinted by Figs. 2 and 3. Its meaning is that the minimum energy trajectory resembles a Hohmann transfer trajectory, even though it cannot be a Hohmann transfer, owing to the disturbing influence of Earth and Mars on the terminal branches of the trajectory.

The fast transfer trajectory does not have a mirror property or an asymptotic parallelism property (Fig. 4). The requirement that Mars be ahead of Earth at spacecraft arrival to Mars is implemented via a midcourse impulse at time $t_1$ during the outgoing trip. The values of $t_1$ and $\Delta V_{\text{MID}}(t_1)$ are obtained via the sequential gradient-restoration

Fig. 5. Fast transfer trajectory in near-planet space, Earth or Mars coordinates. Outgoing trip: $\Delta V_{\text{LEO}} = 4.53 \text{ km/s}$, $\Delta V_{\text{MID}} = 3.00 \text{ km/s}$, $\Delta V_{\text{LMO}} = 5.54 \text{ km/s}$, $\Delta V_{\text{OUT}} = 13.07 \text{ km/s}$, Mars entry velocity $V_{\text{PM}} = 8.99 \text{ km/s}$. 

The fast transfer trajectory, 30.0 days waiting in LMO, and 159.1 days for the return trip. The total characteristic velocity is 20.56 km/s, including 13.07 km/s for the outgoing trip and 7.49 km/s for the return trip. A midcourse velocity impulse of 3.00 km/s is applied in the outgoing trip.
algorithm. The trajectory of the spacecraft in near-planet space (departure from or arrival to Earth/Mars) is no longer parallel to the planet velocity (Figs. 5 and 6). The fact that the flight time has been reduced to less than half is obtained at an expensive price, that of nearly doubling the characteristic velocity (Table 2).

Remark. Table 3 shows that the entry velocity into LEO is 11.18 km/s for the minimum energy trajectory and 11.63 km/s for the fast transfer trajectory; the entry velocity into LMO is 5.55 km/s for the minimum energy trajectory and 8.99 km/s for the fast transfer trajectory. These values are below the 14 km/s limit considered reasonable in [19]. See also the aerobraking studies in [20–22].

6.2. Optimal planetary trajectories

We consider two cases: ascent from Earth and ascent from Mars.

6.2.1. Ascent from Earth. Ascending trajectories from the Earth surface to LEO have been optimized for maximum payload mass, with initial thrust-to-weight ratio $\sigma = 1.4$, spacecraft structural factor $\varepsilon = 0.1$, and engine specific impulse $I_{sp} = 450$ s. The results of [16] show that it is not advisable to use a single-stage rocket to insert a spacecraft into LEO; indeed, the maximum payload is marginally positive. On the other hand, if two stages are used, the ratio of payload mass to initial mass can reach 0.067 (see [16] for details), implying that the inverse ratio of initial mass to payload mass is $\eta = 15$.

The implication is that it is not desirable to launch a spacecraft designed for human mission to Mars directly from the ground. Hence, in this paper, we assume that the spacecraft for human mission to Mars is assembled in LEO and launched from LEO.

6.2.2. Ascent from Mars. Ascending trajectories from the Mars surface to LMO have been optimized for maximum payload mass with initial thrust-to-weight ratio $\sigma = 1.4$, spacecraft structural factor $\varepsilon = 0.1$, and engine specific impulse $I_{sp} = 450$ s. The results show that it is feasible to use a single-stage rocket to insert a spacecraft into LMO; indeed, the ratio of payload mass to initial mass is 0.33, implying that the inverse ratio of initial mass to payload mass is $\eta = 3$. This number is to be further increased to $\eta' = 1.2 \eta = 3.6$, if one accounts for the propellant mass and structural mass needed for the descent from LMO to Mars surface at the end of the outgoing trip from Earth.

The implication is that it is not desirable for the entire spacecraft arriving to LMO from Earth to descend and land on the Mars surface. Thus, an
Table 5. Specific impulse effect on mass ratios of minimum energy solution, $\varepsilon = 0.1$

<table>
<thead>
<tr>
<th>Trip Type</th>
<th>$I_{sp}$</th>
<th>$\eta_{\text{LEO}}$</th>
<th>$\eta_{\text{MID}}$</th>
<th>$\eta_{\text{LMO}}$</th>
<th>$\eta_{\text{OUT}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outgoing</td>
<td>450 s</td>
<td>2.59</td>
<td>1.58</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>900 s</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000 s</td>
<td>1.73</td>
<td>1.31</td>
<td>1.08</td>
<td>4.48</td>
</tr>
<tr>
<td>Return</td>
<td></td>
<td>1.73</td>
<td>1.31</td>
<td>1.08</td>
<td>4.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.59</td>
<td>1.58</td>
<td>1.14</td>
<td>4.48</td>
</tr>
<tr>
<td>Round</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.06</td>
</tr>
</tbody>
</table>

6.3. Effect of engine type

Three different types of engines are considered: (i) a chemical engine with a specific impulse of 450 s; (ii) a thermal-nuclear engine with a specific impulse of 900 s; (iii) a low-thrust ion engine with a specific impulse of 3000 s. Tables 5 and 6 show the effect of the specific impulse on the mass ratios of the minimum energy trajectory and fast transfer trajectory.

The results show that higher values of the specific impulse cause considerable reduction in the total mass ratio of both the minimum energy trajectory and fast transfer trajectory. On a relative basis, more reduction in mass ratio is achieved for the fast transfer trajectory than for the minimum energy trajectory.

It must be noted that the above advantages are achieved at a price. The thermal-nuclear engine has severe safety and pollution problems. The low-thrust ion engine takes too much time to spiral out of the Earth or Mars gravitational fields and therefore is not suitable for human voyage to Mars. Nevertheless, the high specific impulse of the ion engine and the resulting low mass ratios make this engine attractive for cargo transportation.

6.4. Effect of aerobraking

The atmospheres of Mars and Earth provide the opportunity of reducing excess velocity via aerobraking maneuvers in combination with propulsive maneuvers. Tables 7 and 8 show the aerobraking effect on the characteristic velocities of the minimum energy trajectory and the fast transfer trajectory. Tables 9 and 10 show the aerobraking effect on the mass ratios of the minimum energy trajectory and the fast transfer trajectory.

For comparison, three cases are considered: (i) no aerobraking, (ii) partial aerobraking, (iii) full aerobraking.
6.5. Initial LEO mass estimate

In Section 6.2 dealing with optimal planetary trajectories, two conclusions were reached:

(i) Due to the large gravitational constant of Earth, it is not desirable to launch a manned spacecraft directly from the ground. Instead, the spacecraft should be assembled in LEO and launched from LEO.

(ii) Due to the relative large gravitational constant of Mars, it is not desirable for the entire spacecraft arriving at LMO to descend and land on the Mars surface. Instead, the spacecraft arriving at LMO should consist of three modules: Earth return module (ERM), habitation module (HBM), and Mars excursion module (MEM).

For the ERM + HBM + MEM spacecraft, the sequence of operations is as follows:

(iii) MEM is the only component to descend from LMO, land on Mars, and then ascend to LMO for the subsequent rendezvous with the ERM+HBM spacecraft, followed by crew transfer to the ERM+HBM spacecraft.

(iv) With rendezvous and crew transfer accomplished, MEM is discarded and the ERM+HBM spacecraft initiates the long journey back from LMO to LEO. In the vicinity of LEO, the crew transfers from HBM to ERM, and then HBM is discarded.

(v) With HBM is discarded, the ERM spacecraft circularizes its motion into LEO, prior to the subsequent descent and landing on Earth.

In light of (i)–(v), the following formula allows one to estimate the initial LEO mass, namely, the mass prior to departure from LEO:

\[ m_{\text{TOT}} = m_{\text{ERM}} \eta_{\text{OUT}} \eta_{\text{RET}} + m_{\text{HBM}} \eta_{\text{OUT}} \eta_{\text{LMO}} \eta_{\text{RET}} + m_{\text{MEM}} \eta_{\text{OUT}}. \]  

where

\[ \eta_{\text{OUT}} = \eta_{\text{LEO}} \eta_{\text{MID}} \eta_{\text{LMO}}, \quad \text{outgoing trip}, \] (52a)

\[ \eta_{\text{RET}} = \eta_{\text{LMO}} \eta_{\text{LEO}}, \quad \text{return trip}. \] (52b)

Let the following dimensionless ratios be introduced:

\[ \eta_{\text{TOT}} = m_{\text{TOT}} / m_{\text{ERM}}, \] (53a)

\[ \eta_{\text{ERM}} = m_{\text{ERM}} / m_{\text{ERM}} = 1, \] (53b)

\[ \eta_{\text{HBM}} = m_{\text{HBM}} / m_{\text{ERM}}. \] (53c)

\[ \eta_{\text{MEM}} = m_{\text{MEM}} / m_{\text{ERM}}. \] (53d)

Then, eqn (51) can be rewritten as

\[ \eta_{\text{TOT}} = \eta_{\text{OUT}} (\eta_{\text{ERM}} \eta_{\text{RET}} + \eta_{\text{HBM}} \eta_{\text{LMO}} \eta_{\text{RET}} + \eta_{\text{MEM}}). \] (54)

For minimum energy transfer, typical value of the constants in eqn (54) are

\[ \eta_{\text{ERM}} = 1, \quad \eta_{\text{HBM}} = 7.8, \]

\[ \eta_{\text{MEM}} = 9.7(1 - C); \] (55a)

### Table 10. Aerobraking effect on mass ratios of fast transfer solution, \( I_{\text{SP}} = 450 \, \text{s}, \varepsilon = 0.10, \varepsilon_{\text{AB}} = 0.15 \)

<table>
<thead>
<tr>
<th></th>
<th>No AB</th>
<th>Partial AB</th>
<th>Full AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outgoing trip</td>
<td>( \eta_{\text{LEO}} )</td>
<td>3.49</td>
<td>3.49</td>
</tr>
<tr>
<td></td>
<td>( \eta_{\text{MID}} )</td>
<td>2.21</td>
<td>2.21</td>
</tr>
<tr>
<td></td>
<td>( \eta_{\text{LMO}} )</td>
<td>4.87</td>
<td>3.86</td>
</tr>
<tr>
<td></td>
<td>( \eta_{\text{OUT}} )</td>
<td>37.65</td>
<td>29.80</td>
</tr>
<tr>
<td>Return trip</td>
<td>( \eta_{\text{LMO}} )</td>
<td>2.56</td>
<td>2.56</td>
</tr>
<tr>
<td></td>
<td>( \eta_{\text{LEO}} )</td>
<td>2.95</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>( \eta_{\text{RET}} )</td>
<td>7.55</td>
<td>4.17</td>
</tr>
<tr>
<td>Round trip</td>
<td>( \eta_{\text{TOT}} )</td>
<td>284.34</td>
<td>124.26</td>
</tr>
</tbody>
</table>
Table 11. Effect of various options on mass ratios of minimum energy solution, LEO–LMO–Mars–LMO–LEO,
$I_{SP} = 450$ s, $\epsilon = 0.10$, $\epsilon_{AB} = 0.15$

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
<th>Option 4</th>
<th>Option 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{LEO}$</td>
<td>$\eta_{MID}$</td>
<td>$\eta_{LMO}$</td>
<td>$\eta_{OUT}$</td>
<td>$\eta_{LEO}$</td>
</tr>
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Table 12. Effect of various options on mass ratios of fast transfer solution, LEO–LMO–Mars–LMO–LEO,
$I_{SP} = 450$ s, $\epsilon = 0.10$, $\epsilon_{AB} = 0.15$

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
<th>Option 4</th>
<th>Option 5</th>
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<td>862.7</td>
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<td>205.6</td>
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</table>

for fast transfer, typical value of the constants in eqn (54) are

$$\eta_{ERM} = 1, \quad \eta_{HBM} = 5.9, \quad \eta_{MEM} = 9.7(1 - C).$$

See [3,5]. In eqns (55), $C = 0$ in the absence of a cargo spacecraft, and $C = 1$ if a cargo spacecraft is employed to resupply the manned spacecraft in LMO prior to the return to Earth.

With the help of eqns (54) and (55), the initial LEO mass can be estimated in terms of the mass of the Earth return module. This is done in Table 11 for the minimum energy trajectory and in Table 12 for the fast transfer trajectory. In both cases, the engine specific impulse (chemical engine) is set at $I_{SP} = 450$ s and the spacecraft structural factor is set at $\epsilon = 0.10$; if there is aerobraking, the aerobraking structural factor is set at $\epsilon_{AB} = 0.15$.

For both the minimum energy trajectory and the fast transfer trajectory, five options must be considered:

(i) Option 1 is a purely propulsive option;

(ii) Option 2 involves partial aerobraking (PAB);

(iii) Option 3 involves full aerobraking (FAB);

(iv) Option 4 involves partial aerobraking and cargo (PABC);

(v) Option 5 involves full aerobraking and cargo (FABC).

Concerning cargo transportation, the basic idea is as follows: before the crew leaves Earth, any equipment or propellant not required during the outgoing trip is sent ahead using a low-thrust ion engine; this results in a low mass ratio for the cargo spacecraft due to the high specific impulse of the ion engine. Specifically, the cargo spacecraft brings to LMO the Mars excursion module and the propellant required for the return trip. After this material is safely in LMO, the manned spacecraft departs from Earth and upon reaching the vicinity of LMO executes a rendezvous maneuver with the waiting cargo spacecraft.

The results of Tables 11 and 12 show that: (i) aerobraking and cargo transportation can cause considerable reduction in the initial LEO mass required for a Mars mission; (ii) a larger reduction in the mass ratio can be achieved for a fast transfer mission than for a minimum energy mission. Indeed, comparing the extreme options (Options 1 and 5), we see that the mass ratio can be reduced
by a factor of 5 for a minimum energy mission and by a factor of 20 for a fast transfer mission.

7. SUMMARY AND CONCLUSIONS

In this paper, a mathematical model for the assessment of launch vehicle advances to enable human Mars excursions has been developed. The model is divided into two parts: interplanetary flight and planetary flight. The interplanetary flight model is based on a restricted four-body scheme and covers the spacecraft transfer from a low Earth orbit to a low Mars orbit and back; the planetary flight model concerns the spacecraft ascent from Earth/Mars surface to low Earth/Mars orbit. The sequential gradient-restoration algorithm is employed to solve optimal trajectory problems of interplanetary flight in mathematical programming problem format and optimal trajectory problems of planetary flight in optimal control format.

From computation and analysis, the following main conclusions emerge:

(a) The planetary model study shows that, due to the large gravitational constant of Earth, it is best to assemble the spacecraft in low Earth orbit and launch it from there, rather from the Earth surface. To further reduce the initial LEO mass, it is best to design the spacecraft as consisting of three modules: Earth return module, habitation module, and Mars excursion module, the latter being the only spacecraft component which descends to the Mars surface and then ascends to LMO.

(b) The interplanetary model study shows that, for a minimum energy Mars mission including landing on Mars, the required mass ratio (ratio of initial LEO mass to Earth return module mass) is 120 if a chemical engine is used with specific impulse of 450 s; the round-trip time is 970 days, including a stay of 454 days on Mars while waiting for the optimal return date. For a fast Mars mission, the required mass ratio is 1200; the round-trip time is 446 days, including a stay of 30 days on Mars.

(c) Aerobraking techniques can help reducing the required mass ratio: excess velocity on arrival to Mars (outgoing trip) and on arrival to Earth (return trip) can be depleted via aerobraking maneuvers instead of solely propulsive maneuvers. Numerical computation shows that, if aerobraking techniques are employed, the mass ratio can be brought down from 120 to 30 for a minimum energy mission and from 1200 to 60 for a fast transfer mission.

(d) The development of engines with high specific impulse is another key factor for reducing the mass ratio. A low-thrust engine (ion engine) with a very high specific impulse is suitable for cargo transportation. Cargo transportation means the following: before the crew leaves Earth, equipment and propellant not required for the outgoing trip are sent with a cargo spacecraft using a low-thrust engine with high specific impulse. Numerical computation shows that, if both aerobraking techniques and cargo transportation techniques are employed, the mass ratio can be brought down from 120 to 30 for a minimum energy mission and from 1200 to 60 for a fast transfer mission.

To sum up, at this time we must look at a round-trip Earth–Mars–Earth by humans as a formidable undertaking. There are no easy options: any advantage in a particular area implies some disadvantage in some other area. This paper merely indicates some useful directions.

REFERENCES


