COORDINATED CONTROL OF
TETHERED SATELLITE CLUSTER SYSTEMS

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Abstract

We propose the concept of Tethered Satellite Cluster Systems, which keep and change the constellation with the tension/length control of tethers. The purpose of the system is the formation flight to perform the observation, communication and rendezvous-docking missions. The system is applied to tethered service satellites, which perform the missions, for example an automatic casting, capture, recovery, moorage and deorbit of an uncontrolled satellite.

In this paper, we consider the rotating motion of this system for the interferometry mission. This system can save the fuel of the thruster in the rotating motion around the center of the mass of the system in orbit, because the tether tension supplies the control force. By numerical simulations, we establish the coordinated control method and analyze the case where three satellites are joined by three tethers. By two-dimensional ground experiments, we indicate that the tether tension save the fuel of the thruster for driving the rotating motion.

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**Introduction**

We propose the concept of Tethered Satellite Cluster Systems\(^\text{[1]}\), which keep and change the constellation with the tension/length control of tethers. The purpose of the system is the formation flight to perform the observation, communication and rendezvous-docking missions as shown in figure 1. The system is applied to tethered service satellites, which perform the missions, for example an automatic casting, capture, recovery, moorage and deorbit of an uncontrolled satellite as shown in figure 2\(^\text{[2][3]}\).

NASA considers the satellite formation flying for the interferometry mission in Space Technology 3, and Randal W. Beard and Fred Y. Hadaegh derived control laws for rotating a constellation of spacecraft using on/off thrusters\(^\text{[4]}\). Existing system, however, needs a lot of fuel of the thruster for rotating motion. On the other hand, tether satellite cluster systems can save the fuel, because the tether tension supplies the control force. In this paper, we analyze the rotating motion around the center of the mass of the system in orbit and establish the coordinated control method by numerical simulations. In addition, we outline a ground experiment system including the reel mechanism for the tethered satellite cluster systems, and verify the advantage of saving the fuel of the thruster with use of tether tension by two-dimensional experiments. Finally, we indicate the future works.

**Tethered Satellite Cluster Systems**

Figure 3 shows the concept of tethered satellite cluster systems. The features of the system are as follows.

1. The system keeps and changes the constellation by satellites joint/separation with tether.
2. The tether tension/length is controlled by a reel mechanism, and the satellites
joint/separation is conducted by joint mechanism.

(3) The reel or joint mechanism is installed in the arms on each satellite.

(4) Each satellite conducts coordinated control of position and attitude.

As most fundamental motion of the system, we consider following two motions.

(a) Translating motion: to move the object to arbitrary position.

This motion is applied to the rendezvous-docking mission. We already analyzed the dynamics of two satellites joined by a tether in inertial coordinate system and established the tension control method for equalizing two satellites in the velocity magnitude without exciting the attitude vibration of each satellite[5].

(b) Rotating motion: to spin the system for keeping or changing the constellation.

This motion is applied to the interferometry mission. Existing system needs a lot of fuel of the thruster for rotating motion. This system, however, save the fuel, because the tether tension supplies the control force. In this paper, we analyze the rotating motion of this system in orbit and verify the advantage.

Combining two motions, the system performs various applied missions, for example attitude control of an uncontrolled satellite.

**Modeling and Formulation**

The dynamical model and formulation of the tethered satellite cluster systems is as follows. Figure 4 shows an analytical model. The motion of the \( n \) satellites joined by tethers in earth-centered inertial coordinate system \( \{i\} \) is considered. The following assumptions are made.
Satellite \( j \) (\( j = 1, \ldots, n \)):

The mass is \( m_j \); the position of the center of the mass is \( \mathbf{q}_j \); the moment of inertia is \( \mathbf{I}_j \); the angular velocity with respect to \( \{i\} \) is \( \omega_j \); and the body-fixed coordinate system is \( \{b_j\} \).

Tether \( jk \) (Tether \( kj \)):

The tether joins satellite \( j \) and satellite \( k \). The tension is \( T_{jk} (= T_{kj}) \); the mass, twist and bending moment, and tensile strain are ignored.

Arm \( jk \):

The arm joins tether \( jk \) on the satellite \( j \). The arm controls the joint (reel or joint mechanism) position \( \mathbf{a}_{jk} (\neq \mathbf{a}_{ij}) \) arbitrarily.

By Newton-Euler equations, the motion of each satellite in circular orbit is formulated as follows.

\[
m_j \ddot{\mathbf{q}}_j = \frac{\mu m_j}{|\mathbf{q}_j|^3} \mathbf{q}_j + \sum_k T_{jk}
\]

\[
\mathbf{I}_j \cdot \dot{\omega}_j + \mathbf{\omega}_j \times \mathbf{I}_j \cdot \dot{\mathbf{w}}_j = \frac{2\Omega^2}{|\mathbf{q}_j|^3} \mathbf{q}_j \times \mathbf{I}_j \cdot \mathbf{q}_j + \sum_k \mathbf{\tau}_{jk}
\]

where \( \Omega \) is the orbital angular velocity; \( T_{jk} \) and \( \mathbf{\tau}_{jk} \) are tension and torque vector working on satellite \( j \) by tether \( jk \).

\[
T_{jk} = T_{jk} \frac{(\mathbf{q}_j + \mathbf{r}_{ij}) - (\mathbf{q}_j + \mathbf{r}_{ik})}{\| (\mathbf{q}_j + \mathbf{r}_{ij}) - (\mathbf{q}_j + \mathbf{r}_{ik}) \|^3}
\]

\[
\mathbf{\tau}_{jk} = \mathbf{a}_{jk} \times T_{jk}
\]
Rotating Motion

We assume that all satellites in the system are joined by tethers, and establish the coordinated control for the rotating motion where all satellites rotate around the center of the mass of the system in same plane at same angular velocity.

Rotating Coordinate System

Figure 5 shows the simulation model \((n = 3)\). We define the rotating coordinate system \(\{r\}\) as follows.

The origin is the center of the mass of the system.

\[
q_{c.m.} = \frac{\sum_{j=1}^{n} m_j q_j}{\sum_{j=1}^{n} m_j}
\]  

(5)

\(r_z\) axis shows the direction of the angular momentum of the system on the origin.

\[
P = \sum_{j=1}^{n} \left\{ m_j \left( q_j - q_{c.m.} \right) \times \left( q_j - q_{c.m.} \right) + I_j \cdot \omega_j \right\}
\]  

(6)

\(r_x\) axis shows the direction of satellite 1 in \(r_y r_x\) plane.

The relationship between \(\{r\}\) and \(\{i\}\) is defined by Euler angles.

\[
\{r\} = C^3(\varphi)C^2(\theta)C^1(\psi)\{i\}
\]  

(7)

We call \(r_x r_y\) plane rotating plane.

Position/Attitude

The position and attitude of each satellite in \(\{r\}\) is defined as follows.
Position: \[ \mathbf{q}_j - \mathbf{q}_{\text{c.m.}} = \left\{ \begin{array}{c} d_j \cos \lambda_j \\ d_j \sin \lambda_j \\ h_j \end{array} \right\} \] (8)

Attitude: \[ \left\{ \mathbf{b}_j \right\} = C^1(\psi_j)C^2(\theta_j)C^3(\varphi_j)\{\mathbf{r}\} \] (9)

Where, \( d_j \) is the distance between satellite \( j \) and \( \mathbf{r}_z \) axis; \( \lambda_j \) is the angle between satellite \( j \) and \( \mathbf{r}_z \) axis in rotating plane; and \( h_j \) is the height of satellite \( j \) out of rotating plane. Because of definition of \( \{\mathbf{r}\} \), these values satisfy the following equations.

\[
\sum_{j=1}^{n} m_j \begin{bmatrix} d_j \cos \lambda_j \\ d_j \sin \lambda_j \\ h_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \] (10)

\[ \lambda_i = 0 \] (11)

The objective position/attitude is denoted by a superscript \( d \) : \( d_j^d \), for example. In objective rotating motion, the distance \( L_{jk} \) between satellite \( j \) and satellite \( k \), and the angular velocity \( \omega_r \) are represented as follows.

\[ L_{jk} = d_j^2 + d_k^2 - 2d_j d_k \cos (\lambda_k^d - \lambda_j^d) \] (12)

\[ \omega_r \mathbf{r}_z = \frac{\mathbf{P}}{\sum_{j=1}^{n} \left( m_j d_j^2 + \left| \mathbf{r}_j^d \cdot \mathbf{r}_z \right| \right)} \] (13)

Because of the condition of the rotating motion, the following equation is satisfied.

\[ h_j^d = 0 \] (14)

**Coordinated Control Method**

We establish the control method for rotating motion. First, we consider the conditions
that each satellite is stable in objective position/attitude dynamically.

Condition 1: The sum of the tether tensions working on each satellite is equal to the centrifugal force for rotating motion in objective position. The centrifugal tension is represented as follows.

\[
T^d_{jk} = \frac{m_j m_k L_{jk} \omega^2}{\sum_{i=1}^{n} m_i}
\]

(15)

Note that the condition 1 is satisfied in case of all satellites in the system are joined by tethers.

Condition 2: The arm direction is equal to the tether direction in objective position/attitude in order that the tether torque working on the satellite is equal to zero. The joint vector satisfies following equation.

\[
a^d_{jk} \times T_{jk} \left( q^d_k - q^d_j \right) = 0
\]

(16)

Figure 7 shows the equilibrium of satellite 1 in case of \( n = 3 \).

By the items mentioned above, we propose the coordinated control method for transition to objective position/attitude.

Tension control: \( T_{jk} = T^d_{jk} \left[ 1 + k_1 \Delta L + k_2 \Delta \dot{L} - k_3 \Delta \omega \right] \)

(17)

Arm control: \( a^d_{jk} = l_{jk} \frac{q^d_k - q^d_j}{\left| q^d_k - q^d_j \right|} \)

(18)

Where, \( k_1 \) and \( k_2 \) are the coefficients for correcting position, and \( k_3 \) is that for correcting attitude. The errors of distance, velocity and angular velocity between satellite \( j \) and satellite \( k \) are as follows.
\[ \Delta L = \left( q_k - q_j \right) - \Delta L \]

\[ \Delta \dot{L} = (q_k - q_j) \cdot (q_k - q_j) / |q_k - q_j| \]

\[ \Delta \omega = \left[ \tau_{ik} \times (\omega_j - \omega_i) + \tau_{ij} \times (\omega_k - \omega_i) \right] r_z \]

\( l_{jk} \) shows the distance between tether joint and center of the satellite \( j \). Note that the tether torque working on each satellite is proportional to \( l_{jk} \).

**Numerical Simulations**

We analyze the rotating motion in case of \( n = 3 \) and verify the coordinated control method by numerical simulations.

**Simulation Conditions**

The simulation conditions are as follows.

Condition 1: The orbital angular velocity of the center of the system (height: 350 km, period: 5492 s) is

\[ \Omega = 0.001144 \text{ rad/s} \]  \hfill (22)

Condition 2: The shape of each satellite is

\[ m_j = 50 \text{ kg}, \quad I_j = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{ kgm}^2. \]  \hfill (23)

Condition 3: The objective position and attitude of each satellite are

\[ d_1^d = 20 \text{ m}, \quad d_2^d = d_3^d = 10\sqrt{2} \text{ m} \]  \hfill (24)

\[ \lambda_1^d = 3\pi / 4 \text{ rad}, \quad \lambda_2^d = 5\pi / 4 \text{ rad} \]  \hfill (25)
\[ \varphi_j^d = \theta_j^d = \psi_j^d = 0 \text{ rad} \].

The objective formation is the isosceles triangle.

Condition 4: The initial attitude of the rotating plane is
\[ \psi_{r0} = \pi / 6 \text{ rad}, \quad \theta_{r0} = \pi / 6 \text{ rad}, \quad \varphi_{r0} = 0 \text{ rad}. \] (27)

The initial position/attitude of each satellite is defined by \( d_{j0}, h_{j0}, \varphi_{j0}, \theta_{j0}, \psi_{j0} \). The initial angular velocities of all satellites are same: \( \omega_{j0} \). Thus we consider the following \((6n+4)\) parameters in numerical simulations. When we do not show these values, we set them as follows.

\[ \Omega = \begin{bmatrix} 20 \Omega, d_j^d, h_j^d, \varphi_j^d, \theta_j^d, \psi_j^d, 0.5, 0.5, 1000 \end{bmatrix}^T \] (28)

In numerical simulations, we assume that each arm is controlled and equations (18) and (29) are satisfied in order to verify tension control method.

\[ l_{jk} = 1 \text{ m} \] (29)

**Rotating Plane Motion**

First, we analyze the rotating plane motion with respect to various \( \omega_0 \). Figure 8 shows the transition of \( \psi_r, \theta_r \) and \( \varphi_r \) in an orbital period. Each line shows data when \( \omega_0 \) is set as follows.

- Solid line: \( \omega_0 = 20 \Omega \) (30)
- Broken line: \( \omega_0 = 10 \Omega \) (31)
- Dotted line: \( \omega_0 = 5 \Omega \) (32)

The solid line satisfies the following equations.
These lines show that, when $\omega_r \gg \Omega$ is satisfied, the influence of gravitation can be ignored and rotating plane motion becomes constant. In following section, we set $\omega_0 = 20\Omega$ in order to simplify the rotating motion. Note that Steven G. Tragesser considered the stability based on the conical Likins-Pringle relative equilibrium for keeping the spin axis of the formation roughly pointed toward the Earth\textsuperscript{[6]}.

**Position Control**

Next, we consider the position control when the initial formation is the following triangle leaning from rotating plane.

\begin{align}
    d_{10} &= d_{20} = d_{30} = 10 \text{ m} \\
    \lambda_{10} &= 0 \text{ rad} \quad \lambda_{20} = 2\pi/3 \text{ rad} \quad \lambda_{30} = 4\pi/3 \text{ rad} \quad \text{(35)} \\
    h_{10} &= -1 \text{ m} \quad h_{20} = 0.5 \text{ m} \quad h_{30} = 0.5 \text{ m} \quad \text{(36)}
\end{align}

Figure 9 shows the graphs of $d_2$, $\lambda_2$, and $h_2$ in an orbital period. Each line shows the following case.

- **Solid line**: $k_1 = 0.5$, $k_2 = 0.5$ \text{(37)}
- **Broken line**: $k_1 = 0$, $k_2 = 0$ \text{(38)}

Figure 10 shows the constellation transition in former case. The solid line shows that satellite 2 can change the objective position in rotating plane. This is the effect of the tension for correcting position. The vibration out of rotating plane, however, cannot be damped, because tether direction is perpendicular to vibration direction. The transitional inclination of satellites 1 and 3 is nearly equal to that of satellite 2.
**Attitude Control**

Finally, we investigate the attitude control when the initial attitude of satellite 2 is not equal to the objective attitude as follows.

\[
\varphi_2 = \pi/9, \quad \theta_2 = \pi/9, \quad \psi_2 = \pi/9
\]  

(39)

Figure 11 shows the data of \( \varphi_2, \theta_2 \) and \( \psi_2 \). We analyze two cases as follows.

- **Solid line**: \( k_3 = 1000 \)
  
  (40)

- **Broken line**: \( k_3 = 0 \)
  
  (41)

Figure 12 shows the attitude transition of satellite 2 in former case. The attitude vibration on center of objective attitude can be decreased by the tension for correcting attitude. We assume that tether torque working on the satellite is equal to zero in this simulation. Therefore, the attitude of satellite can change objective attitude by the coordinate control of arm and tension.

**Thruster Control**

This system has the advantages of supplying the driving force to each satellite by tension. The thrust for keeping object position/attitude of each satellite in rotating motion is

\[
F_j = m_j d_j \cdot \omega_i^2.
\]  

(42)

In numerical simulation, the thrust for rotating motion of satellite 2 is 0.370 N. When the specific impulse is 220 s, the fuel of thruster per an orbital period is 0.943 kg. It is about as much as 2 percent of mass of satellite 2, and it can be saved by the system with tether tension. Tether tension is, however, the internal force of the whole system, thus the thruster is required for the following control.

1. Control of the orbital motion of the system.
2. Control of the rotating plane motion.

3. Control of the motion of out of rotating plane.

In numerical simulations, we analyze the case of $n=3$. In the other case, following problem is occurred.

In case of $n=2$: Each satellite is joined by only one tether as shown in figure 13. The rotating motion on the tether cannot be controlled, because the twist moment of the tether is ignored. Thus, the torque is necessary for control of the rotating motion.

In case of $n \geq 4$: The interaction between tethers cannot be ignored, because tethers are crossed as shown in figure 14.

**Ground Experiments**

We verify the advantage of saving the fuel of the thruster for rotating motion in case of $n=2$ by two-dimensional ground experiments.

**Ground Experiment System**

Figure 15 shows the concept of two-dimensional ground experiment system. This system consists of three subsystems: satellites 1, 2 and ground station.

Figure 16 shows the picture of each subsystem. The outline is as follows.

Satellites 1 and 2: Each satellite is the rectangular solid ($0.6 \text{ m} \times 0.6 \text{ m} \times 0.45 \text{ m}$), floating on the flat floor ($3 \text{ m} \times 5 \text{ m}$) with air pad. It obtains the driving force with tether tension and thruster. The air for floating pad thrust is supplied by tank ($20 \text{ atm}$). The gyro acquires the attitude data of the satellite and the PC communicates with the ground station by wireless LAN (TCP/IP protocol). In addition, the reel mechanism, installed on satellite 1 controls
tether tension. The mass of each satellite is

\[ m_1 = 58\text{kg}, \quad m_2 = 42\text{kg}. \] (43)

Ground Station: The CCD camera gains the pictures of the whole flat floor and the PC calculates the position of satellites 1 and 2 by pattern matching. The Joy Stick controls the position/attitude of each satellite through the wireless network.

**Reel Mechanism**

Tether control system consists of reel and joint mechanisms. In this paper, the reel mechanism is considered.

The most fundamental function of the reel mechanism is to deploy and retract the tether. But the reel mechanism for the tethered satellite cluster systems needs the following functions as well.

(1) To avoid jam in the case where tension is nearly zero.

(2) To measure tension and direction of the tether.

(3) To wind the tether on the reel uniformly.

(4) To measure deployed length of the tether.

Taking into account those requirements, we developed the reel mechanism for ground experiment as shown in Figure 17. The size is 140 mm \( \times \) 400 mm \( \times \) 200 mm and the weight is 3.95 kg. Each function is realized as follows.

(1) The reel mechanism is separated into two parts: an inner mechanism and an outer mechanism. In the inner mechanism, motor A deploys and retracts the tether under the specified constant tension. In the outer mechanism, motor B conducts tension/length control of the tether.
(2) Strain gage A measures the inner tension, strain gage B measures the outer tension, and strain gage C measures the direction of deployed tether.

(3) By a level winder, a winded point moves periodically and winds the tether on the reel uniformly.

(4) A laser displacement sensor measures tether-winding thickness and an encoder measures the reel rotation.

**Results of Ground Experiment**

We conduct the ground experiment in which two satellites face each other and rotate around the center of the system as follows.

Angular velocity: \( \omega = 0.0698 \text{rad/s} \) (Period: 90 s) \( \quad (44) \)

Rotating radius: \( d_1^d = 0.88 \text{m} \), \( d_2^d = 1.22 \text{m} \) \( \quad (45) \)

The initial velocity and angular velocity of each satellite satisfy the condition of the rotating motion. Figure 18 shows the attitude angle and fuel consumption of satellite 1 in one round, respectively. We conduct two experiments as follows.

Solid line (this system): \( T = m_r d_1^d \omega^2 = 0.25 \text{ N} \) \( \quad (46) \)

Broken line (existing system): \( T = 0 \text{ N} \) \( \quad (47) \)

In figure 18 (a), two lines are nearly equal and increase linearly to 2\( \square \) in about 90 s. Thus, both cases satisfy the objective rotating motion. On the other hand, in figure 18 (b), the value of the solid line is smaller than that of the broken line. The former is not equal to zero, for the friction between each satellite and flat floor damps the rotating motion. The difference between the latter and the former is nearly equal to the gas consumption of thruster for driving the central force. Therefore, we conclude that the tension can save the
fuel of thruster.

**Conclusions**

In this paper, we consider the rotating motion of tethered satellite cluster systems as follows.

1. By numerical simulations, we established the coordinated control method and analyzed the case where three satellites were joined by tether.
2. By two-dimensional ground experiments, we indicated that the tether tension saved the fuel of the thruster for driving the rotating motion in this system.

**Future Works**

Finally, we show the future works.

(1) Formation flight experiment:
As shown in figure 19, we will conduct the rotating experiment of three satellites joined by three tethers in order to verify the advantage of saving the fuel of the thruster.

(2) Reconfigurable analysis
We will analyze the motion of the system transition from close-loop configuration to open-loop configuration by tether separation.

(3) Tether service experiment:
We are developing the arm system for ground experiment. We will conduct the application experiments for attitude control of uncontrolled satellite with use of tension and arm control as shown in figure 20.
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