AOE 2104: Sample Exam 1 Solution

1. There are four problems worth 25 points each.
2. Do each problem on a separate sheet of paper, and be sure your name and the problem number are clearly indicated on each sheet.
3. For each problem, set up the required calculations, carefully indicating the units and verifying that the units combine to produce correct units for the required answer. **You must also explain clearly every step of your work.**
4. Print your name, and sign and date the Honor Pledge below.

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1. Explain the following terms and the relationships between them: flow field, pressure, temperature, density, velocity, steady flow, unsteady flow, and aerodynamic forces. Use complete sentences, correct grammar and neat handwriting.

   Solution: Standard information from text and lectures.

2. In the test section of a subsonic wind tunnel, the average air pressure and temperature are $p = 1.1\ \times 10^5$ N/m^2 and $T = 288$ K, respectively. Set up the calculation for the density, ρ, and verify that the units are correct.

   Solution:

   This problem requires a straightforward application of the equation of state:

   $$p = \rho RT$$

   Solve for ρ

   $$\rho = p/(RT)$$

   Substituting the values given:

   $$\rho = 1.1 \times 10^5 [\mathrm{N/m^2}]/(287\ [\mathrm{J/[kg\ K]}] \times 288[\mathrm{K}])$$

   Recalling that the Joule unit can be written as [J] = [N m], we can rewrite this equation as

   $$\rho = \frac{1.1 \times 10^5}{287 \times 288} \frac{\mathrm{N/m^2\ kg\ K}}{\mathrm{N\ m\ K}} = \frac{1.1 \times 10^5}{287 \times 288} \mathrm{kg/m^3}$$

   If the test section is 2 m $\times$ 2 m $\times$ 4 m, what is the weight of the air in the test section? Again, set up the calculation and verify the units.

   The volume of the air is $v = 16$ m^3, and the density was calculated as ρ in the previous calculation. The mass of air is then

   $$m = \rho v = 16\rho$$

   And the weight is $w = mg_0$ or

   $$w = \rho v g_0 = 9.81[\ \mathrm{m/s^2}] \times 16[\ \mathrm{m^3}] \times \rho[\ \mathrm{kg/m^3}] = 16 \times 9.81\rho[\mathrm{kg\ m/s^2}]$$

3. A flight test engineer flying at a geometric altitude of 2500 m measures the ambient temperature and finds it to be 10% hotter than the standard value (given in Kelvins). Simultaneously she measures the ambient pressure and finds that the pressure altitude is also 2500 m. Set up all equations required to compute the density of the air at this point.

   Solution:

   The geometric altitude is 2500 m. The standard temperature at this altitude, T_{2500} can be found by linear interpolation as

$$T_{2500} = T_{2000} + \frac{T_{3000} - T_{2000}}{3000 - 2000} \times 500$$

   Looking the numbers up in the given table, and noticing that 500/(3000-2000)=1/2, we find

$$T_{2500} = 275.16 + \frac{271.92 - 275.16}{2}$$

   The measured temperature is 10% hotter than the standard temperature, so

$$T = 1.1 T_{2500}$$

   This is the temperature of the air at the specified point.

   Since the pressure altitude is 2500 m, the pressure can be found by using the standard atmosphere equation, or by using linear interpolation. Using the standard atmosphere pressure equation leads to

$$p_{2500} = p_{2000} \times \left(\frac{T_{2500}}{T_{2000}} \right)^{-g_0/(aR)}$$

   where $g_0 = 9.81$ m/s^2, $a = (T_{2500} - T_{2000})/500$ K/m, and $p_{2000} = 7.9501 \times 10^4$ N/m^2. For comparison, this calculation leads to

   Using linear interpolation instead leads to

$$p_{2500} = p_{2000} + \frac{p_{3000} - p_{2000}}{3000 - 2000} \times 500$$

   which, with the values from the table gives

$$p_{2500} = 7.9501 \times 10^4 + \frac{7.4692 \times 10^4 - 7.9501 \times 10^4}{2}$$

   Either of these two approaches is acceptable, and the pressure is $p = p_{2500}$.

   Now, knowing p and T, we can compute the density ρ using

$$\rho = p/(RT)$$

   Note that the linear interpolation is halfway between the two endpoints. In this case, you can just take the average of the two endpoint values. Many people made this observation and used it effectively.

4. A hot air balloon pilot finds that his balloon is neutrally buoyant, and makes measurements of atmospheric pressure and temperature. He determines that his pressure altitude is 1000 m and that his density altitude is 1100 m. He knows that his balloon with all its cargo has a mass of 200 kg (excluding the air in the balloon), and displaces a volume of 5000 m^3. Assuming that the pressure inside the balloon is the same as that outside the balloon, set up all equations to determine the temperature of the air inside the balloon.

Solution:

From the buoyancy principle, we want the weight of the balloon system (balloon+payload+air) to be equal to the weight of the atmospheric air displaced by the balloon. The balloon volume is $v = 5000$ m^3.

First compute the weight of the atmospheric air displaced by the balloon, w_v:

$$w_v = \rho v g$$

We can get $\rho = \rho_{1100}$ from the table, but we don't know what value of g to use. However, we will find that it isn't required in the calculation. If it were, we could safely assume that $g \approx g_0$. In any case, we have

$$w_v = 1.1008 \times 5000 \times g$$

Now compute the weight of the balloon system, w_{bs}:

$$w_{bs} = \left(m_{\text{b}} + \rho_{\text{air}} v\right) g$$

where $m_b = 200$ kg is the mass of the balloon, ρ_{air} is the unknown density of the air, $v = 5000$ m^3 is the volume of the balloon, and g is the acceleration due to gravity.

Setting $w_{bs} = w_v$, we see that the g terms cancel, leaving

$$m_{\text{b}} + \rho_{\text{air}} v = \rho v$$

which we can solve for ρ_{air}:

$$\rho_{\text{air}} = \frac{\rho v - m_{\text{b}}}{v}$$

Finally, using the equation of state, we get

$$T = \frac{pv}{R(\rho v - m_{\text{b}})}$$

Some Useful Constants and Equations

Constants

Symbol	Value	Description
g	9.81 m/s^2	gravitational acceleration at Earth's surface (metric)
	32.2 ft/s^2	(English)
p_s	1.01325 $\times 10^5$ N/m^2	standard atmosphere pressure at sea level (metric)
	2116.2 lb/ft^2	(English)
ρ_2	1.2250 kg/m^3	standard atmosphere density at sea level (metric)
	0.002377 $slug/ft^3$	(English)
r	6378 km	Earth's radius (metric)
	3961 mi	(English)
R	287 J/(kg K)	specific gas constant for air (metric)
	1716 ft lb/(slug °R)	(English)
T_s	288.16 K	standard atmosphere temperature at sea level (metric)
	518.69 °R	(English)

Equations

$g = g_0 r^2/(r + h_G)^2$ gravitational acceleration *vs* geometric altitude

$h = r h_G/(r + h_G)$ geopotential altitude *vs* geometric altitude

$p = \rho R T$ equation of state

$dp = -\rho g\, dh_G$ hydrostatic equation

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ equation of continuity

Standard atmosphere isothermal regions

$$\frac{p}{p_1} = e^{-(g_0/(RT))(h-h_1)} \qquad \frac{\rho}{\rho_1} = e^{-(g_0/(RT))(h-h_1)}$$

Standard atmosphere gradient regions

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/(aR)} \qquad \frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-(g_0/(aR)+1)} \qquad T = T_1 + a(h - h_1)$$

Standard Atmosphere

Altitude, h_G, m	Temperature, T, K	Pressure, p, N/m^2	Density ρ, kg/m^3
0	288.16	1.0132×10^5	1.225
1000	281.66	8.9876×10^4	1.1117
1100	281.01	8.8792×10^4	1.1008
2000	275.16	7.9501×10^4	1.0066
3000	271.92	7.4692×10^4	9.5696×10^{-1}
4000	262.18	3.1660×10^4	8.1935×10^{-1}
5000	255.69	5.4048×10^4	7.3643×10^{-1}