AOE 2104: Homework Assignment 3 Solution

1. What are the two sources of all aerodynamic forces? Name the two sources and provide a sketch of an airfoil, showing the two sources. Explain how these two sources lead to lift and drag.

   Solution: The two sources of all aerodynamic forces are the pressure distribution (p) and the shear stress distribution (τ_w). The figure is in the handouts and in the text. Integration of the pressure distribution over the surface of a body results in a net force; normally the component normal to the direction of the flow is called *lift*, and the component in the direction of flow is called *drag*. Integration of the shear stress distribution over the surface of the body results in a net force; normally this force is in the direction of flow and contributes to the drag.

2. Carry out the integration of vertical pressure forces on a spherical balloon. Use the notation we developed in class.

   Solution: This exercise was intended to give you incentive to "fill in the missing steps." If you have trouble carrying out this integration, please come see me as soon as possible.

3. Define, in your own words, the six different types of "altitude" used in this course. Explain the use of each.

   Solution:

   Geometric altitude is the altitude measured from the mean sea level surface of the Earth. This measurement is the principal altitude used in describing position of aerospace vehicles.

   Absolute altitude is the altitude measured from the center of the Earth. It is more important for spacecraft than aircraft, but is used in describing how gravity varies with altitude.

   Geopotential altitude is a fictitious altitude defined to make standard atmosphere calculations easier. It is only used in doing these calculations. Whenever one refers to altitude, it is almost certainly the geometric altitude, using the geometric altitude column of the standard atmosphere tables.

   Pressure altitude is the geometric altitude corresponding to a specific pressure. That is, one measures the pressure and looks in a standard atmosphere table to find the corresponding geometric altitude.

   Temperature altitude is the geometric altitude corresponding to a specific temperature. That is, one measures the temperature and looks in a standard atmosphere table to find the corresponding geometric altitude.

   Density altitude is the geometric altitude corresponding to a specific density. That is, one measures the density and looks in a standard atmosphere table to find the corresponding geometric altitude.

4. An F-15 supersonic fighter aircraft is in a rapid climb. At the instant it passes through a standard altitude of 25,000 ft, its time rate of change of altitude is 500 ft/s,

which by definition is the *rate of climb*, which we will discuss later in the course. Corresponding to this rate of climb at 25,000 ft is a time rate of change of ambient pressure. Calculate this rate of change of pressure in units of pounds per square foot per second.

Solution: Imagine that you have a pressure gauge on the aircraft as it climbs through a "standard" atmosphere. What pressure will it measure? Answer: the standard atmosphere pressure corresponding to the given altitude. If you plotted that pressure as a function of time, its slope at 25,000 ft would be the answer to this question. Now, how do you calculate it?

You want to calculate dp/dt, so the pattern recognition thing to do is look around for an equation with dp in it, perhaps even involving altitude. The hydrostatic equation will work (though you could also use the $p(h)$ developed for the standard atmosphere)1:

$$dp = -\rho g \, dh_G$$

where ρ and g both depend on h_G. "Dividing" by dt, we find

$$\frac{dp}{dt} = -\rho(h_G)g(h_G)\frac{dh_G}{dt}$$

We can look up $\rho(h_G)$, compute $g(h_G)$, and use the given value of dh_G/dt to get the desired rate of change of pressure. For the given data, we have

$$\frac{dp}{dt} = -1.0663 \times 10^{-3}[\text{slugs/ft}^3] \times 32.1[\text{ft/s}^2] \times 500[\text{ft/s}] = -17.11 \text{ lb/ft}^2/s$$

5. A hot air balloon has a structural mass of 50 kg, and the payload mass is 10 kg. The mass of the air inside the balloon depends on the temperature and pressure. For this problem, assume that the pressure is the same as the standard ambient pressure for the given geometric altitude, h. Assume the balloon is spherical with radius of 10 m, and neglect the volume of the payload.

   Determine the temperature of the "hot" air such that the balloon is neutrally buoyant at a geometric altitude of 10 km.

   Solution: Recall that Archimides' principle says that the buoyancy force on an object immersed in a fluid is equal to the weight of the volume of fluid displaced by the object. Here the weight of the volume of fluid can be written as

$$W_v = \frac{4}{3}\pi r^3 \rho g$$

   where $r = 10$ m, ρ depends on altitude, and g also depends on altitude. Note that assuming ρ is constant is a bad assumption, whereas assuming g is constant is not so bad. Even in low-Earth orbit (LEO), gravity is about 90% of its surface value.

   To make these relationships clear, let's rewrite the equation for the buoyancy force as

$$W_v = V_b \rho(h_G) g(h_G)$$

   where $V_b = 4\pi r^3/3 = 4189$ m^3 for the given balloon dimensions.

We want the weight of the volume of displaced fluid, W_v, to be equal to the weight of the balloon with its payload *and the air inside it.* The weight of the balloon, its payload, and its air can be written as

$$W_b = W_{\text{balloon}} + W_{\text{payload}} + W_{\text{air}}$$

or

$$W_b = 50g + 10g + m_{\text{air}}g$$

Applying Archimides' principle and the neutral buoyancy constraint, we get

$$m_{\text{air}}g(h_G) = V_b\rho(h_G)g(h_G) - 60g(h_G)$$

which we can rewrite, upon dividing by $g(h_G)$ as

$$m_{\text{air}} = V_b\rho(h_G) - 60$$

Remember that we want to find the temperature of the air in the balloon, under the assumption that the pressure is the same as the local standard atmosphere pressure. Also note that the mass of the air is related to the density and volume by

$$m_{\text{air}} = V_b\rho_{\text{air}}$$

Thus we can solve for ρ_{air} to get

$$\rho_{\text{air}} = \rho(h_G) - 60/V_b$$

Clearly the density of air can never be negative, so if the balloon is at an altitude such that $\rho(h_G) < 60/V_b$, then no heating of ordinary air can provide neutral buoyancy. The value of geometric altitude where this inequality is reached is of course of interest to us the balloon designers. We can find it by scanning the table to see where the density first becomes less than $60/V_b = 0.014323$, which turns out to be somewhere around 31,400 km.

Using the equation of state for a perfect gas, we can calculate the temperature from pressure and density as

$$T_{\text{air}} = p(h_G)/(\rho(h_G) - 60/V_b)/R$$

For the specific point problem given here, we can calculate the temperature, using the pressure and density at $h_G = 10$ km. The values we need are $p = 2.6500 \times 10^4$ N/m^2 and $\rho = 4.1351 \times 10^{-1}$ kg/m^3. We also need to know that $R = 287$ J/(kg K). Turning the crank leads to the neutral buoyancy temperature of

$$T_{nb} = 231.31 \text{ K}$$

Extra Credit (10 points to be added to your homework total before averaging): Use Matlab to make a plot of the neutrally buoyant temperature T_{nb} for this balloon as a function of altitude. Make sure that your plot is "well-constructed;" that is, the labels and lines should be clear, the caption should be descriptive, and a paragraph should be included to describe the figure as if to someone who is unable to see it.

The plot looks like this:

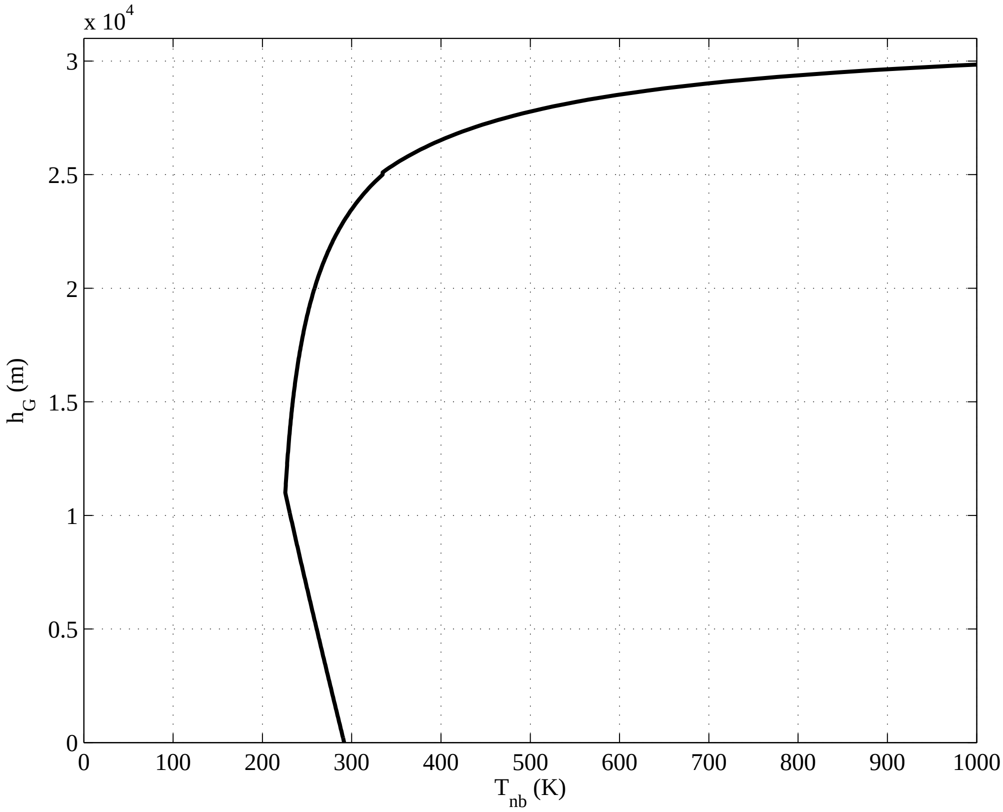

Figure 1: Neutral Buoyancy Temperature as Function of Altitude