AOE 2104: Homework Assignment 2
Due: Thursday, 9/14 in class

Do each problem on a separate sheet, with name, problem number, problem statement.

I will only collect and grade two of the problems, and will post solutions for all four problems.

1. At 12 km in the standard atmosphere, the pressure, density and temperature are 1.9399×10^4 N/m^2, 3.1194 $\times 10^-1$ kg/m^3, and 216.66 K, respectively. Using these values, calculate the standard atmospheric values of pressure, density, and temperature at an altitude of 18 km, and check with the standard altitude tables.

   Solution:

   An examination of the standard temperature distribution through the atmosphere given in Fig. 3.3 of the text shows that both 12 km and 18 km are in the same isothermal region. Hence, the equations that apply are Eqs. (3.9) and (3.10) in the text. Since the two altitudes are in the same isothermal region we use the same base values of p and ρ, and the equations can be written as

   $\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-(g_0/[RT])(h_2-h_1)}$

   where points 1 and 2 are any two arbitrary points in the isothermal region. Hence, with $g_0 = 9.8$ m/s^2 and $R = 287$ J/(kg K), and letting points 1 and 2 correspond to 12 km and 18 km, respectively, we have

   $\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-(9.8/[287 \times 216.66])(6000)} = 0.3884$

   Hence

   $p_2 = 0.3884 \times 1.9399 \times 10^4 = 7.53 \times 10^3$ N/m^2

   $\rho_2 = 0.3884 \times 3.1194 \times 10^{-1} = 0.121$ kg/m^3

   and, of course,

   $T_2 = 216.66$ K

   These answers agree with the table in Appendix A within round-off error.

2. During a flight test of a new airplane, the pilot radios to the ground that she is in level flight at a standard altitude of 35,000 ft. What is the ambient pressure far ahead of the airplane?

   Solution:

   At 35,000 ft, from Appendix B, we find that $p = 4.99 \times 10^2 = 499$ lb/ft^2.

3. At what value of the geometric altitude is the difference $h - h_G$ equal to 2% of the geopotential altitude, h?

   Solution:

   $|h - h_G|/h = 0.02 = |1 - h_G/h|$

   Using Eq. (3.6), this relationship becomes

   $|1 - (r + h_G)/r| = |1 - 1 - h_G/r| = 0.02$

   Therefore

   $h_G = 0.02r = 0.02 \times 6378 \times 10^3 = 127.56$ km

4. The atmosphere of Jupiter is essentially made up of hydrogen, H_2. For H_2, the specific gas constant is 4157 J/(kg K). The acceleration of gravity of Jupiter is 24.9 m/s^2. Assuming an isothermal atmosphere with a temperature of 150 K and assuming that Jupiter has a definable surface, calculate the altitude above that surface where the pressure is one-half the surface pressure.

Solution:

$\ln \frac{p}{p_1} = -\frac{g}{RT}(h - h_1)$

$h - h_1 = -\frac{1}{g} RT \ln \frac{p}{p_1} = -\frac{1}{24.9} \times 4157 \times 150 \times \ln 0.5$

Letting $h_1 = 0$ (the surface),

$h = 17,358$ m $= 17.358$ km.