The Standard Atmosphere

- The Standard Atmosphere
  - Some definitions
    - Absolute altitude
    - Geometric altitude
    - Geopotential altitude
  - Some physics – The hydrostatic equation
  - Construction of the standard atmosphere – Variation of p, T and ρ with altitude
  - Definitions of pressure, density, and temperature altitudes

Mysterious Fishy-Looking Thingy

Tourist Transportation?

Upside down?

NASA Dryden flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC03-0154-3 Date: June 9, 2003 Photo By: Tom Tschida

The remotely-piloted Altair unmanned aerial vehicle (UAV) took to the air on its first checkout flight on June 9, 2003 at El Mirage, California.

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Altitude (Six Different Flavors)

- Geometric altitude

  h_G: altitude measured from sea level

- Absolute altitude

  h_a: altitude measured from center of the earth

  $h_a = h_G + r$, where r is the radius of the earth

- Local acceleration due to gravity depends on altitude

  $g = g_0(r/h_a)^2 = g_0\,(r/[r+h_G])^2$

  This variation has an effect on the pressure

Virginia Tech

AOE

Hydrostatic Equation

- Compute the balance of forces acting on an element of fluid at rest.
- The element of fluid has unit width in both horizontal directions and height dh_G in the vertical direction
- Newton's law: sum of forces = zero

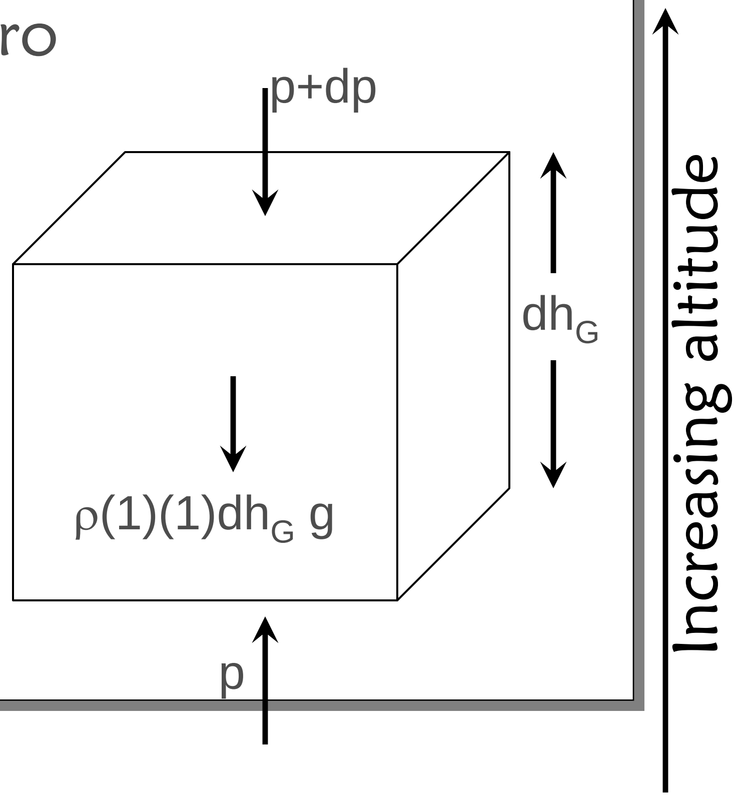
$$p = p + dp + \rho\, g\, dh_G$$

$$dp = -\rho\, g\, dh_G$$

- The hydrostatic equation applies to any fluid of density ρ
- The H.E. is a differential equation:

p, ρ, and g depend on h_G

- First step: assume $g = g_0$



Integrating the Hydrostatic Eq.

- Making the assumption that $g = g_0$, the H.E. becomes

  $dp = -\rho \; g_0 \, dh$

- Note that we have replaced h_G with h

  Since g is not really g_0, we use h that's not really h_G

  The variable h is called the geopotential altitude

- What is the relationship between the geopotential and geometric altitudes?

  $dp = -\rho \; g \, dh_G$ and $dp = -\rho \; g_0 \, dh$

  Should both be true for the same differential change in pressure, so divide them:

  $1 = (g_0/g)(dh/dh_G) \quad \Rightarrow \quad dh = (g/g_0)\, dh_G$

Geopotential and Geometric h

- We assumed $g = g_0$, and defined the geopotential altitude h, based on that assumption, leading to

  $$dh = (g/g_0)\ dh_G$$

- Need to recall that gravity depends on altitude:

  $$g = g_0(r/h_a)^2 = g_0\,(r/[r+h_G])^2$$

- Thus

  $dh = (g/g_0)\ dh_G = (r/[r+h_G])^2\ dh_G$ [Exercise] and hence

$$h = r\,h_G/(r+h_G)$$

Finally,
defining the Standard Atmosphere

- We want to know
  p(h), T(h), and ρ(h)
- The fundamental idea behind the standard atmosphere is a *defined* variation of temperature   T = T(h)
- The figure to the right defines the temperature variation
  - Isothermal and gradient regions
- Most aircraft fly below 20 km, but balloons, sounding rockets, and launch vehicles traverse the entire range

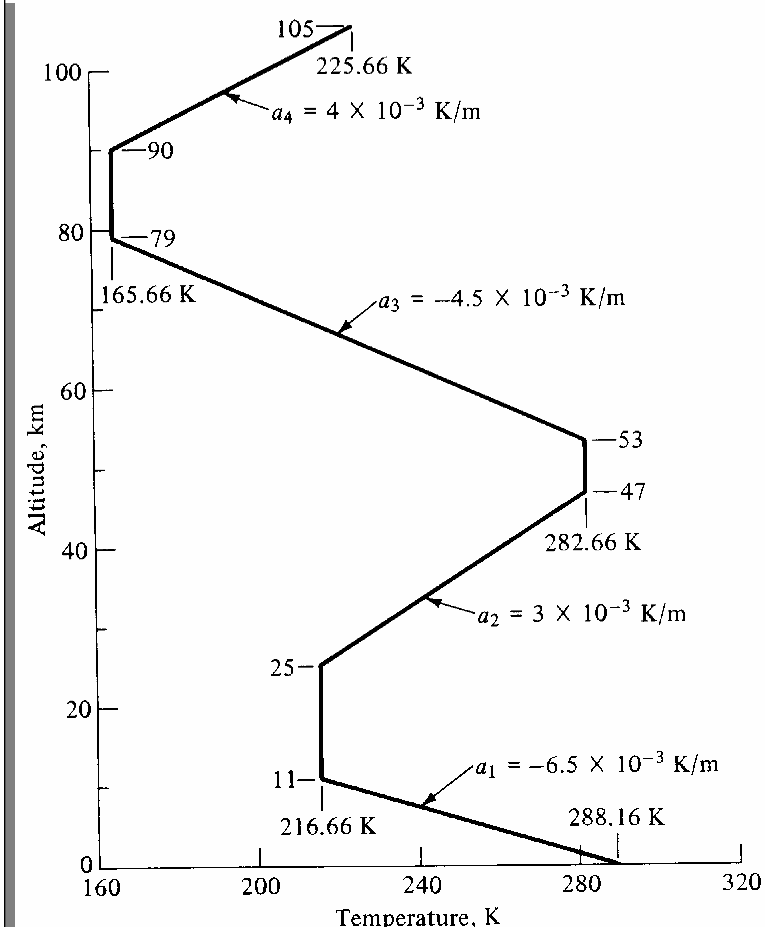


Figure 3.3 Temperature distribution in the standard atmosphere.

Integrating Pressure

- Given the definition of T(h), revisit the equation:

  $dp = - \rho\, g_0\, dh$

- Remember the equation of state?

  $p = \rho RT$

- Divide the hydrostatic equation by the equation of state:

  $dp/p = - g_0\, dh/(RT)$

- In the isothermal regions, T is constant, so [Exercise]

  $p/p_1 = e^{-[g_0/(RT)](h-h_1)}$

  $\rho/\rho_1 = e^{-[g_0/(RT)](h-h_1)}$

- In the gradient regions, T depends on h, so the integration is more difficult

Integrating Pressure (2)

- Begin with:

  $dp/p = -\, g_0\, dh/(RT)$

- In the gradient regions, T depends linearly on h

  $(T-T_1)/(h-h_1) = dT/dh = a$ (a = lapse rate)

- So,

  $dh = dT/a \quad \Rightarrow \quad dp/p = -\, g_0\, dT/(aRT)$

- Integrating [Exercise] leads to

  $p/p_1 = (T/T_1)^{-g_0/(aR)}$

  $\rho/\rho_1 = (T/T_1)^{-[g_0/(aR)+1]}$

- Since T=T(h), we can write p=p(h) and $\rho=\rho(h)$
- And it all starts at sea level, where $h=h_G=0$

At Sea Level

- At s.l., $h=h_G=0$

  $p_s = 1.01325 \times 10^5$ N/m^2 = 2116.2 lb/ft^2

  $\rho_s = 1.2250$ kg/m^3 = 0.002377 slug/ft^3

  $T_s = 288.16$ K = 518.69°R
- From sea level, we can calculate the rest
- The calculated values are available in Appendices A & B in the text, in SI and English units, respectively

Summary of Standard Atmosphere

- Temperature distribution defined by Fig. 3.4
- Equation of state relates pressure, density and temperature
- Integration of hydrostatic equation and use of equation of state leads to:

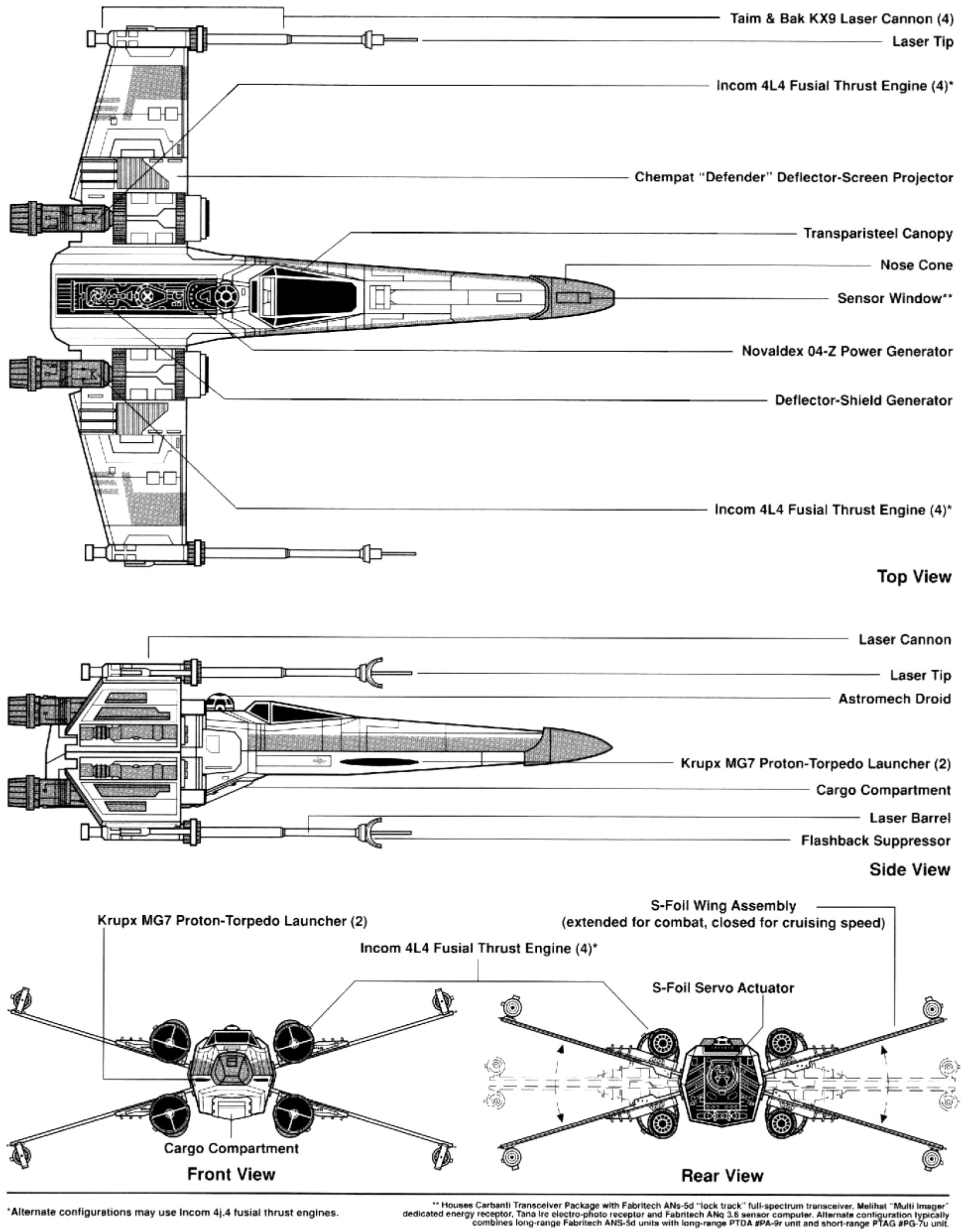
Isothermal	Gradient
$p/p_1 = e^{-[g_0/(RT)](h-h_1)}$	$p/p_1 = (T/T_1)^{-g_0/(aR)}$
$\rho/\rho_1 = e^{-[g_0/(RT)](h-h_1)}$	$\rho/\rho_1 = (T/T_1)^{-[g_0/(aR)+1]}$
$T = \text{constant}$	$T = T_1 + a(h-h_1)$

- See Appendices A & B in text

Who Is This Guy and What Is He Doing?

From Where to Here?

Is It Real?



Taim & Bak KX9 Laser Cannon (4)
Laser Tip
Incom 4L4 Fusial Thrust Engine (4)*
Chempat "Defender" Deflector-Screen Projector
Transparisteel Canopy
Nose Cone
Sensor Window**
Novaldex 04-Z Power Generator
Deflector-Shield Generator
Incom 4L4 Fusial Thrust Engine (4)*
Top View
Laser Cannon
Laser Tip
Astromech Droid
Krupx MG7 Proton-Torpedo Launcher (2)
Cargo Compartment
Laser Barrel
Flashback Suppressor
Side View
Krupx MG7 Proton-Torpedo Launcher (2)
S-Foil Wing Assembly
(extended for combat, closed for cruising speed)
Incom 4L4 Fusial Thrust Engine (4)*
S-Foil Servo Actuator
Cargo Compartment
Front View
Rear View
*Alternate configurations may use Incom 4j.4 fusial thrust engines.
** Houses Carbanti Transceiver Package with Fabritech ANs-5d "lock track" full-spectrum transceiver, Melihat "Multi Imager" dedicated energy receptor, Tana Ire electro-photo receptor and Fabritech ANq 3.6 sensor computer. Alternate configuration typically combines long-range Fabritech ANS-5d units with long-range PTDA #PA-9r unit and short-range PTAG #PG-7u unit.

Motion Sickness Generator

Example 3.1

Calculate the standard atmosphere values of T, p, and ρ at a geopotential altitude of 14 km.

From Fig. 3.4, T = 216.66 K (isothermal region)

Begin at sea level and compute values at the "corner":

Gradient region from h = 0 to h = 11.0 km,

with lapse rate a=-6.5 K/km

So $p = p_1(T/T_1)^{-g_0/(aR)}$ and $\rho = \rho_1(T/T_1)^{-[g_0/(aR)+1]}$

$p_1=p_s=1.01325 \times 10^5$ N/m^2 and $\rho_1=\rho_s=1.2250$ kg/m^3

$T_1=T_s=288.16$ K

$p = 2.26 \times 10^4$ N/m^2 and $\rho = 0.367$ kg/m^3

at h = 11.0 km

Example 3.1 continued

From 11.0 km to 14.0 km is an isothermal region, so

$p = p_1 e^{-[g_0/(RT)](h-h_1)}$ and $\rho = \rho_1 e^{-[g_0/(RT)](h-h_1)}$

Here the subscript "1" refers to the values at h=11.0 km

Doing the calculations leads to

$p = 1.41 \times 10^4$ N/m^2 and $\rho = 0.23$ kg/m^3
at h = 14.0 km

Three More Altitudes

- Pressure, Temperature and Density Altitudes
- Remember Geometric, Absolute, and Geopotential Altitudes
- Suppose you're flying and you have a set of instruments that can measure Pressure, Temperature and Density
- You could look them up in the standard atmosphere tables, right?
- Would they all agree?   Not likely.

Sample Table

1976 Standard Atmosphere
(from http://www.digitaldutch.com/atmoscalc/tableoptions1.htm)

Altitude [km]	**Temperature** [Kelvin]	**Pressure** [pascal]	**Density** [kg/m^3]
0	288.15	101325	1.225
1	281.65	89874.5705	1.1116
2	275.15	79495.2155	1.0065
3	268.65	70108.5447	0.9091
4	262.15	61640.2353	0.8191
5	255.65	54019.9121	0.7361

- Suppose measured pressure is 6.16×10^4 N/m^2
  - Then the pressure altitude is $\approx$ 4 km
- Suppose the measured temperature is 269 K
  - Then the temperature altitude is $\approx$ 3 km
- What is the geometric altitude?

Example 3.2

If an airplane is flying at an altitude where the actual pressure and temperature are

4.72×10^4 N/m^2 and 255.7 K

respectively, what are the pressure, temperature and density altitudes?

Use Appendix A to find that

pressure altitude = 6 km

temperature altitude = 5 km (or 38.2 or 59.5 km) Why?

Use equation of state to find that the density is

$\rho = 0.643$ kg/m^3

And then use Appendix A to find that

density altitude = 6.24 km

Summary

- Standard atmosphere is defined in order to relate flight tests, wind tunnel results, and general airplane design and performance to a common reference
- Temperature variation is defined
  - Isothermal and gradient regions
- Hydrostatic equation is applied

  $dp = -\rho \; g \; dh_G$
- Pressure and density variations are derived
- There are six different flavors of altitude:

  Geometric, absolute, geopotential

  Pressure, temperature, density

Additional Topics

- Geometric vs geopotential altitudes
- Linear interpolation

- Geopotential altitude is only used as a convenience for computing the standard atmosphere tables
- Always use the geometric altitude column of the tables when referring to pressure, density, and temperature altitudes

Linear Interpolation

- Usually looking up numbers in tables requires *linear interpolation*
  - Select the two rows that "bracket" the given value
  - Use a straight-line approximation between the values in the two rows

h [km]	T [K]	P [N/m^2]	ρ [kg/m^3]
1	281.65	89874	1.1116
2	275.15	79495	1.0065

Example: you measure pressure at

$p^* = 8.0 \times 10^4$ N/m^2

Then the pressure altitude is found by

$h^* = h_1 + (h_2-h_1)(p^*-p_1)/(p_2-p_1)$

$= 1.9514$ km