The Standard Atmosphere

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Some definitions

Absolute altitude

Geometric altitude

Geopotential altitude

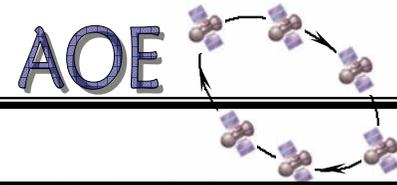Some physics – The hydrostatic equation

Construction of the standard atmosphere – Variation of p, T and ρ with altitude

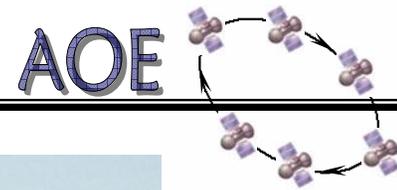Definitions of pressure, density, and temperature altitudes

Mysterious Fishy-Looking Thingy

NASA Dryden flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC03−0154−3   Date: June 9, 2003   Photo By: Tom Tschida

The remotely−piloted Altair unmanned aerial vehicle (UAV) took to the air on its first checkout flight on June 9, 2003 at El Mirage, California.

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Definitions of pressure, density, and temperature altitudes

Altitude (Six Different Flavors)

- Geometric altitude

  h_G: altitude measured from sea level
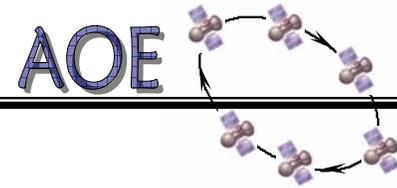
- Absolute altitude

  h_a: altitude measured from center of the earth

  $h_a = h_G + r$, where r is the radius of the earth

- Local acceleration due to gravity depends on altitude

  $g = g_0(r/h_a)^2 = g_0 (r/[r+h_G])^2$

 This variation has an effect on the pressure

Hydrostatic Equation

- Compute the balance of forces acting on an element of fluid at rest.

- The element of fluid has unit width in both horizontal directions and height dh_G in the vertical direction

- Newton's law: sum of forces = zero

$$p = p + dp + \rho \, g \, dh_G$$
$$dp = - \rho \, g \, dh_G$$

- The hydrostatic equation applies to any fluid of density ρ

- The H.E. is a differential equation:

  p, ρ, and g depend on h_G

- First step: assume $g = g_0$

$p+dp$

dh_G

$\rho(1)(1)dh_G \, g$

p

Increasing altitude
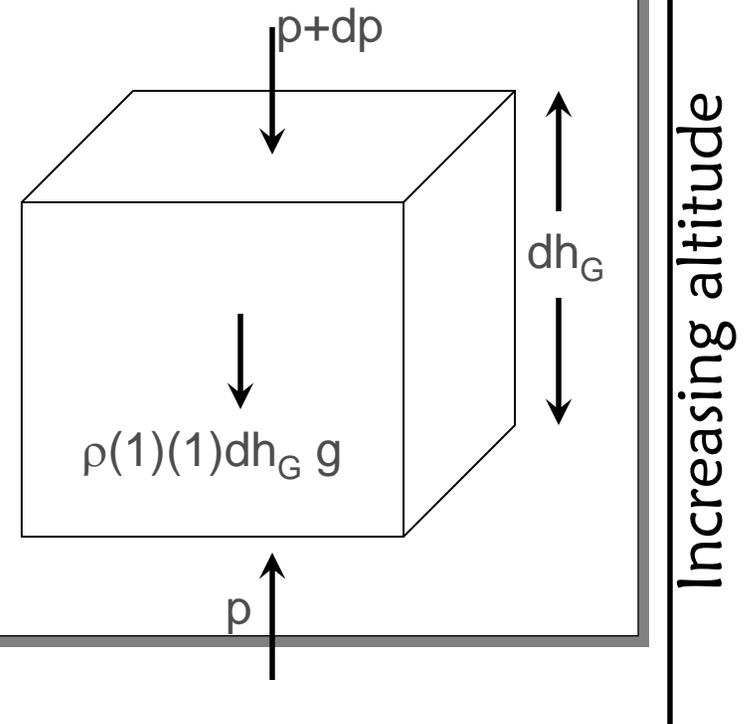
Integrating the Hydrostatic Eq.

- Making the assumption that $g = g_0$, the H.E. becomes

  $dp = - \rho \, g_0 \, dh$

- Note that we have replaced h_G with h

  Since g is not really g_0, we use h that's not really h_G

  The variable h is called the **geopotential altitude**

- What is the relationship between the geopotential and geometric altitudes?

  $dp = - \rho \, g \, dh_G$ and $dp = - \rho \, g_0 \, dh$

  Should both be true for the same differential change in pressure, so divide them:

  $1 = (g_0/g)(dh/dh_G)$ $\Rightarrow$ $dh = (g/g_0) \, dh_G$

Geopotential and Geometric h

- We assumed $g = g_0$, and defined the geopotential altitude h, based on that assumption, leading to

$$dh = (g/g_0) \, dh_G$$

- Need to recall that gravity depends on altitude:

$$g = g_0(r/h_a)^2 = g_0 \, (r/[r+h_G])^2$$

- Thus

$$dh = (g/g_0) \, dh_G = (r/[r+h_G])^2 \, dh_G \quad \text{[Exercise] and hence}$$

$$h = r \, h_G/(r+h_G)$$

Finally,
defining the Standard Atmosphere

- We want to know p(h), T(h), and ρ(h)
- The fundamental idea behind the standard atmosphere is a *defined* variation of temperature   T = T(h)
- The figure to the right defines the temperature variation
  - Isothermal and gradient regions
- Most aircraft fly below 20 km, but balloons, sounding rockets, and launch vehicles traverse the entire range
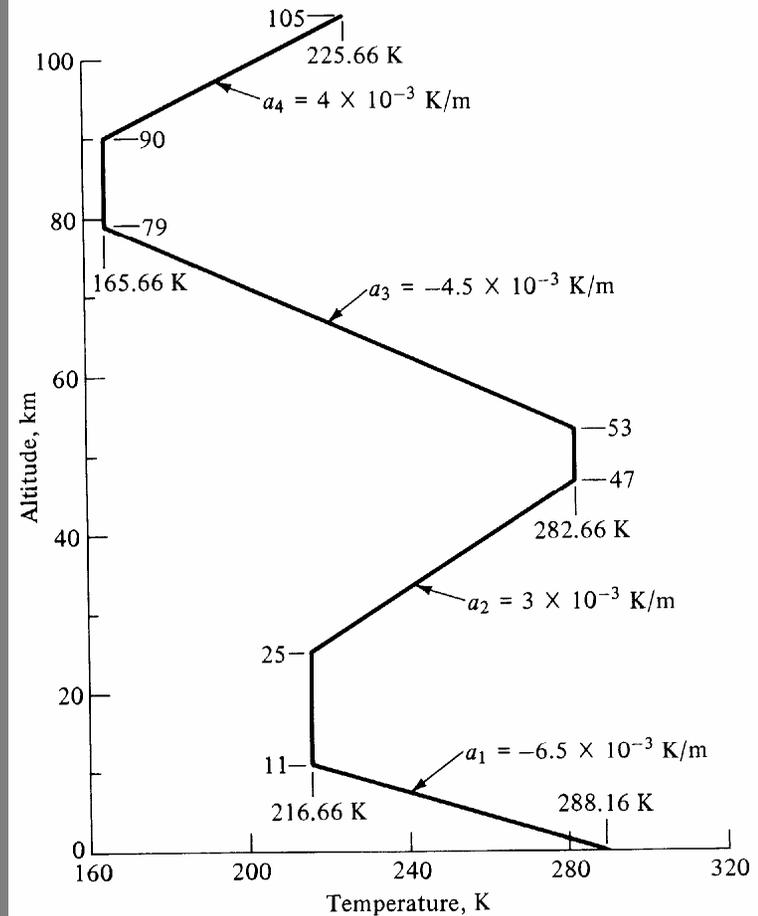

Figure 3.3 Temperature distribution in the standard atmosphere.

Integrating Pressure

- Given the definition of T(h), revisit the equation:

$$dp = -\rho g_0 \, dh$$

- Remember the equation of state?

$$p = \rho R T$$

- Divide the hydrostatic equation by the equation of state:

$$dp/p = -g_0 \, dh/(RT)$$

- In the **isothermal regions**, T is constant, so [Exercise]

$$p/p_1 = e^{-[g_0/(RT)](h-h_1)}$$

$$\rho/\rho_1 = e^{-[g_0/(RT)](h-h_1)}$$

- In the gradient regions, T depends on h, so the integration is more difficult

Integrating Pressure (2)

- Begin with:

$$dp/p = - g_0 \, dh/(RT)$$

- In the **gradient regions**, T depends linearly on h

$$(T-T_1)/(h-h_1) = dT/dh = a \qquad (a = \text{lapse rate})$$

- So,

$$dh = dT/a \qquad \Rightarrow \qquad dp/p = - g_0 \, dT/(aRT)$$

- Integrating [Exercise] leads to

$$p/p_1 = (T/T_1)^{-g_0/(aR)}$$

$$\rho/\rho_1 = (T/T_1)^{-[g_0/(aR)+1]}$$

- Since $T=T(h)$, we can write $p=p(h)$ and $\rho=\rho(h)$
- And it all starts at sea level, where $h=h_G=0$

At Sea Level

- At s.l., h=h$_G$=0

  p$_s$ = 1.01325 × 10^5 N/m^2 = 2116.2 lb/ft^2

  ρ$_s$ = 1.2250 kg/m^3 = 0.002377 slug/ft^3

  T$_s$ = 288.16 K = 518.69°R

- From sea level, we can calculate the rest

- The calculated values are available in Appendices A & B in the text, in SI and English units, respectively

Summary of Standard Atmosphere

- Temperature distribution defined by Fig. 3.4
- Equation of state relates pressure, density and temperature
- Integration of hydrostatic equation and use of equation of state leads to:

Isothermal

$p/p_1 = e^{-[g_0/(RT)](h-h_1)}$

$\rho/\rho_1 = e^{-[g_0/(RT)](h-h_1)}$

$T = $ constant

Gradient

$p/p_1 = (T/T_1)^{-g_0/(aR)}$

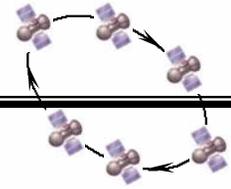$\rho/\rho_1 = (T/T_1)^{-[g_0/(aR)+1]}$

$T = T_1 + a(h-h_1)$

- See Appendices A & B in text

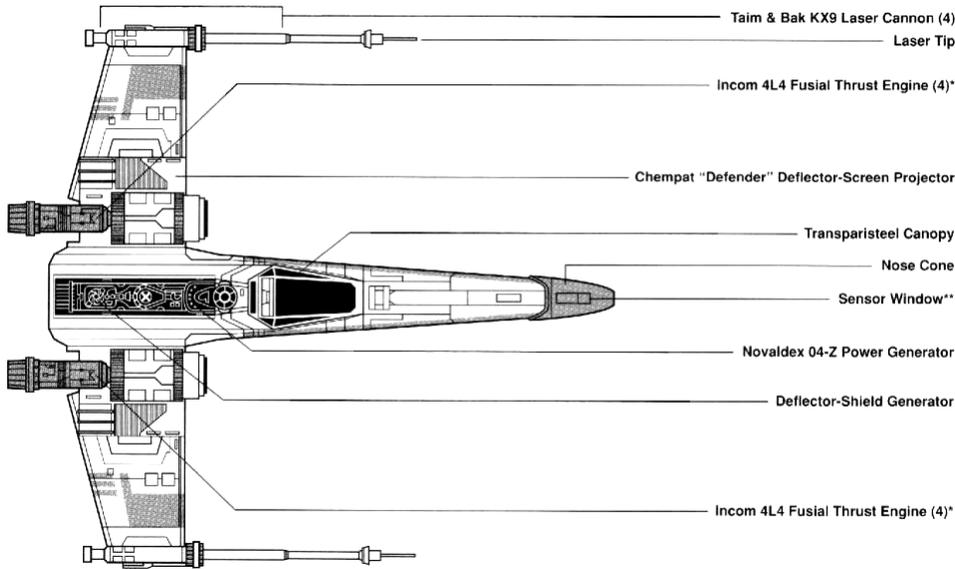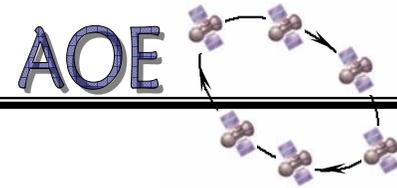# Who Is This Guy and What Is He Doing?

Taim & Bak KX9 Laser Cannon (4)
Laser Tip

Incom 4L4 Fusial Thrust Engine (4)*

Chempat "Defender" Deflector-Screen Projector

Transparisteel Canopy
Nose Cone
Sensor Window**

Novaldex 04-Z Power Generator

Deflector-Shield Generator

Incom 4L4 Fusial Thrust Engine (4)*

Top View

Laser Cannon
Laser Tip
Astromech Droid

Krupx MG7 Proton-Torpedo Launcher (2)
Cargo Compartment
Laser Barrel
Flashback Suppressor

Side View

Krupx MG7 Proton-Torpedo Launcher (2)

Incom 4L4 Fusial Thrust Engine (4)*

S-Foil Wing Assembly
(extended for combat, closed for cruising speed)

S-Foil Servo Actuator

Cargo Compartment
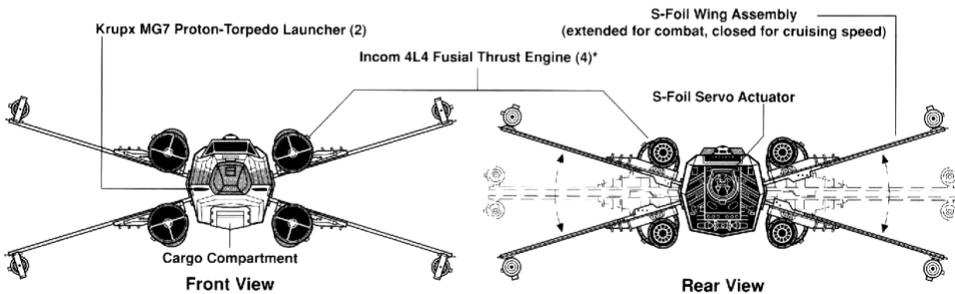**Front View**

Rear View

*Alternate configurations may use Incom 4j.4 fusial thrust engines.

** Houses Carbanti Transceiver Package with Fabritech ANs-5d "lock track" full-spectrum transceiver, Melihat "Multi Imager" dedicated energy receptor, Tana Ire electro-photo receptor and Fabritech ANq 3.6 sensor computer. Alternate configuration typically combines long-range Fabritech ANS-5d units with long-range PTDA #PA-9r unit and short-range PTAG #PG-7u unit.

Motion Sickness Generator

Example 3.1
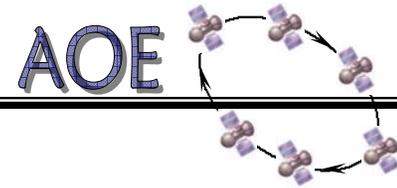
Calculate the standard atmosphere values of T, p, and ρ at a geopotential altitude of 14 km.

From Fig. 3.4, T = 216.66 K (isothermal region)

Begin at sea level and compute values at the "corner":

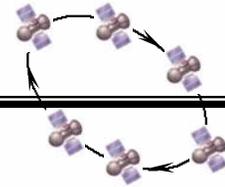**Gradient region** from h = 0 to h = 11.0 km,

with lapse rate a=-6.5 K/km

So $p = p_1(T/T_1)^{-g_0/(aR)}$ and $\rho = \rho_1(T/T_1)^{-[g_0/(aR)+1]}$

$p_1=p_s=1.01325 \times 10^5$ N/m² and $\rho_1=\rho_s=1.2250$ kg/m³

$T_1=T_s=288.16$ K

$\qquad$ p = 2.26 × 10⁴ N/m²   and ρ = 0.367 kg/m³

$\qquad$ at h = 11.0 km

Example 3.1 continued AOE

From 11.0 km to 14.0 km is an isothermal region, so

$$p = p_1 e^{-[g_0/(RT)](h-h_1)} \quad \text{and} \quad \rho = \rho_1 e^{-[g_0/(RT)](h-h_1)}$$

Here the subscript "1" refers to the values at h=11.0 km

Doing the calculations leads to

$$p = 1.41 \times 10^4 \ \text{N/m}^2 \quad \text{and} \quad \rho = 0.23 \ \text{kg/m}^3$$
$$\text{at } h = 14.0 \ \text{km}$$

Three More Altitudes

- Pressure, Temperature and Density Altitudes
- Remember Geometric, Absolute, and Geopotential Altitudes
- Suppose you're flying and you have a set of instruments that can measure Pressure, Temperature and Density
- You could look them up in the standard atmosphere tables, right?
- Would they all agree?   Not likely.

Sample Table

1976 Standard Atmosphere
(from http://www.digitaldutch.com/atmoscalc/tableoptions1.htm)

Altitude [km]	Temperature [Kelvin]	Pressure [pascal]	Density [kg/m^3]
0	288.15	101325	1.225
1	281.65	89874.5705	1.1116
2	275.15	79495.2155	1.0065
3	268.65	70108.5447	0.9091
4	262.15	61640.2353	0.8191
5	255.65	54019.9121	0.7361

- Suppose measured pressure is 6.16×10^4 N/m^2
  - Then the pressure altitude is $\approx$ 4 km
- Suppose the measured temperature is 269 K
  - Then the temperature altitude is $\approx$ 3 km
- What is the geometric altitude?

Example 3.2
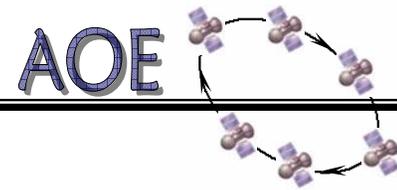
If an airplane is flying at an altitude where the actual pressure and temperature are

$$4.72 \times 10^4 \text{ N/m}^2 \quad \text{and} \quad 255.7 \text{ K}$$

respectively, what are the pressure, temperature and density altitudes?

Use Appendix A to find that

pressure altitude = 6 km
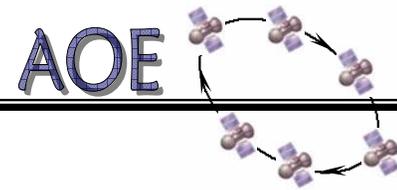
Why?

temperature altitude = 5 km (or 38.2 or 59.5 km)

Use equation of state to find that the density is

$$\rho = 0.643 \text{ kg/m}^3$$

And then use Appendix A to find that

density altitude = 6.24 km

- Standard atmosphere is defined in order to relate flight tests, wind tunnel results, and general airplane design and performance to a common reference
- Temperature variation is defined
  - Isothermal and gradient regions
- Hydrostatic equation is applied

  $dp = -\rho\, g\, dh_G$

- Pressure and density variations are derived
- There are six different flavors of altitude:

  Geometric, absolute, geopotential

  Pressure, temperature, density

Additional Topics

- Geometric vs geopotential altitudes
- Linear interpolation

- Geopotential altitude is only used as a convenience for computing the standard atmosphere tables
- Always use the geometric altitude column of the tables when referring to pressure, density, and temperature altitudes
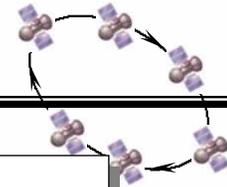
- Usually looking up numbers in tables requires *linear interpolation*
    - Select the two rows that "bracket" the given value
    - Use a straight-line approximation between the values in the two rows

h [km]	T [K]	P [N/m^2]	ρ [kg/m^3]
1	281.65	89874	1.1116
2	275.15	79495	1.0065

Example: you measure pressure at

$$p^* = 8.0 \times 10^4 \text{ N/m}^2$$

Then the pressure altitude is found by

$$h^* = h_1 + (h_2 - h_1)(p^* - p_1)/(p_2 - p_1)$$

$$= 1.9514 \text{ km}$$