Aircraft Flight

Chapter 5: Airfoils, Wings and Other Aerodynamic Shapes

AOE 3014 Fall Junior Year

- Lift, Drag, and Moment (§5.1-5.3)
- Lift, Drag, and Moment Coefficients (§5.3)
- Drag Polar (§5.14)

Chapter 6: Elements of Airplane Performance

AOE 3104 Spring Sophomore Year

- Equations of Motion (§6.2)
- Static Performance (§6.1-6.6)

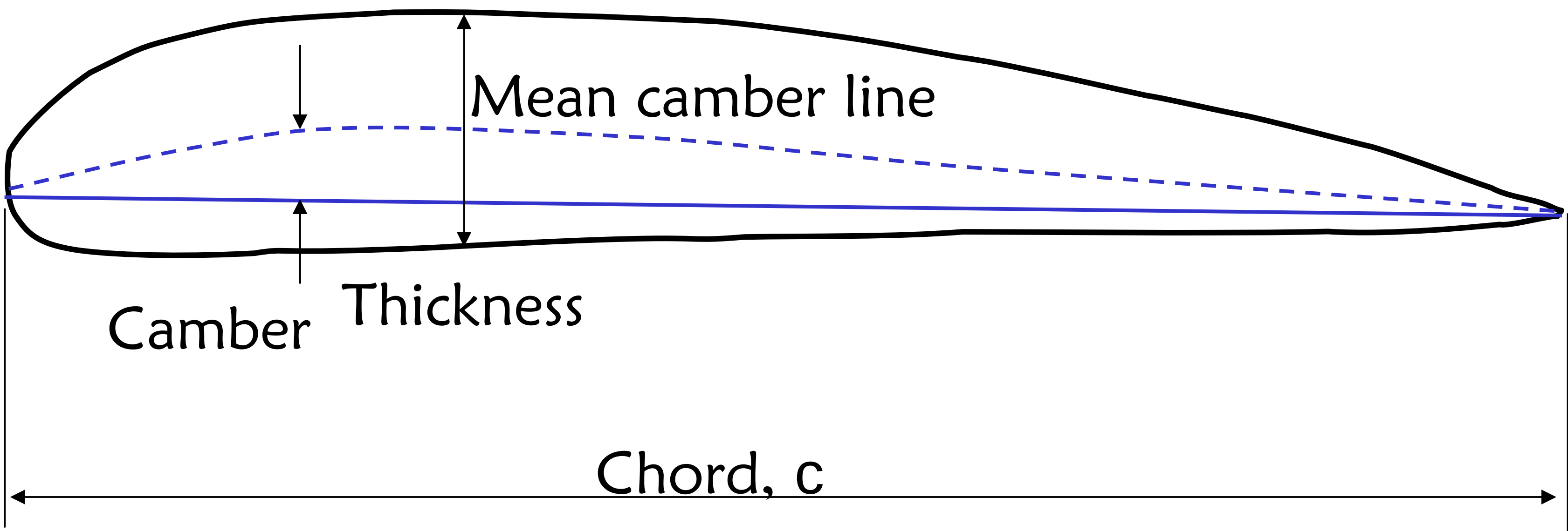
Chapter 7: Principles of Stability and Control

AOE 3134 Spring Junior Year (or Spacecraft)

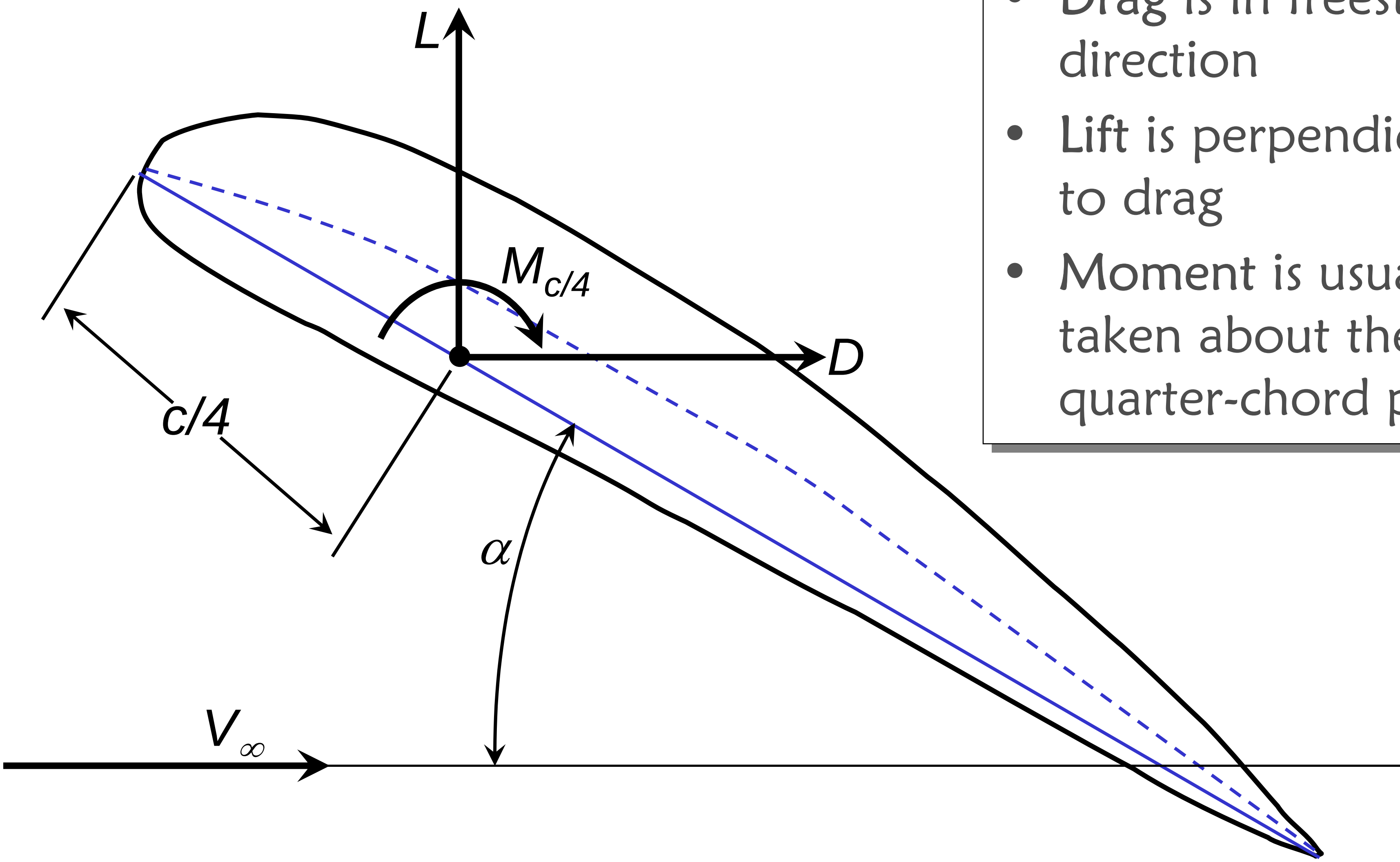
- Introduction and Definitions (§7.1-7.4)

Airfoil Nomenclature

- Airfoil is a two-dimensional cross-section of a wing
- Camber is the maximum distance between the mean camber line and the chord line
- Camber, shape of mean camber line, and thickness determine the lift and moment characteristics of the airfoil

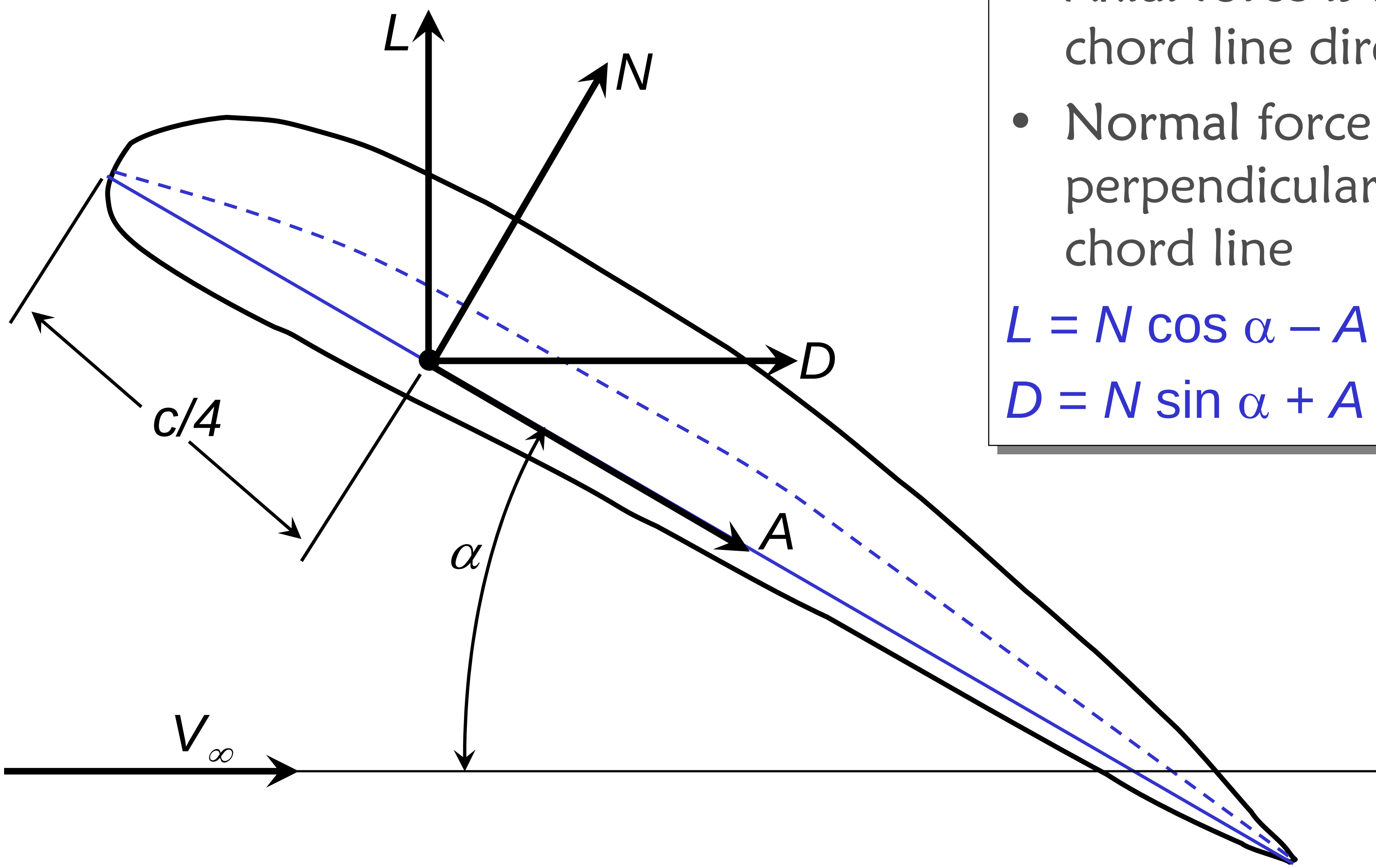


Lift and Drag



- Drag is in freestream direction
- Lift is perpendicular to drag
- Moment is usually taken about the quarter-chord point

Normal and Axial Forces



- Axial force is in chord line direction
- Normal force is perpendicular to chord line

$L = N \cos \alpha - A \sin \alpha$

$D = N \sin \alpha + A \cos \alpha$

Lift, Drag, and Moment Coefficients

- Applying dimensional analysis to the forces and moments leads to the definitions of these coefficients

$$L = q_\infty S c_l$$
$$D = q_\infty S c_d$$
$$M = q_\infty S c c_m$$

- Here q_∞ is the dynamic pressure, S is the wing area, and c is the chord length
- The three coefficients c_l, c_d, and c_m are dimensionless numbers that depend on angle of attack, Mach number, and Reynolds number (also dimensionless numbers)

Lift, Drag, and Moment Coefficients

- These dimensionless coefficients depend on angle of attack, Mach number, and Reynolds number:

  $c_l = f_1(\alpha, M_\infty, Re)$

  $c_d = f_2(\alpha, M_\infty, Re)$

  $c_m = f_3(\alpha, M_\infty, Re)$

- These three numbers are also dimensionless:

  α = angle of attack (units = radians, dimensionless)

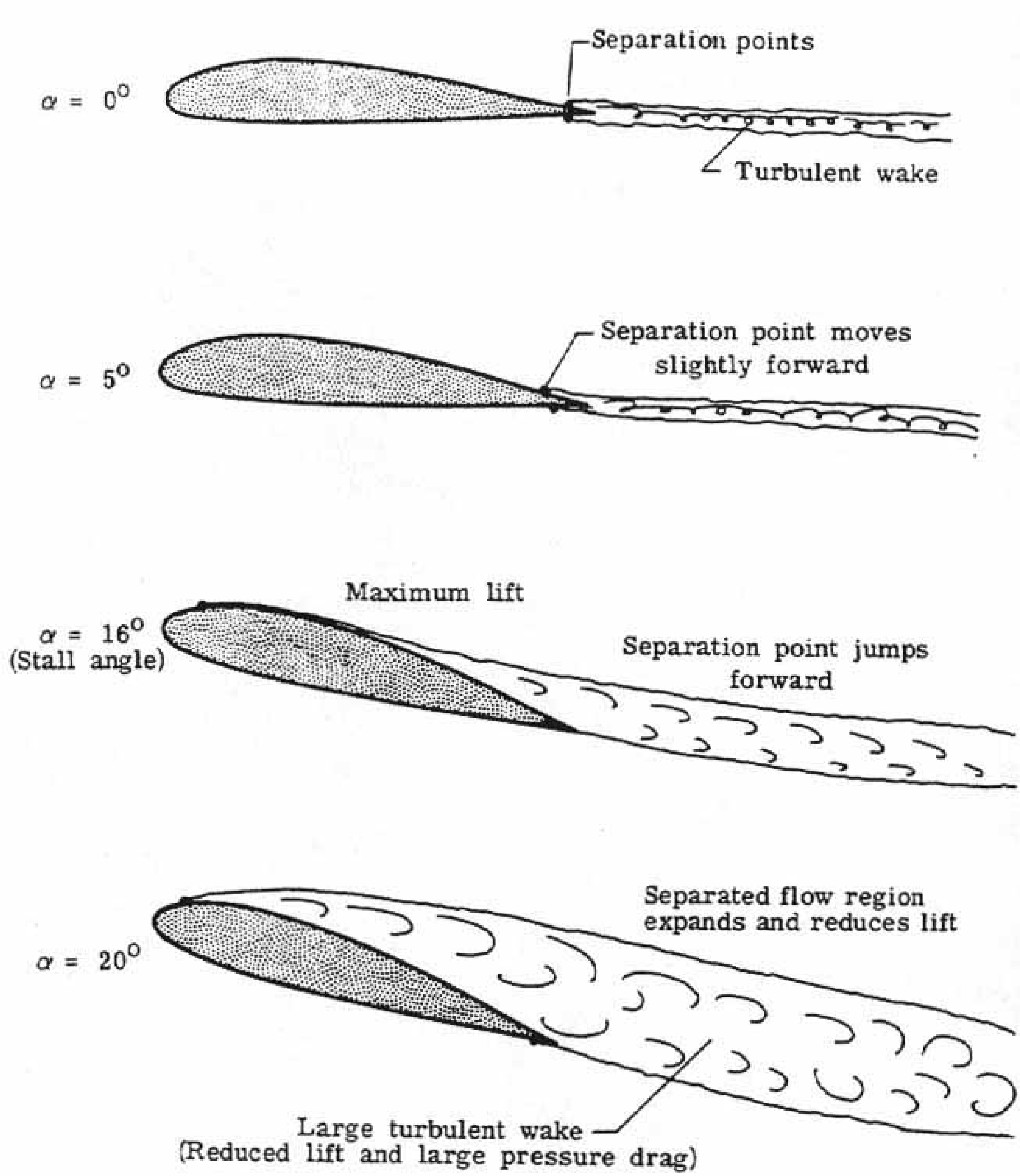
  M_∞ = Mach number = V_∞ / a_∞ (a_∞ = speed of sound)

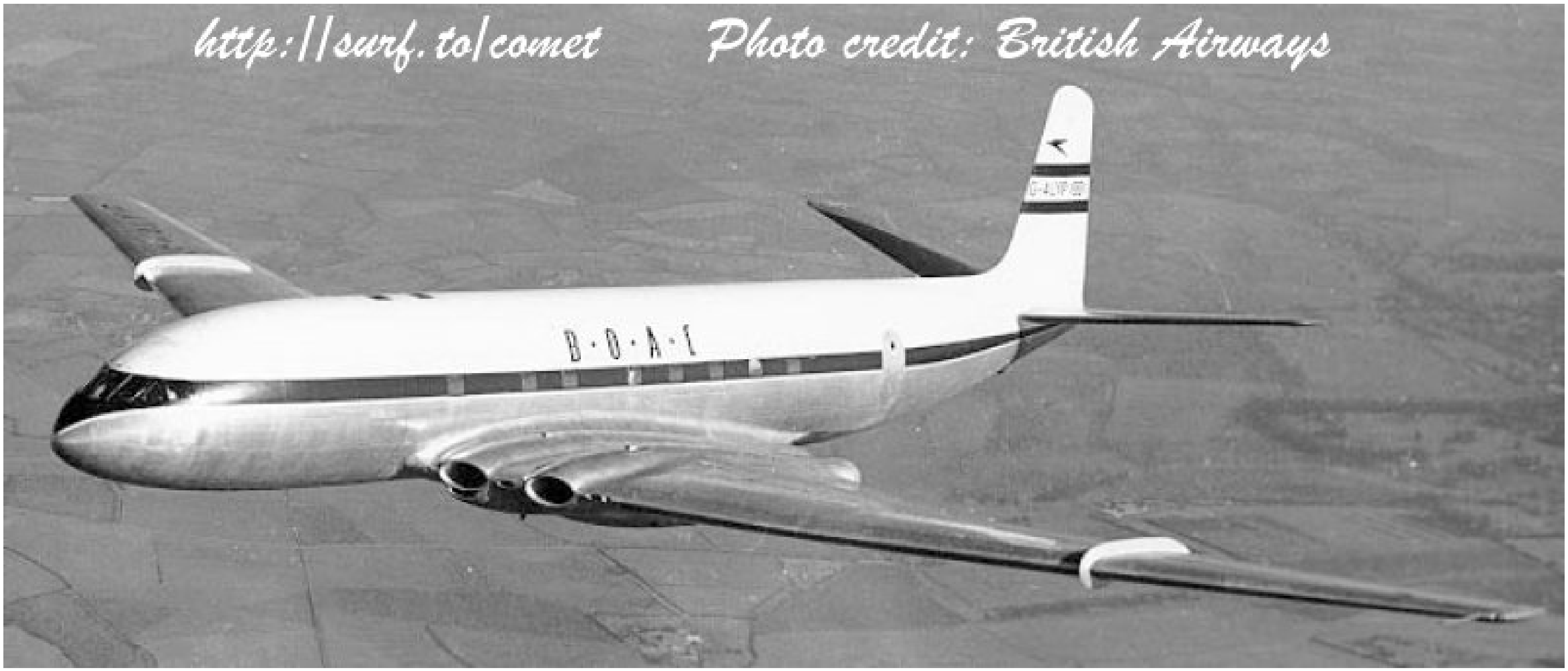
  Re = Reynolds number = $\rho_\infty V_\infty c / \mu_\infty$ (μ=viscosity)

- For subsonic incompressible flow, M_∞ is "small" and Re is "large" $\Rightarrow c_l = f_1(\alpha)$, *etc.*

Flow Separation

- Low angle of attack ⇒ minimal flow separation, at trailing edge
- As angle of attack increases point of flow separation moves slightly forward
- At stall angle, separation point moves forward dramatically
- Separated flow increases pressure on upper surface ⇒ reduced lift




http://surf.to/comet
Photo credit: British Airways
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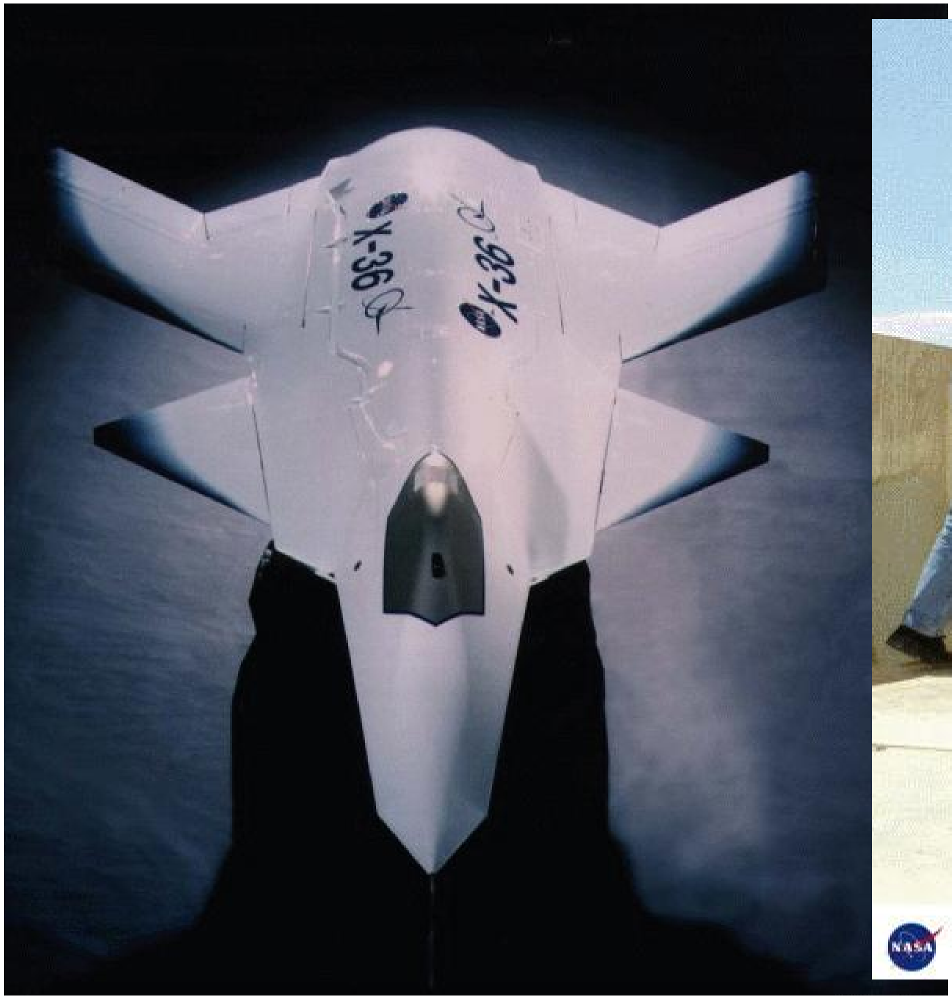
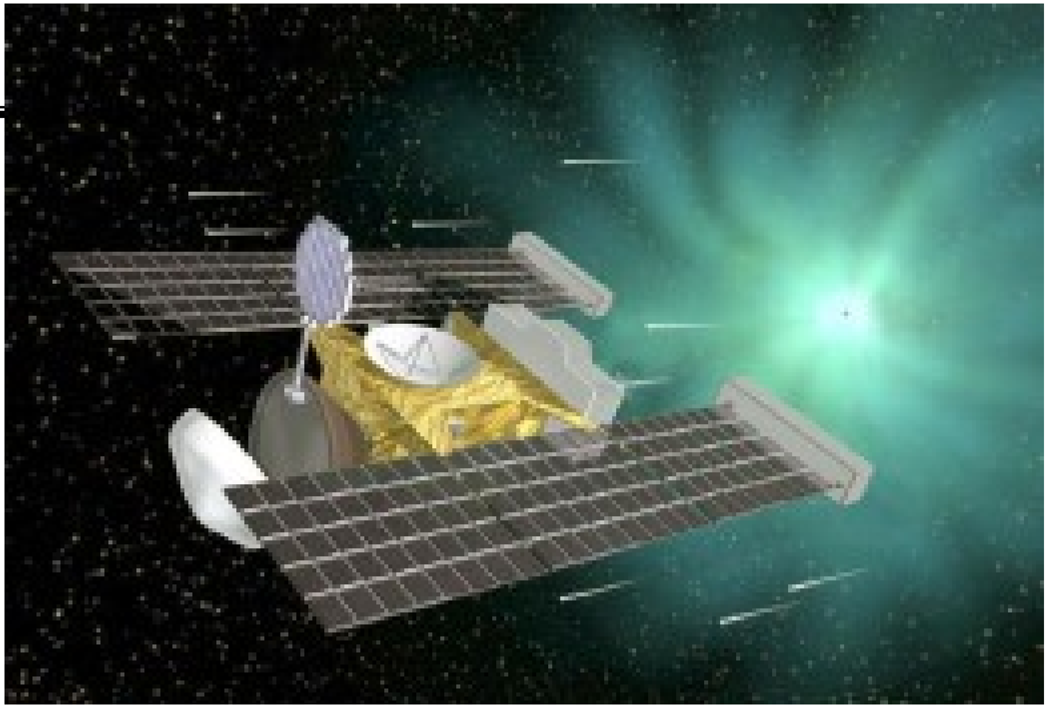
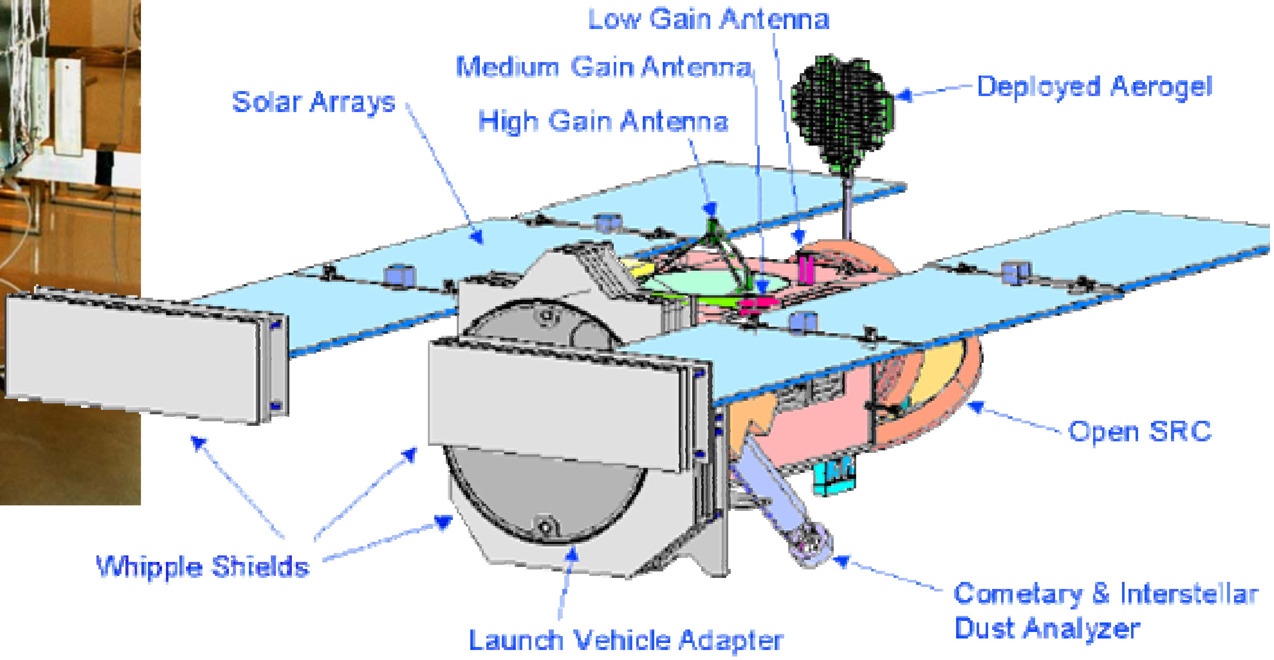


Dryden Flight Research Center EC96-43641-5 Photographed 7/2/96
X-36 arrival at Dryden, Sequence #3. NASA photo by Dennis Taylor

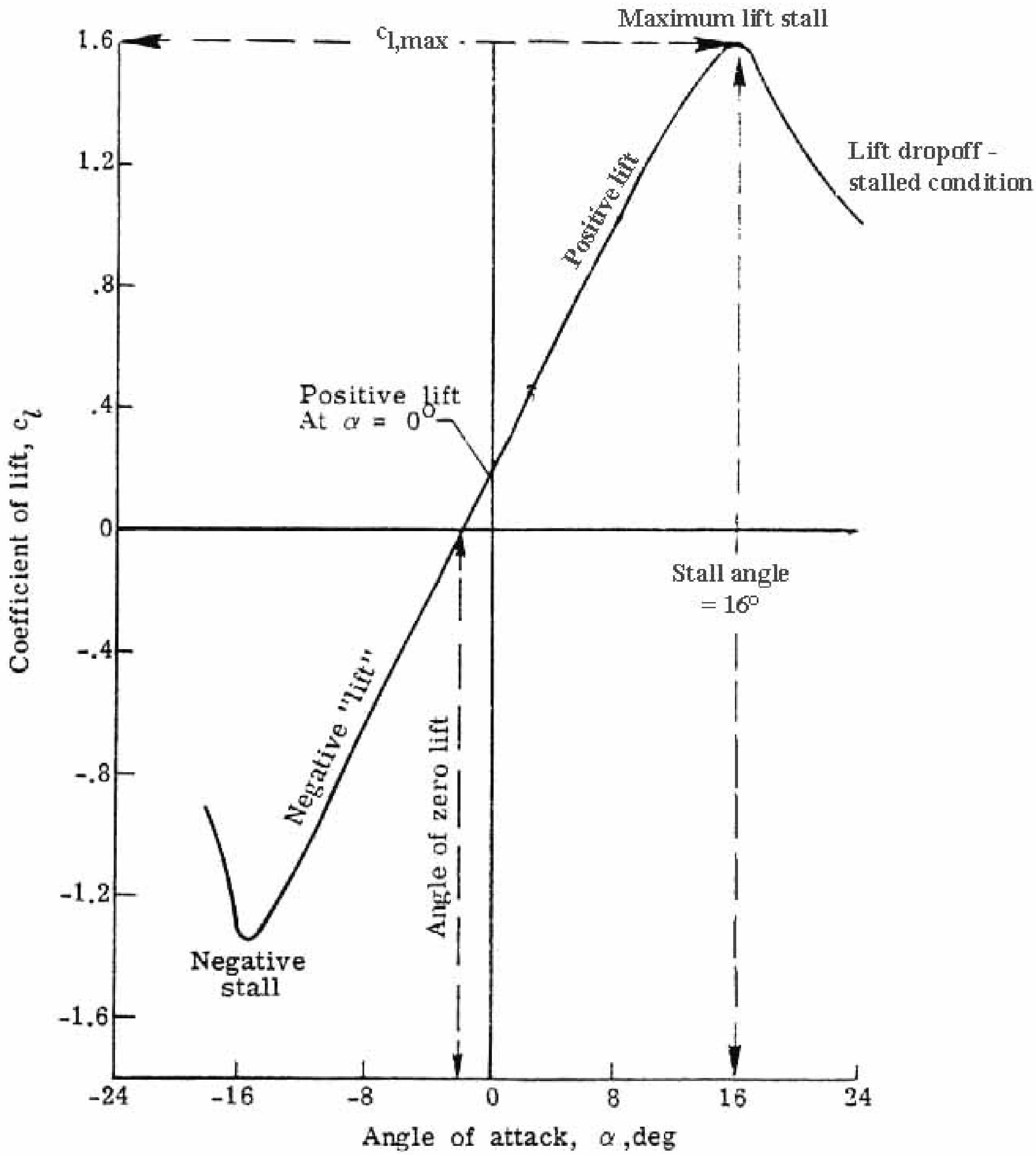



Low Gain Antenna
Medium Gain Antenna
Deployed Aerogel
Solar Arrays
High Gain Antenna
Open SRC
Whipple Shields
Cometary & Interstellar
Dust Analyzer
Launch Vehicle Adapter

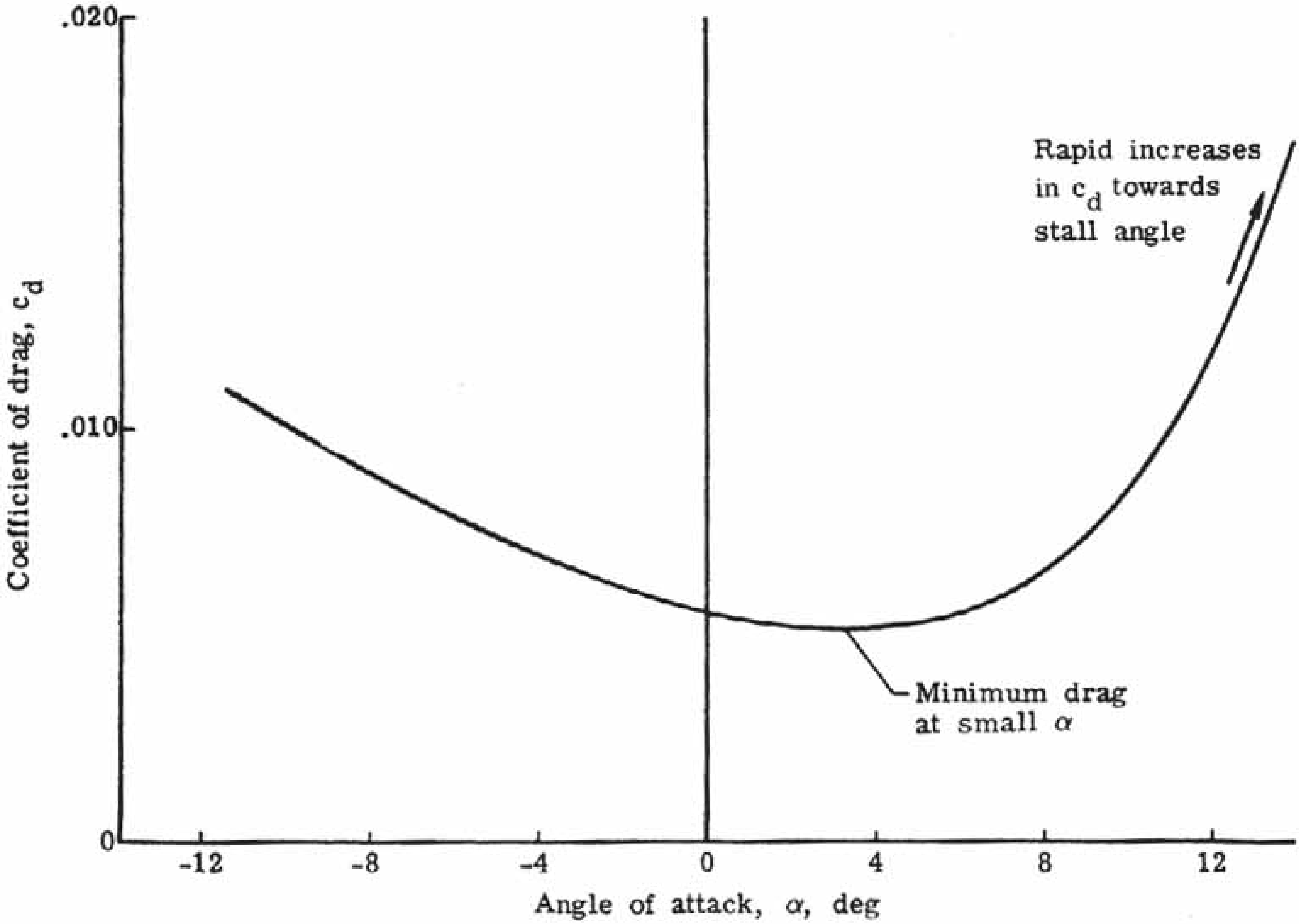
Lift Coefficient

$c_l(\alpha)$

- Nearly constant slope, $dc_l/d\alpha$, between the stall angles
- Positive lift at $\alpha = 0$
- Stall corresponds to "flow separation"



Drag Coefficient



Drag Polar

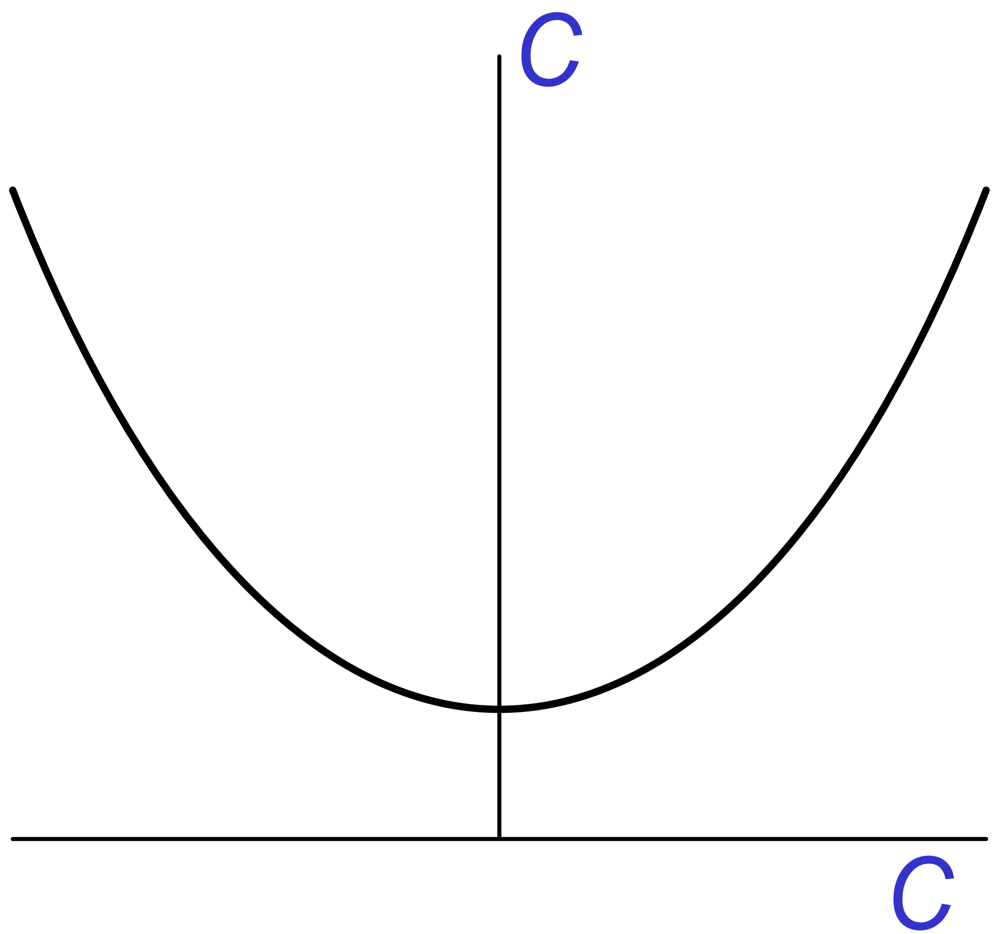
- For finite wings at subsonic speeds, the drag coefficient can be written as

  $C_D = c_d + C_L^2/(\pi e AR)$

- The "little" c_d denotes the *profile drag* or the airfoil section drag
- The "big" C_D denotes the *total drag* on the finite wing
- The "big" C_L denotes the total lift on the finite wing (as compared with c_l)
- The term $0 < e < 1$ denotes a *planform efficiency factor*. Elliptical wing $\Rightarrow e = 1$
- The term is called the *induced drag*

Drag Polar

- The profile drag includes drag due to skin friction and pressure drag due to separation
- This plot is an essential tool in the design of airplanes, and we will see one application a bit later



Example 5.14

Consider the Northrop F-5 fighter airplane, which has a wing area of 170 ft^2. The wing is generating 18,000 lb of lift. For a flight velocity of 250 mi/h at standard sea level, calculate the lift coefficient.

Example 5.15

The wingspan of the F-5 is 25.25 ft. Calculate the induced drag coefficient and the induced drag for the conditions of Ex. 5.14. Use e=0.8.

Example 5.16

Consider a "flying wing" with a wing area of 206 m^2, an aspect ratio of 10, a span effectiveness factor of 0.95, and a NACA 4412 airfoil. The weight of the airplane is 7.5×10^5 N. If the density altitude is 3 km and the flight velocity is 100 m/s, calculate the total drag on the aircraft.

We've seen this before, but this is a nice picture


BOEING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Airplane Performance

Equations of Motion

Static Performance (zero acceleration)

- Thrust required
- Thrust available
- Power required
- Power available
- Maximum velocity
- Rate of climb
- Time to climb
- Maximum altitude
- Range and endurance

Dynamic Performance (finite acceleration)

- Takeoff
- Landing
- Turning flight
- *V-n* diagram
- Accelerated rate of climb

Reading: Chapter 6

Preview of AOE 3104

Equations of Motion

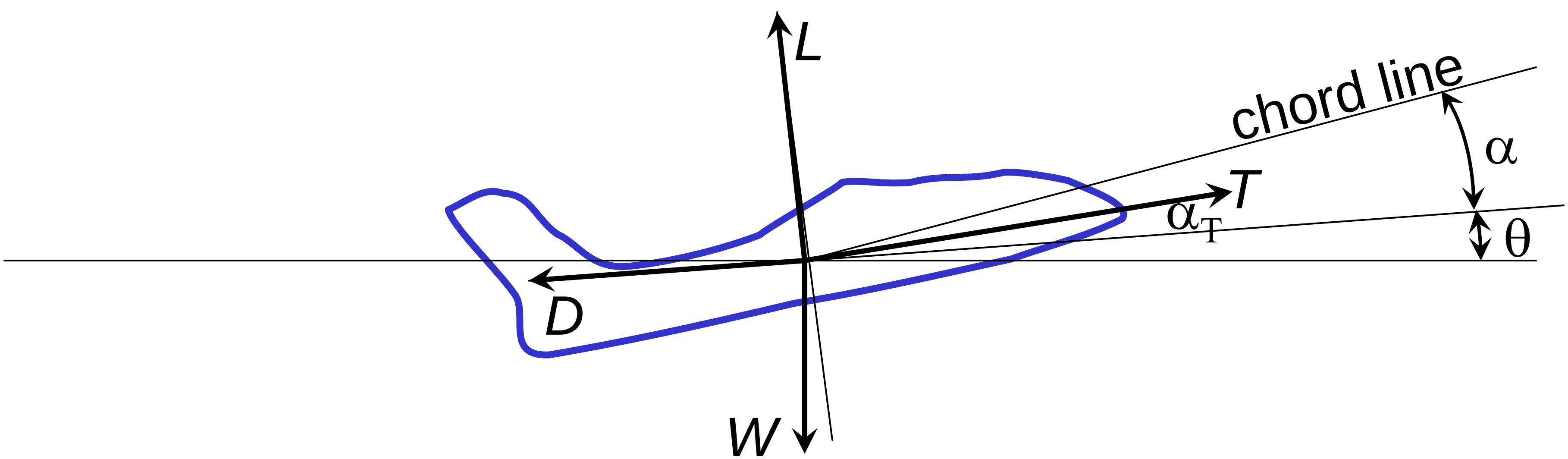
- Newton's Second Law, $F = ma$
- The four forces:

  Lift L, perpendicular to flight path

  Drag D, parallel to flight path

  Weight W, toward center of Earth

  Thrust T, generally inclined wrt flight path



Equations of Motion

$F = ma$ (this equation is a vector equation)

- Velocity is always along flight path

$\Sigma F_{\parallel} = m\, dV/dt$ (a scalar equation)

- Acceleration perpendicular to flight path is centripetal acceleration, which depends on velocity and radius of curvature, r_c

$\Sigma F_{\perp} = m\, V^2/r_c$ (a scalar equation)

- The preceding two equations are the *kinematics* equations; next we must determine the two force summations

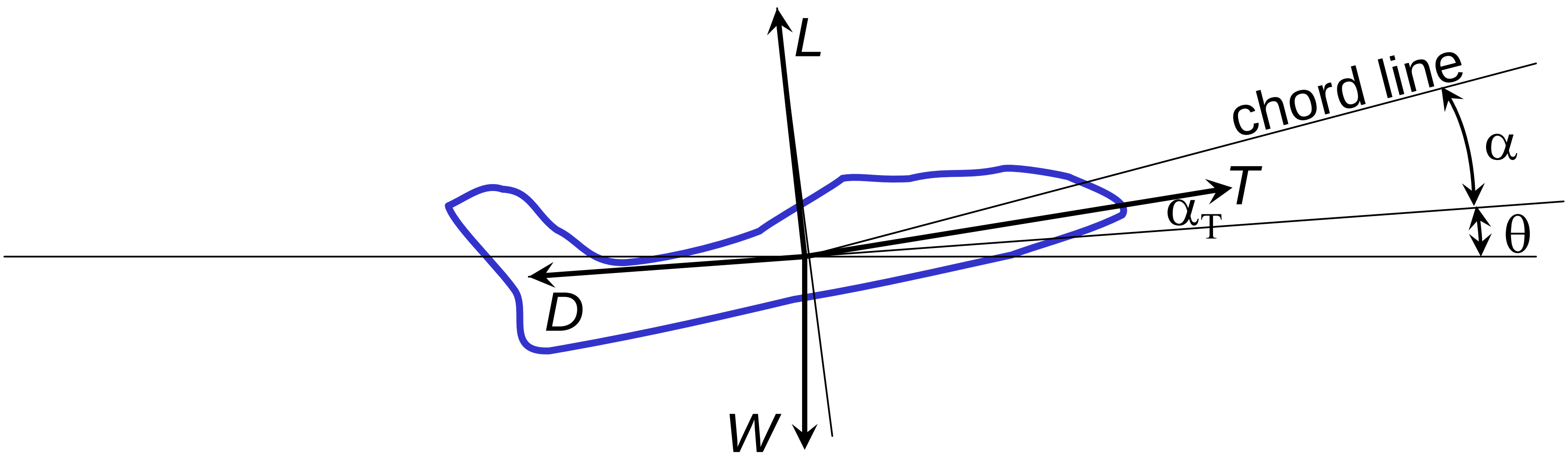
Equations of Motion

- Examination of the figure below leads to

$$\Sigma F_{\parallel} = T \cos \alpha_T - D - W \sin \theta = m\, dV/dt$$

$$\Sigma F_{\perp} = L + T \sin \alpha_T - W \cos \theta = m\, V^2/r_c$$

- These are the equations of motion for an airplane in 2-D translational flight
- Rotational motion is not included here



Level Unaccelerated Flight

- Velocity is constant, radius of curvature is infinite, $\theta = 0$
- Equations of motion reduce to

$$T \cos \alpha_T = D$$
$$L + T \sin \alpha_T = W$$

- Assuming that $\alpha_T = 0$, these equations further reduce to

$$T = D \quad \text{(thrust = drag)}$$
$$L = W \quad \text{(lift = weight)}$$

- Since lift and drag are related by the drag polar, we can use the drag polar to determine the required thrust for straight level flight

Thrust Required for Straight, Level Flight

$T = D$ (thrust = drag)

$T = D = q_\infty S\, C_D$

$L = W$ (lift = weight)

$L = W = q_\infty S\, C_L$

Thrust-to-weight ratio:

$T / W = C_D / C_L$

Required thrust:

$T_R = W\, C_D / C_L = W / (C_L / C_D) = W / (L/D)$

Thrust-Required Curve

1. Choose value of V_∞
2. Calculate lift coefficient C_L

   $L = W \Rightarrow C_L = W/(q_\infty S) = 2W/(\rho_\infty V_\infty^2 S)$
3. Calculate drag coefficient C_D from drag polar

   $C_D = C_{D,0} + C_L^2/(\pi e\, AR)$
4. Calculate the lift-to-drag ratio,

   $L/D = C_L/C_D$
5. Calculate the thrust required

   $T_R = W/(C_L/C_D)$

This procedure can be used to compute $T_R(V_\infty)$ for a specific V_∞, or to compute for a range of speeds

Example 6.1

Given

span: $b = 35.8$ ft

area: $S = 174$ ft^2

weight: $W = 2950$ lb

parasite drag coeff: $C_{D,0} = 0.025$

Oswald efficiency factor: $e = 0.8$

Look up density: $\rho = 0.002377$ slug/ft^3

Compute

aspect ratio: $AR = b^2/S$

induced drag denominator: "pear" = $\pi e AR$

Follow procedure from previous slide . . .

A Matlab Code to Compute T_R

```
% Treq.m
% thrust required vs V_\infty
% using data from example 6.1
% this section just does the initialization
clear
close all

b    = 35.8;
S    = 174;
AR   = b^2/S;
W    = 2950;
Cdo  = 0.025;
e    = 0.8;
pear = pi*e*AR;

rho=0.002377;
```

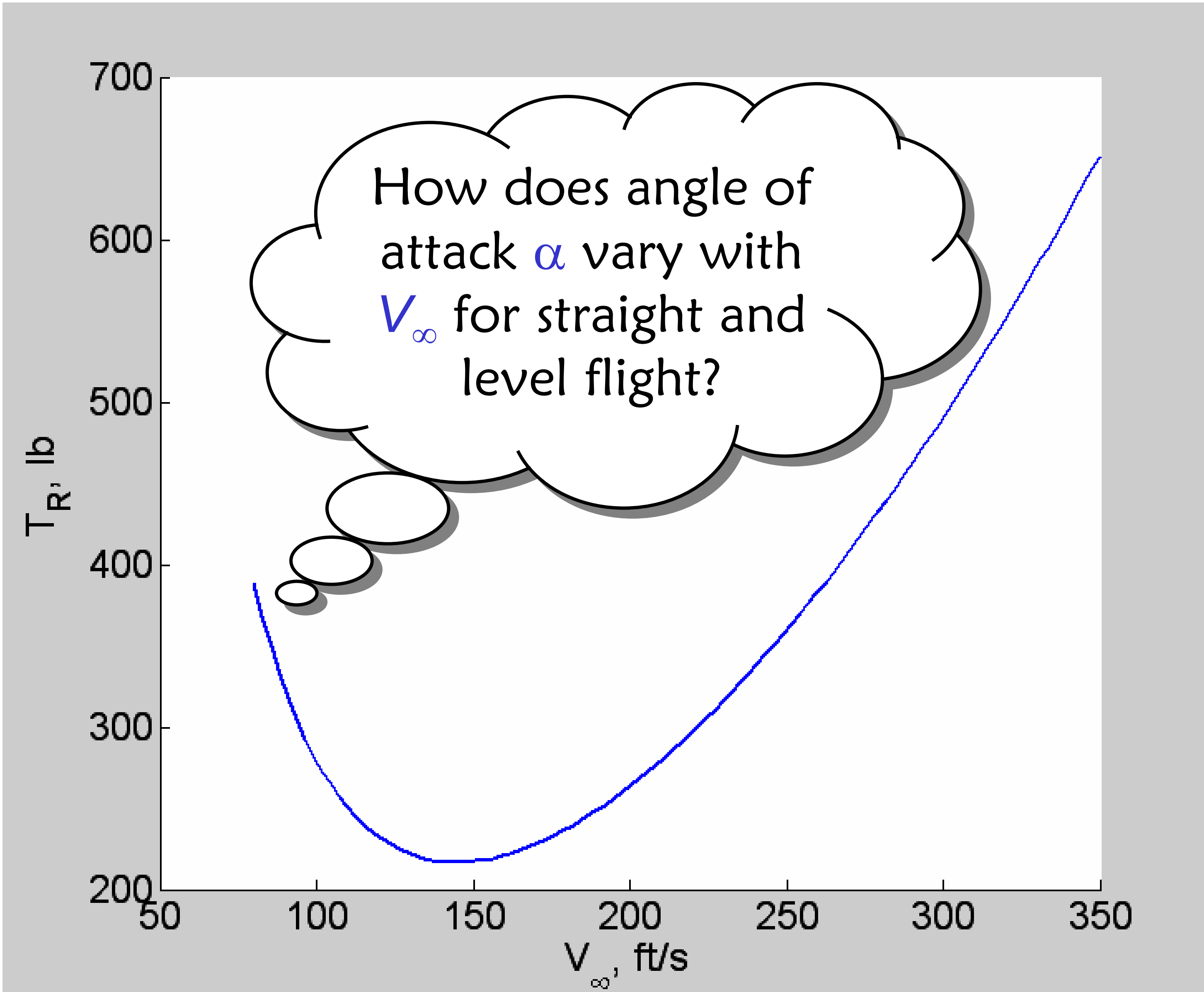
```
%Treq.m  continued

N=100;
Vmin=80;
Vmax=350;
Vi=linspace(Vmin,Vmax,N);   % N points [Vmin, Vmax]
CLv=zeros(size(Vi));        % save Lift Coefficients
CDv=CLv;                    % save Drag Coefficients
TRv=CLv;                    % save Thrust Required
for i=1:N
    CL=2*W/(rho*Vi(i)^2*S); % Steps 1& 2
    CD=Cdo+CL^2/pear;       % Step 3
    LoD=CL/CD;              % Step 4
    TR=W/LoD;               % Step 5
    CLv(i) = CL;            % Save everything
    CDv(i) = CD;
    TRv(i) = TR;
end
```

```
%Treq.m continued   Make the TR vs Vinfty plot
figure;   hold on
hndl=plot(Vi,TRv);
set(hndl,'linewidth',2);
hndl=xlabel('V_\infty, ft/s');
set(hndl,'fontsize',18)
hndl=ylabel('T_R, lb');
set(hndl,'fontsize',18)
set(gca,'fontsize',18)
```

This code snippet opens the figure window, makes the plot, changes the line thickness, makes x&y axis labels, and changes the fontsizes

T_R vs V_∞

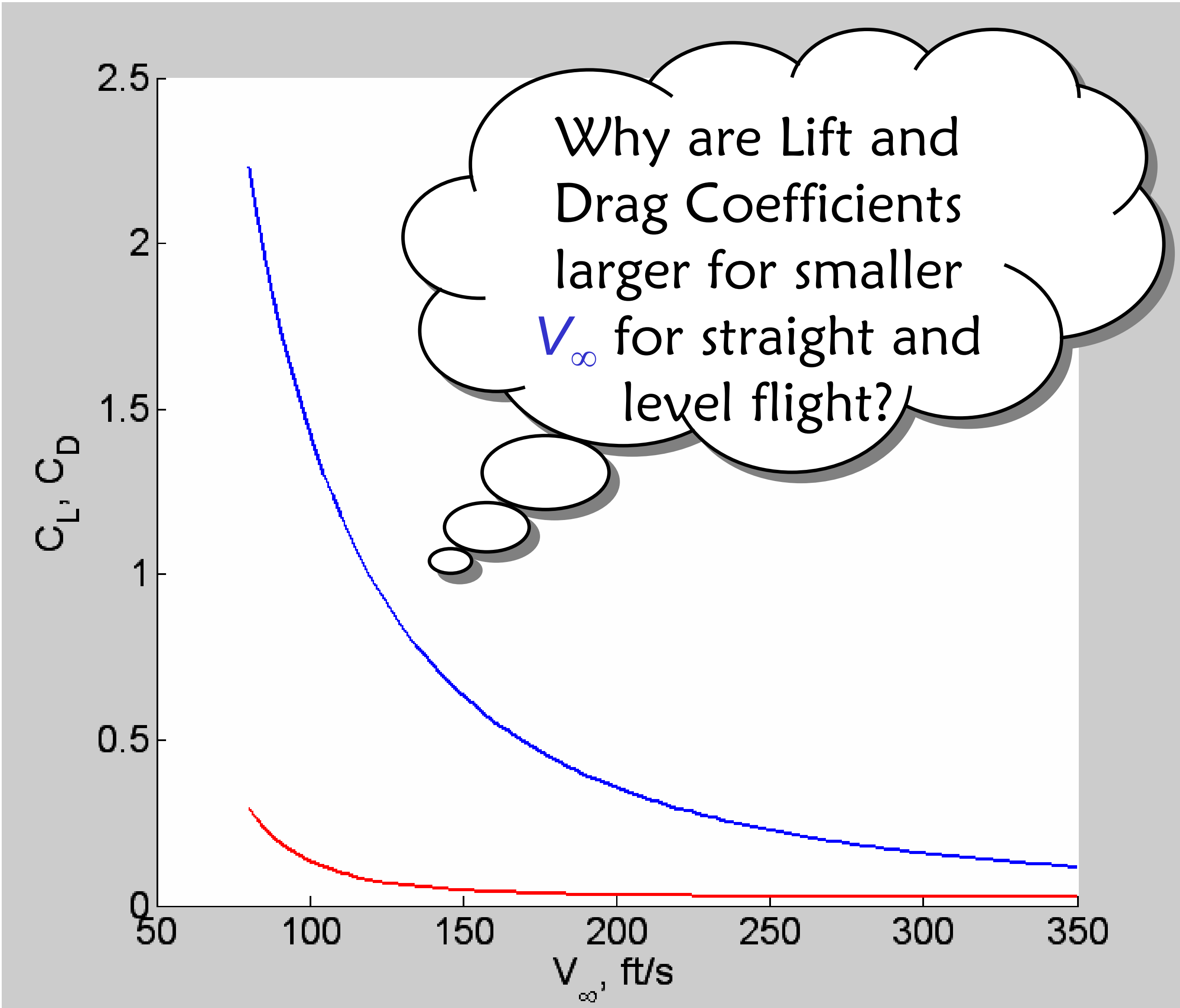


How does angle of attack α vary with V∞ for straight and level flight?
700
600
500
400
300
200
50
100
150
200
250
300
350
T_R, lb
V∞, ft/s

```
figure;   hold on
hndl=plot(Vi,CLv,’b’);
set(hndl,'linewidth',2);
hndl=plot(Vi,CDv,’r’);
set(hndl,'linewidth',2);
hndl=xlabel('V_\infty, ft/s');
set(hndl,'fontsize',18)
hndl=ylabel('C_L, C_D');
set(hndl,'fontsize',18)
set(gca,'fontsize',18)
```

This code snippet opens the figure window, makes two plots, changes the line thickness, makes x&y axis labels, and changes the fontsizes

C_L and C_D vs V_∞



Some Questions

- How does angle of attack α vary with V_∞?
- What is special about the minimum Thrust Required point on the T_R vs V_∞ curve?
- Why are lift and drag coefficients larger for smaller V_∞?
- Begin by recalling that thrust = drag, $T_R = D$

Analysis of T_R vs V_∞

$$T_R = D = q_\infty S C_D = q_\infty S (C_{D,0} + C_{D,i})$$

$$T_R = q_\infty S \left(C_{D,0} + \frac{C_L^2}{\pi e AR} \right)$$

$$T_R = q_\infty S C_{D,0} + \frac{q_\infty S C_L^2}{\pi e AR}$$

- First term is **parasite thrust** required (zero-lift)
- Second term is **induced thrust** required
- Recall that C_L also depends on q_∞ :

$$C_L = \frac{W}{q_\infty S} \qquad \text{so that}$$

$$T_R = q_\infty S C_{D,0} + \frac{W^2}{q_\infty S \pi e AR}$$

Continued Analysis of T_R vs V_∞

$$T_R = q_\infty S C_{D,0} + \frac{W^2}{q_\infty S \pi e AR}$$

$$\frac{dT_R}{dq_\infty} = \frac{dT_R}{dV_\infty}\frac{dV_\infty}{dq_\infty} \text{ (chain rule)}$$

$$\frac{dT_R}{dV_\infty} = 0 \Rightarrow \frac{dT_R}{dq_\infty} = 0$$

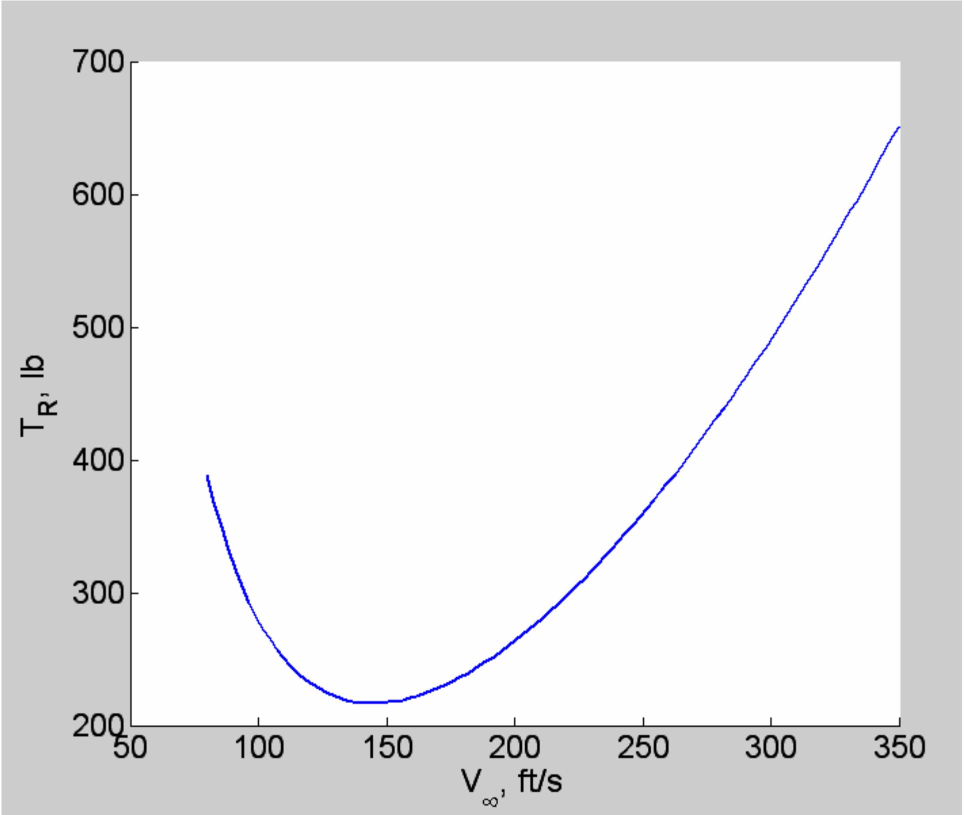
$$\frac{dT_R}{dq_\infty} = 0 \Rightarrow S C_{D,0} - \frac{W^2}{q_\infty^2 S \pi e AR} = 0$$

A little manipulation [Exercise] leads to:

$C_{D,0} = C_{D,i}$ Parasite drag = Induced drag

Conclusions Regarding T_R vs V_∞

- T_R has two components: a "zero-lift" term and a "lift-induced" term
- The minimum occcurs where the two terms are equal
- Available thrust must be ≥ required thrust to maintain straight level flight



Further Topics

Equations of Motion

Static Performance (zero acceleration)

- Thrust required
- Thrust available
- Power required
- Power available
- Maximum velocity
- Rate of climb
- Time to climb
- Maximum altitude
- Range and endurance

Dynamic Performance (finite acceleration)

- Takeoff
- Landing
- Turning flight
- ***V-n*** diagram
- Accelerated rate of climb

Reading: Chapter 6

Preview of AOE 3104

Stability & Control

Stability

- Static
  - Longitudinal
    - Criteria
    - Moments about c.g.
    - Equations for stability
    - Neutral point
    - Static margin
  - Directional
  - Lateral
- Dynamic

Control

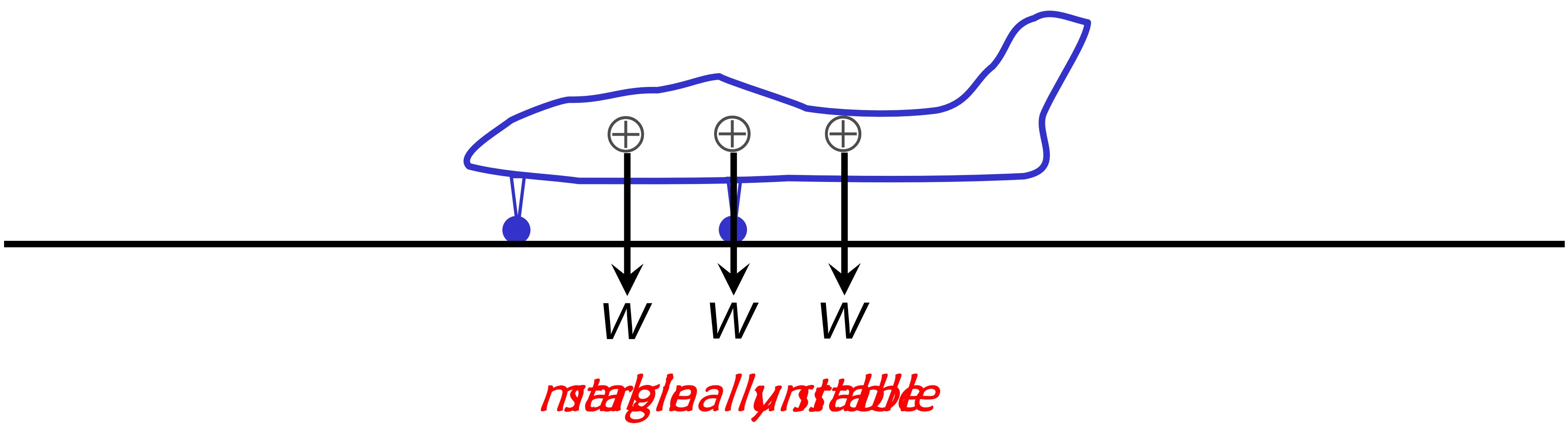
- Static
  - Longitudinal
  - Directional
  - Lateral
- Dynamic

Reading: Chapter 7

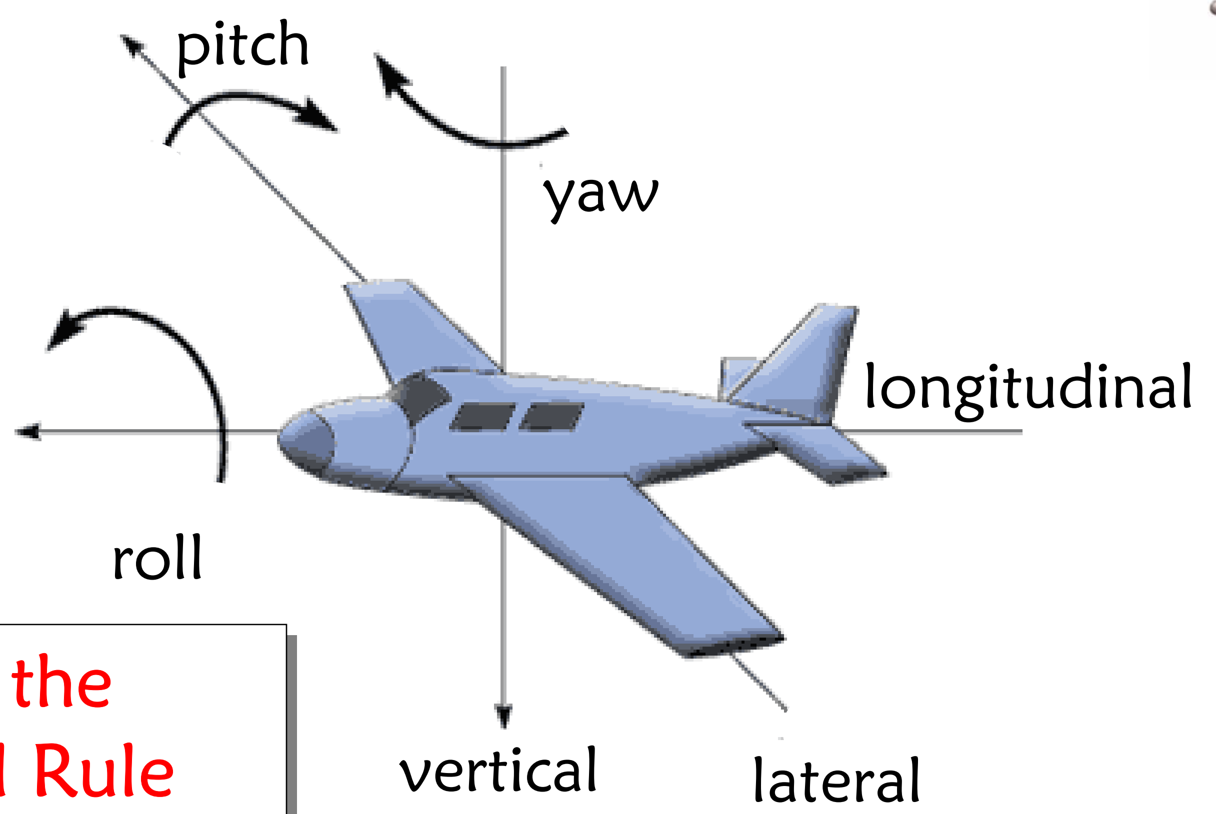
Preview of AOE 3134

"Parking" Stability

- If mass center (c.g. ⊕) is between landing gear, then the parked aircraft is *stable*
- If c.g. is aft of aft landing gear, then the parked aircraft is *unstable*
- If c.g. is aligned with aft landing gear, then the parked aircraft is *marginally stable*



Roll, Pitch and Yaw

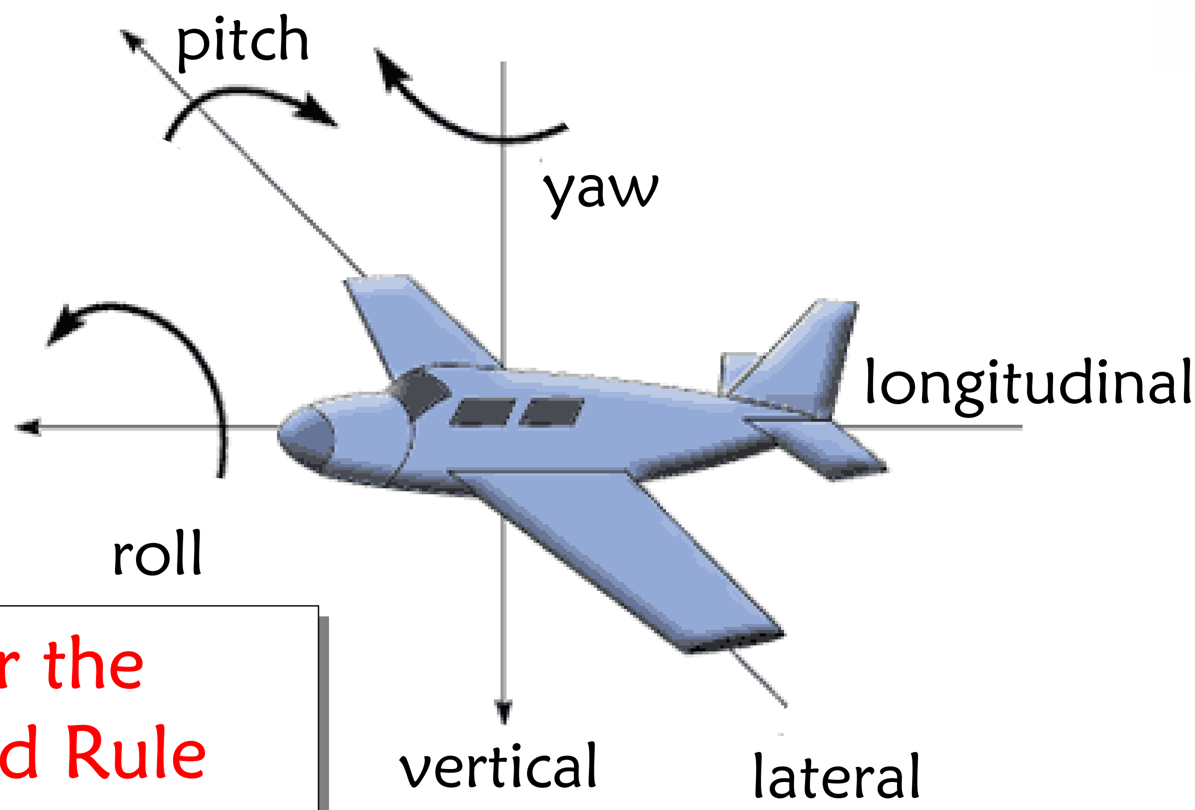


Remember the Right Hand Rule

Stability & Control deals with rotational motion

- Roll about longitudinal axis
- Pitch about lateral axis
- Yaw about vertical axis

Roll, Pitch and Yaw



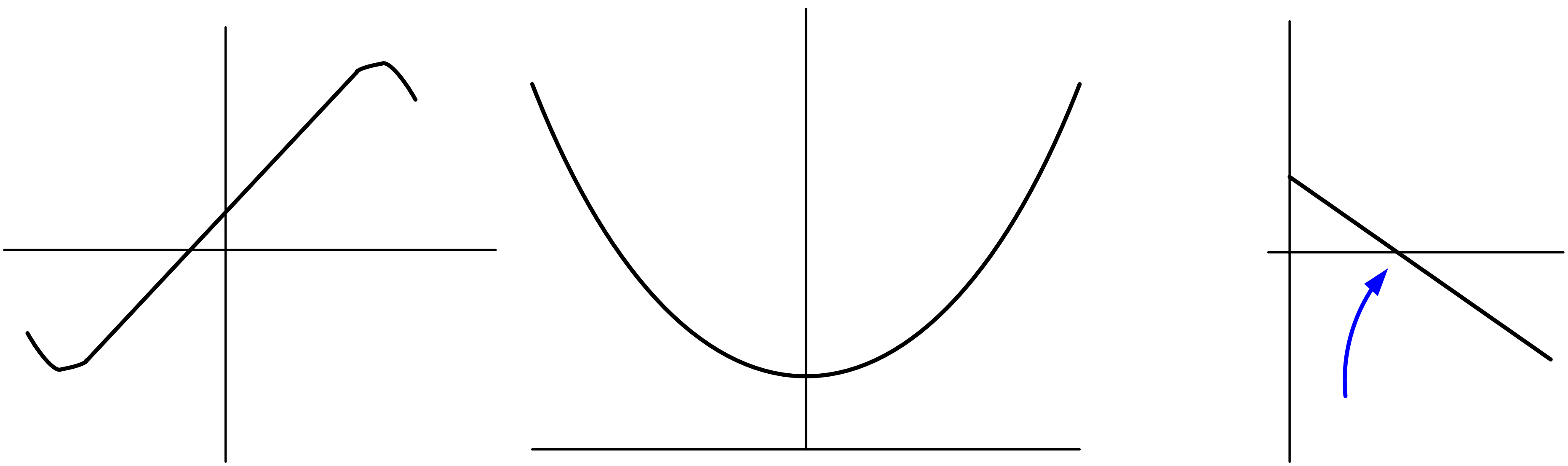
Remember the Right Hand Rule

- Roll angle is positive when right wingtip rotates down
- Pitch angle is positive when nose rotates up
- Yaw angle is positive when right wingtip rotates aft

These are all *conventions*

Longitudinal Stability

- Given an airplane's aerodynamic properties, determine whether it is stable in straight and level flight

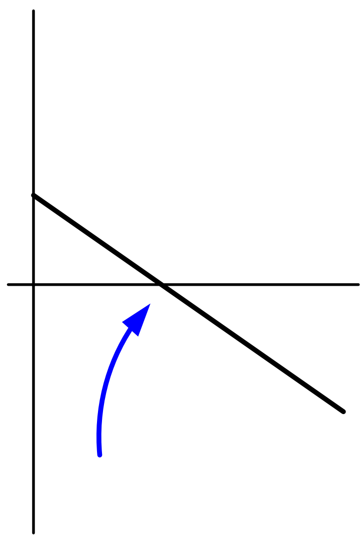


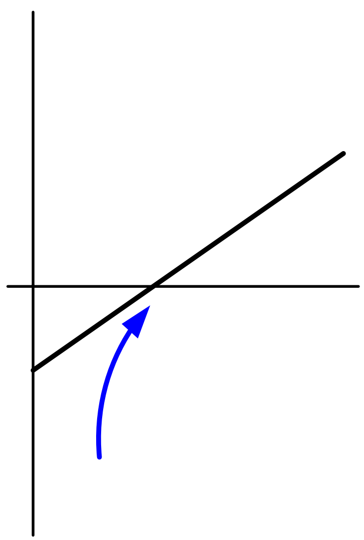
Flight conditions determine lift and drag coefficients. Tail controls (typically) are used to make the moment coefficient $C_{M,cg} = 0$.

Moment Coefficient Possibilities

- Slope could be negative or positive
- Generally, the symbol used for the slope of the moment coefficient for small changes of angle attack from the trim condition is

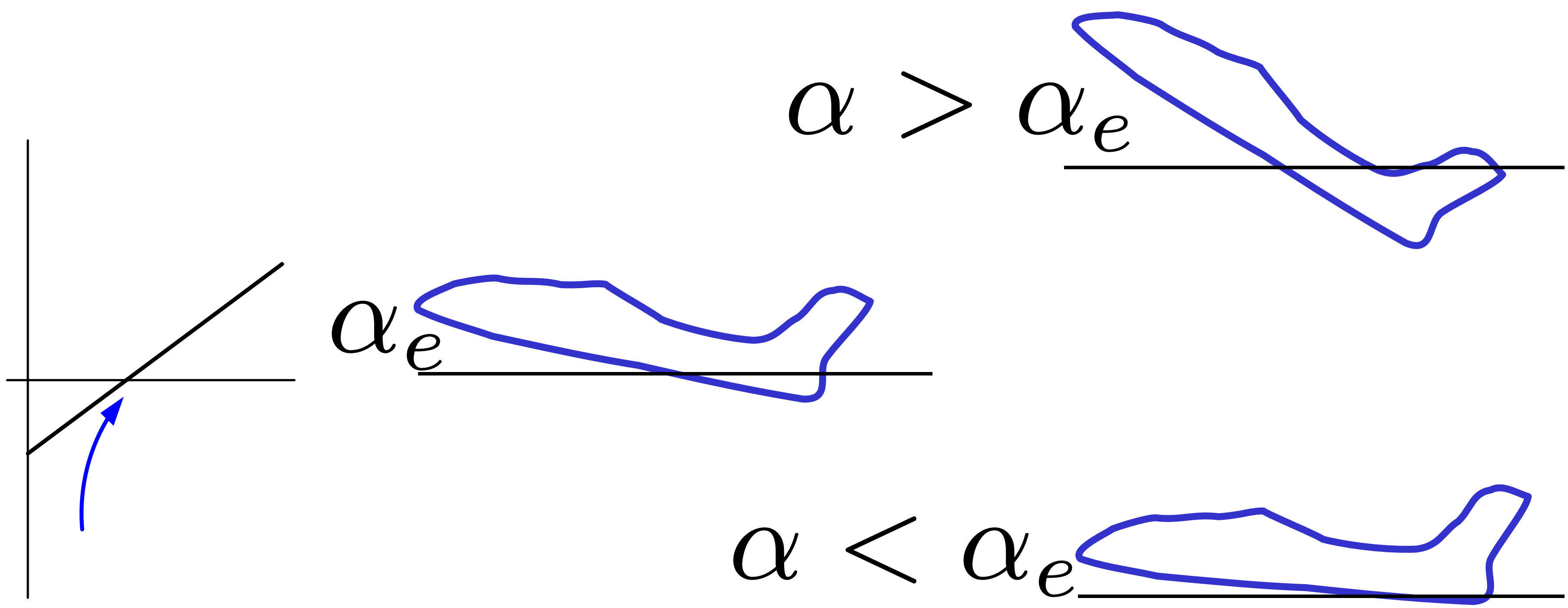
$$\frac{\partial C_{M,cg}}{\partial \alpha} = C_{M\alpha}$$





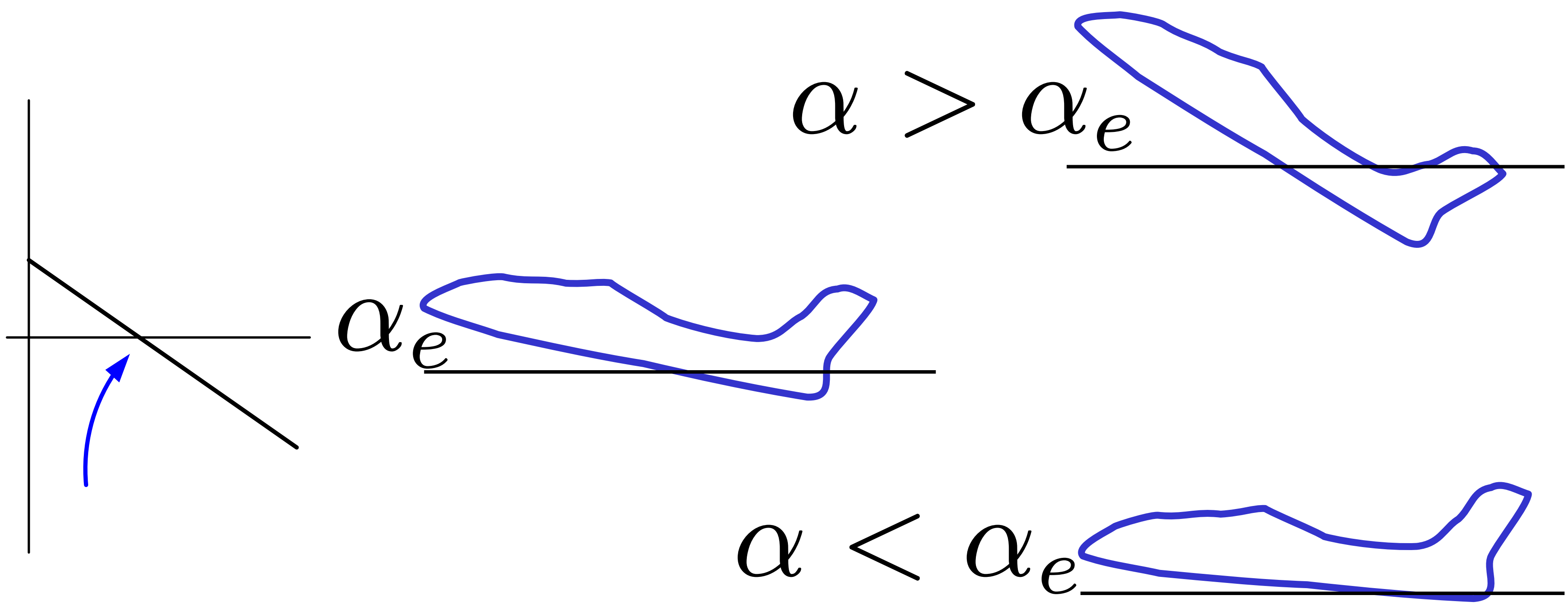
Positive Slope: $C_{M_\alpha}>0$

- Disturbance (gust) could cause $\alpha\uparrow$ or $\alpha\downarrow$
- $\alpha\uparrow$ implies that $C_{M,cg}$ becomes positive
- $\alpha\downarrow$ imples that $C_{M,cg}$ becomes negative



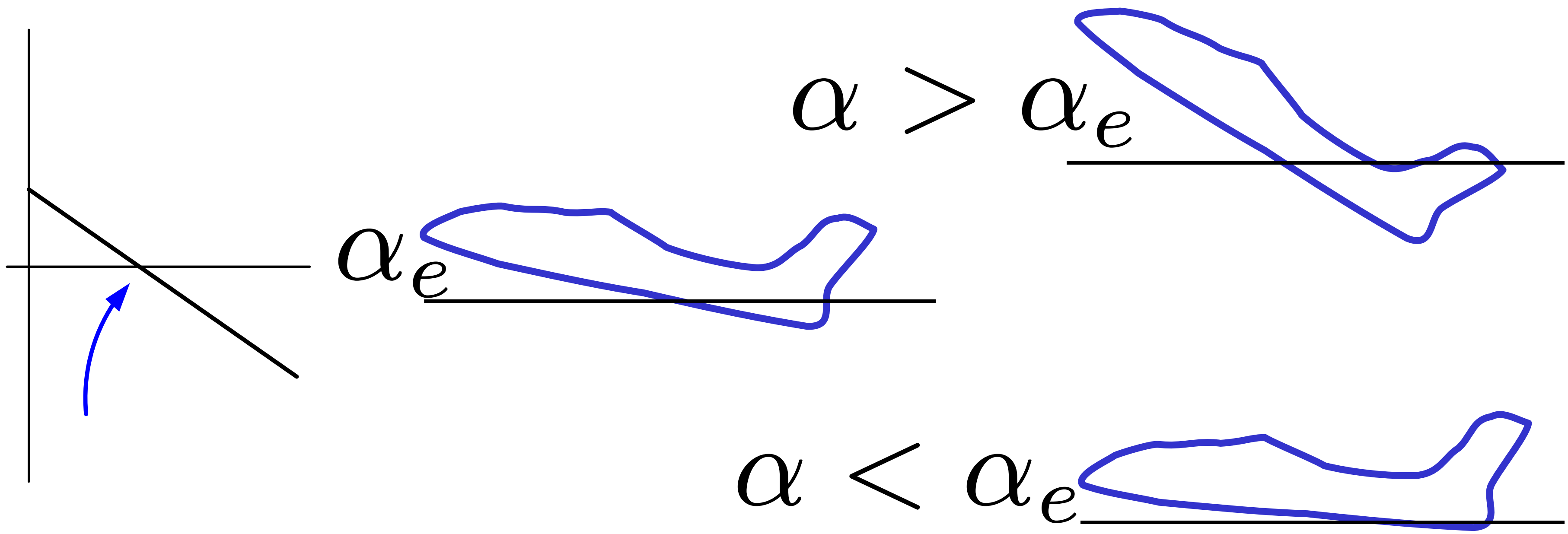
Negative Slope: $C_{M_\alpha} < 0$

- Disturbance (gust) could cause α↑ or α↓
- α↑ implies that $C_{M,cg}$ becomes negative
- α↓ imples that $C_{M,cg}$ becomes positive



Static Longitudinal Stability

- Disturbance (gust) could cause $\alpha\uparrow$ or $\alpha\downarrow$
- $\alpha\uparrow$ implies that $C_{M,cg}$ becomes negative
  - Pitch moment negative $\Rightarrow \alpha\downarrow \Rightarrow$ stable
- $\alpha\downarrow$ imples that $C_{M,cg}$ becomes positive
  - Pitch moment positive $\Rightarrow \alpha\uparrow \Rightarrow$ stable



Further topics in Stability & Control

- Consideration of all contributions to the pitch moment: Wing, Body, Tail
- Neutral point (location of c.g. where $C_{M_\alpha}=0$)
- Static margin (distance between c.g. and n.p.)
- Lateral stability (roll stability, dihedral effect)
- Control (use of actuators such as elevator, rudder and trim tabs to achieve stability)

AOE 3134: Stability & Control, Spring Junior Year covers all these topics, and will likely include some sort of demonstration using the aircraft flight simulator