Aerodynamics

Basic Aerodynamics

Flow with no friction (inviscid)

Continuity equation (mass conserved)

Flow with friction (viscous)

Momentum equation ($F = ma$)
1. Euler’s equation
2. Bernoulli’s equation

Some thermodynamics

Energy equation (energy conserved)

Equation for isentropic flow

Some Applications

Boundary layer concept
Laminar boundary layer
Turbulent boundary layer
Transition from laminar to turbulent flow
Flow separation

Reading: Chapter 4
Recall: Aerodynamic Forces

• “Theoretical and experimental aerodynamicists labor to calculate and measure flow fields of many types.”

• ... because “the aerodynamic force exerted by the airflow on the surface of an airplane, missile, etc., stems from only two simple natural sources:

  Pressure distribution on the surface (normal to surface)
  Shear stress (friction) on the surface (tangential to surface)
Fundamental Principles

- Conservation of mass
  ⇒ Continuity equation (§§ 4.1-4.2)
- Newton’s second law \( F = ma \)
  ⇒ Euler’s equation & Bernoulli’s equation (§§ 4.3-4.4)
- Conservation of energy
  ⇒ Energy equation (§§ 4.5-4.7)
First: Buoyancy

• One way to get lift is through Archimedes’ principle of buoyancy

• The buoyancy force acting on an object in a fluid is equal to the weight of the volume of fluid displaced by the object

• Requires integral
  (assume \( \rho_0 \) is constant)
  \[
  p = p_0 - \rho_0 g_0 (r - r \cos \theta)
  \]

  Force is
  \[
  p \, dA = [p_0 - \rho_0 g_0 (r - r \cos \theta)] \, dA
  \]

  \[
  dA = 2 \pi r^2 \sin \theta \, d\theta
  \]

  Integrate using “shell element” approach
Buoyancy: Integration Over Surface of Sphere

- Each shell element is a ring with radius $r \sin \theta$, and width $r \, d\theta$
  
  Thus the differential area of an element is
  
  $$dA = 2\pi r^2 \sin \theta \, d\theta$$

- Pressure at each point on an element is
  
  $$p = p_0 - \rho_0 g_0 (r - r \cos \theta)$$

- Force is pressure times area
  
  $$dF = p \, dA = [p_0 - \rho_0 g_0 (r - r \cos \theta)] \, dA$$

- Vertical pressure force is
  
  $$dF \cos \theta = p \, dA \cos \theta = [p_0 - \rho_0 g_0 (r - r \cos \theta)] \cos \theta \, dA$$
Buoyancy: Integration Over Surface of Sphere (continued)

- Total vertical pressure force is found by integrating from $\theta = 0$ to $\theta = \pi$:
  \[ F_{vp} = 2\pi r^2 \int [p_0 - \rho_0 g_0 (r - r \cos \theta)] \cos \theta \sin \theta \, d\theta \]

- Some useful identities:
  \[ \int \cos \theta \sin \theta \, d\theta = \frac{1}{2} \sin^2 \theta \]
  \[ \int \cos^2 \theta \sin \theta \, d\theta = -\frac{1}{3} \cos^3 \theta \]

- Put them together to get
  \[ F_{vp} = \frac{4}{3} \pi r^3 \cdot \rho_0 \cdot g_0 \]

- The first bit is the volume of the sphere; multiplying by density gives mass of fluid displaced; multiplying by gravity gives weight of fluid displaced.
Buoyancy: Forces on a Sphere (continued)

- Total vertical pressure force is
  \[ F_{vp} = \frac{4}{3}\pi r^3 \cdot \rho_0 \cdot g_0 \]
  or
  \[ F_{vp} = W_v \text{ (weight of volume of fluid)} \]

- Thus the total vertical force on the sphere is
  \[ F_v = W_v - W_s \]
  where \( W_s = mg \) is the weight of the sphere

- If \( W_v > W_s \), then the net force is a positive “Lift”
- If \( W_v < W_s \), then the net force is a negative “Lift”
- If \( W_v = W_s \), then the sphere is said to be “neutrally buoyant”
Neutral Buoyancy Tanks

• Neutral buoyancy is useful for simulating the freefall environment experienced by astronauts

• NASA’s Marshall Space Flight Center has a Neutral Buoyancy Simulator
  [Link](http://www1.msfc.nasa.gov/NEWSROOM/background/facts/nbs.htm)

• University of Maryland has a Neutral Buoyancy Tank
  [Link](http://www.ssl.umd.edu/facilities/facilities.html)
What’s In Our Toolbox So Far?

• Four aerodynamic quantities, flow field
• Steady vs unsteady flow
• Streamlines
• Two sources of all aerodynamic forces
• Equation of state for perfect gas
• Standard atmosphere: six different altitudes
• Hydrostatic equation
• Linear interpolation, local approximation
• Lift due to buoyancy
• Viscous vs inviscous flow
Lift from Fluid Motion

- **First:** Airplane wing geometry
- **Span, Chord, Area, Planform, Aspect Ratio, Camber, Leading and Trailing Edges**

Aspect Ratio = \( AR = \frac{s^2}{A} = \frac{s}{c} \)

Symmetric Airfoil

Thickness, Mean Camber Line, Chord Line, Camber

Dihedral Angle
Some Wing Shapes

Rectangular straight wing

Tapered straight wing

Rounded or elliptical straight wing

Slightly swept wing

Moderately swept wing

Highly swept wing

Simple delta wing

Complex delta wing

Wing

Airfoil section
Side-view shape of wing

Wright Brothers

P-36 (Subsonic)

F-51 (Subsonic)

F-104 (Supersonic)
Continuity

Physical principle: Mass can be neither created nor destroyed.

Volume bounded by streamlines is called a stream tube

Assumption: Steady flow

At entry point (1):
\[ \frac{dm}{dt} = \rho_1 A_1 V_1 \]

At exit point (2):
\[ \frac{dm}{dt} = \rho_2 A_2 V_2 \]

Since mass is conserved, these two expressions must be equal; hence

\[ \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \]

This is the continuity equation for steady flow
Remarks on Continuity

• In the stream tube figure, the velocities and densities at points 1 and 2 are assumed to be uniform across the cross-sectional areas.

• In reality, V and ρ do vary across the area and the values represent mean values.

• The continuity equation is used for flow calculations in many applications such as wind tunnels and rocket nozzles.

• Stream tubes do not have to represent physical flow boundaries.
**Compressible vs Incompressible**

- **Compressible flow:** flow in which the density of the fluid changes from point to point
  - In reality, all flows are compressible, but $\Delta \rho$ may be negligible
- **Incompressible flow:** flow in which the density of the fluid is constant
  - Continuity equation becomes $A_1V_1 = A_2V_2$
Compressible vs Incompressible

- Incompressible flow does not exist in reality
- However, many flows are “incompressible enough” so that the assumption is useful
- Incompressibility is an excellent model for
  - Flow of liquids such as water and oil
  - Low-speed aerodynamics (<100 m/s or <225 mph)

- For incompressible flow, the continuity equation can be written as
  \[ V_2 = \frac{A_1 V_1}{A_2} \]

- Thus if \( A_1 > A_2 \) then \( V_1 < V_2 \)
Example 4.1

Consider a convergent duct with an inlet area $A_1 = 5 \text{ m}^2$. Air enters this duct with velocity $V_1 = 10 \text{ m/s}$ and leaves the duct exit with a velocity $V_2 = 30 \text{ m/s}$. What is the area of the duct exit?

First, check that the velocities involved are $< 100 \text{ m/s}$, which implies incompressible flow. Then use

$$A_2 = A_1 \frac{V_1}{V_2} = \frac{(5 \text{ m}^2)(10)}{(30)} = 1.67 \text{ m}^2$$
Example 4.2

Consider a convergent duct with an inlet area $A_1 = 3 \text{ ft}^2$ and an exit area $A_2 = 2.57 \text{ ft}^2$. Air enters this duct with velocity $V_1 = 700 \text{ ft/s}$ and a density $\rho_1 = 0.002 \text{ slug/ft}^3$, and leaves the duct exit with a velocity $V_2 = 1070 \text{ ft/s}$. What is the density of the air at the duct exit?

First, check that the velocities involved are $> 300 \text{ ft/s}$, which implies compressible flow. Then use

$$\rho_2 = \rho_1 A_1 V_1 / (A_2 V_2) = 0.00153 \text{ slug/ft}^3$$
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Reading: Chapter 4
Momentum Equation

- Continuity equation does not involve pressure
- Pressure ⇒ Force ⇒ Change in momentum ⇒ Change in velocity
  - Force = \( \frac{d(\text{momentum})}{dt} \)  \( \quad \) What Newton said
  - Force = \( \frac{d(mv)}{dt} \) but only applies if \( m=\text{const} \)
  - \( F = m \frac{dv}{dt} \)
  - \( F = ma \)

- We apply \( F = ma \) to the fluid by summing the forces acting on a single infinitesimally small particle of fluid
Assume element is moving in \( x \) direction.

Force on element has three sources:

- Normal pressure distribution: \( p \)
- Shear stress distribution: \( \tau_w \)
- Gravity: \( \rho \, dx \, dy \, dz \, g \)

Ignore gravity, smaller than other forces.

Consider force balance in \( x \) direction.

Force = Pressure \( \times \) Area.
Force Balance

- Force on left face: \( F_L = p \, dy \, dz \)
- Force on right face: \( F_R = (p + [dp/dx]dx) \, dy \, dz \)
  
  \[ F = F_L - F_R = p \, dy \, dz - (p + [dp/dx]dx) \, dy \, dz \]
  
  \[ F = -(dp/dx) \, dx \, dy \, dz \]

- Mass of the fluid element is
  
  \[ m = \rho \, dx \, dy \, dz \]

- Acceleration of the fluid element
  
  \[ a = dV/dt = (dV/dx)(dx/dt) = (dV/dx)V \]

- Newton’s second law
  
  \[ F = ma \quad \Rightarrow \quad dp = -\rho \, V \, dV \]  
  
  Euler’s Equation

- Also referred to as the Momentum Equation
  
  - Keep in mind that we assumed steady flow and ignored gravity and friction, thus this is the momentum equation for **steady, inviscid flow**
  
  - However, Euler’s equation applies to compressible and incompressible flows
Incompressible Flow

• If the flow is incompressible, then $\rho$ is constant
• The momentum equation can be written as
  \[ dp + \rho \ V \ dV = 0 \]
• Integrating along a streamline between two points 1 and 2 gives
  \[ p_2 - p_1 + \rho \ (V_2^2 - V_1^2)/2 = 0 \]
• Which can be rewritten as
  \[ p_2 + \rho \ V_2^2/2 = p_1 + \rho V_1^2/2 \]
  Or
  \[ p + \rho \ V^2/2 = \text{constant along a streamline} \]
• This equation is known as Bernoulli’s equation
Euler’s and Bernoulli’s Equations

• Bernoulli’s equation
  \[ p_2 + \rho \frac{V_2^2}{2} = p_1 + \rho \frac{V_1^2}{2} \]
  – Holds for inviscid, incompressible flow
  – Relates properties of different points along a streamline

• Euler’s equation
  \[ dp = -\rho \ V \ dV \]
  – Holds for inviscid flow, compressible or incompressible

• These equations represent Newton’s Second Law applied to fluid flow, and relate pressure, density, and velocity
Euler’s and Bernoulli’s Equations

- Bernoulli’s equation
  \[ p_2 + \rho \frac{V_2^2}{2} = p_1 + \rho \frac{V_1^2}{2} \]
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Example 4.3

Consider an airfoil in a flow of air, where far ahead (upstream) of the airfoil, the pressure, velocity, and density are 2116 lb/ft², 100 mi/h, and 0.002377 slug/ft³, respectively. At a given point A on the airfoil, the pressure is 2070 lb/ft². What is the velocity at point A?

First, we must use consistent units. Using the fact that 60 mi/h ≈ 88 ft/s, we find that $V = 100 \text{ mi/h} = 146.7 \text{ ft/s}$. This flow is slow enough that we can assume it is incompressible, so we can use Bernoulli’s equation:

$$p_1 + \rho \frac{V_1^2}{2} = p_A + \rho \frac{V_A^2}{2}$$

Where “1” is the far upstream condition, and “A” is the point on the airfoil. Solving for velocity at A gives

$$V_A = 245.4 \text{ ft/s}$$
Example 4.4

Consider a convergent duct with an inlet area $A_1 = 5\, \text{m}^2$. Air enters this duct with velocity $V_1 = 10\, \text{m/s}$ and leaves the duct exit with a velocity $V_2 = 30\, \text{m/s}$. If the air pressure and temperature at the inlet are $p_1 = 1.2 \times 10^5\, \text{N/m}^2$ and $T_1 = 330\, \text{K}$, respectively, calculate the pressure at the exit.

First, compute density at inlet using equation of state:

$$\rho_1 = \frac{p_1}{R \, T_1} = 1.27\, \text{kg/m}^3$$

Assuming compressible flow, use Bernoulli’s equation to solve for $p_2$:

$$p_2 = p_1 + \rho(V_1^2 - V_2^2)/2 = 1.195 \times 10^5\, \text{N/m}^2$$
Example 4.5

Consider a long dowel with semicircular cross section

... 

See pages 135-141 in text
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Reading: Chapter 4
§4.10: Low-Speed Subsonic Wind Tunnels

Assumption: *Steady incompressible flow*

Continuity and Bernoulli’s Equation apply

Test section

Settling Chamber (reservoir)

Nozzle

Model Mounted on “Sting”

Diffuser

$A_1, p_1, V_1$

$A_2, p_2, V_2$

$A_3, p_3, V_3$
Wind Tunnel Calculations

Continuity \Rightarrow V_1 = (A_2/A_1)V_2

Bernoulli \Rightarrow V_2^2 = 2(p_1-p_2)/\rho + V_1^2

Combine to get

\[ V_2 = \left\{ \frac{2(p_1-p_2)}{\rho(1-(A_2/A_1)^2)} \right\}^{\frac{1}{2}} \]

The ratio \( A_2/A_1 \) is fixed for a given wind tunnel, and the density \( \rho \) is constant for low-speed tunnels, so the “control” is \( p_1-p_2 \)

How to determine \( p_1-p_2 \)?
Manometer

\[ \rho_f g \Delta h, \quad w = \rho_f g \]

\[ p_1 A = p_2 A + w A \Delta h, \quad w = \rho_f g \]

\[ p_1 - p_2 = A + w \Delta h, \quad \text{So} \quad \Delta h \Rightarrow V_2 \]
Example 4.13

In a low-speed subsonic wind tunnel, one side of a mercury manometer is connected to the reservoir and the other side is connected to the test section. The contraction ratio of the nozzle $A_2/A_1 = 1/15$. The reservoir pressure and temperature are $p_1=1.1$ atm and $T_1=300$ K. When the tunnel is running the height difference between the two columns of mercury is 10 cm. The density of liquid mercury is $1.36 \times 10^4$ kg/m$^3$. Calculate the airflow velocity $V_2$ in the test section.
§4.11: Measurement of Airspeed

- **Total pressure vs static pressure**
  - Static pressure is the pressure we’ve been using all along, and is the pressure you’d feel if you were moving along with the fluid.
  - Total pressure includes the static pressure, but also includes the “pressure” due to the fluid’s velocity, the so-called *dynamic pressure*.
- Imagine a hollow tube with an opening at one end and a *pressure sensor* at the other, and imagine inserting it into a flow in two different ways.
Pitot Tube

• This device is called a Pitot Tube (after Henri Pitot, who invented it in 1732; see §4. 23)
• The orientation on the left measures the static pressure (the pressure in all our calculations so far)
• The orientation on the right measures the total pressure, or the pressure if the flow is reduced to zero velocity
Pitot Tube for Incompressible Flow

- The two tube orientations are used together.
- One measures static pressure $p$, and the other measures total pressure $p_0$.
- Since the total pressure is measured by removing all the velocity, and we're assuming incompressible flow, we can apply Bernoulli's equation to see that
  \[ p + \rho \frac{V^2}{2} = p_0 \]
  Static pressure + Dynamic Pressure = Total Pressure
- Dynamic pressure, the $\rho \frac{V^2}{2}$ term, is frequently denoted by $q = \rho \frac{V^2}{2}$.
Using the Pitot-static Probe

The two pressures are measured by a pressure transducer.

Bernoulli’s equation (incompressible flow only!) can be written as:

\[ p_0 = p + q \quad (q = \rho V^2/2) \]

Solve for velocity:

\[ V = \left[ \frac{2(p_0 - p)}{\rho} \right]^{1/2} \]

A Pitot-static tube provides an airspeed measurement.
Example 4.16

The altimeter on a low-speed Cessna 150 reads 5000 ft. The outside temperature is $T = 505^\circ R$. If a Pitot tube on the wingtip measures $p = 1818$ lb/ft$^2$, what is the true velocity of the airplane? What is the equivalent airspeed?
Overview of the “Rest” of Aerodynamics

- We will not cover the remainder of Ch. 4, but here are some highlights
- First Law of Thermodynamics leads to relationships between energy, temperature, heat, enthalpy, and specific heat
- Energy has units of Joules
- Enthalpy has units of Joules but also accounts for temperature
- Adiabatic ⇒ no heat is added or removed
- Reversible ⇒ no frictional losses
- Isentropic ⇒ adiabatic and reversible